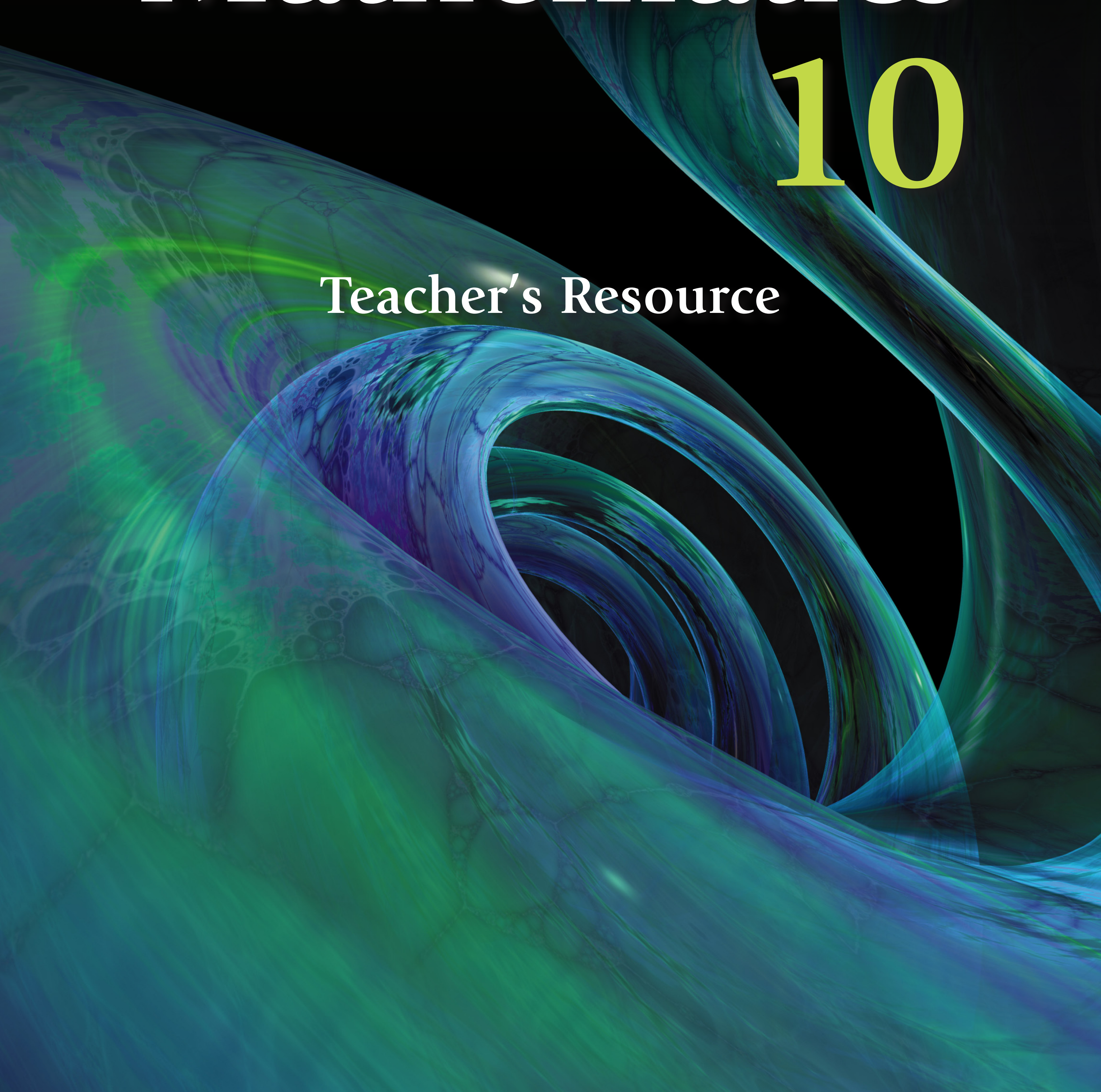
McGraw-Hill Ryerson

Mathematics 10

Teacher's Resource



McGraw-Hill Ryerson

Mathematics

TEACHER'S RESOURCE 10

AUTHORS

Berkley Beingessner
B.Ed.
Calgary Board of Education
Alberta

Scott Carlson
B.Ed., B.Sc.
Golden Hills School Division No. 75
Alberta

Barb Gajdos
B.Ed.
Calgary Roman Catholic Separate School District No. 1
Alberta

Blaise Johnson
B.Sc., B.Ed.
School District 45 (West Vancouver)
British Columbia

Chris Zarski
B.Ed., M.Ed.
Wetaskiwin Regional Division No. 11
Alberta

ASSESSMENT AUTHORS

Bruce McAskill
B.Sc., B.Ed., M.Ed., Ph.D.
Mathematics Consultant
Victoria, British Columbia

Wayne Watt
B.Sc., B.Ed., M.Ed.
Mathematics Consultant
Winnipeg, Manitoba

SUPPORTING LEARNING AUTHORS

Heather Granger
B.Ed.
Prairie South School Division No. 210
Saskatchewan

Shirley Oker-Mykituk
B.Ed.
Edmonton Catholic Separate School District No. 7
Alberta

Christine Purse
B.Ed.
Victoria, British Columbia

Rick Wunderlich
B.Ed.
School District 83 (North Okanagan/Shuswap)
British Columbia

FOLDABLES™ AUTHOR

Dinah Zike
M.Ed.
Educational Consultant
San Antonio, Texas

BLACKLINE MASTERS AUTHORS

Ron Kennedy
B.Ed.
Mathematics Consultant
Edmonton, Alberta

Stephanie Mackay
B.Ed.
Edmonton Catholic Separate School District No. 7
Alberta

Esther Moreno
B.Sc.
School District No. 39 (Vancouver)
British Columbia

Gail Poshtar
B.Ed.
Calgary Roman Catholic Separate School District No. 1
Alberta

McGraw-Hill Ryerson

Toronto Montréal Boston Burr Ridge, IL Dubuque, IA Madison, WI New York San Francisco St. Louis Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City Milan New Delhi Santiago Seoul Singapore Sydney Taipei

McGraw-Hill Ryerson
Mathematics 10 Teacher's Resource

COPIES OF THIS BOOK MAY BE OBTAINED BY CONTACTING:
McGraw-Hill Ryerson Ltd.

WEB SITE:
www.mcgrawhill.ca

E-MAIL:
orders@mcgrawhill.ca

TOLL-FREE FAX:
1-800-463-5885

TOLL-FREE CALL:
1-800-565-5758

OR BY MAILING YOUR ORDER TO:
McGraw-Hill Ryerson
Order Department
300 Water Street
Whitby, ON L1N 9B6

Please quote the ISBN and title when placing your order.

ISBN-13: 978-0-07-012758-6
ISBN-10: 0-07-012758-1

www.mcgrawhill.ca

1 2 3 4 5 6 7 8 9 0 MP 1 9 8 7 6 5 4 3 2 1 0

Printed and bound in Canada

ASSOCIATE PUBLISHER, MATHEMATICS: Kristi Clark
PROJECT MANAGER: Helen Mason
CONTENT MANAGER: Susan Till
DEVELOPMENTAL EDITORS: Susan Till, Jodi Rauch, Rita Vanden Heuvel, Jackie Lacoursiere, Kelly Cochrane
MANAGER, EDITORIAL SERVICES: Crystal Shortt
SUPERVISING EDITOR: Jeanette McCurdy
COPY EDITOR: John Green
EDITORIAL ASSISTANT: Erin Hartley
EDITORIAL COORDINATION: Jennifer Keay, Janie Reeson, Alexandra Savage-Ferr
MANAGER, PRODUCTION SERVICES: Yolanda Pigden
PRODUCTION COORDINATOR: Sheryl MacAdam
COVER DESIGN: Valid Design & Layout
INTERIOR DESIGN: ArtPlus Ltd.
ELECTRONIC PAGE MAKE-UP: ArtPlus Ltd. & LaserWords
COVER IMAGE: Courtesy of Getty Images; Mike Kemp, Photographer

Contents

Time Lines for *Mathematics 10* viii

An Introduction to *Mathematics 10* Teacher's Resource ix

Characteristics of McGraw-Hill Ryerson's *Mathematics 10* Program x

Mathematics: Making Links x

Procedural Fluency and Conceptual Understanding xi
- *Investigate*
- *Link the Ideas*
- *Check Your Understanding*

Problem Solving xii

Differentiating Instruction xii
- *Ten Needs of the Learner*

Assessment xv

Assessment *as* Learning xv

Assessment *for* Learning xv

Assessment *of* Learning xvi
- *Portfolio Assessment*
- *Master 1 Project Rubic*

Concrete Materials xvii

Technology xvii

Capitalizing on Diversity and Real Life xviii

Alternative Learning Environments and Home Connections xviii

Cooperative Learning xviii

- *Types of Groups* xx

Mental Mathematics xxi

- *Estimation* xxi
- *Mental Imagery* xxi
- *Mental Computation* xxi

Chapter Correlation xxiii

Unit 1 Measurement 1

Unit 1 Opener 1

Unit 1 Project Opener 2

Chapter 1 Measurement Systems 3

Opening Matter and Charts 3

Chapter Opener 7

1.1 SI Measurement 9

1.2 Imperial Measurement 16

1.3 Converting Between SI and Imperial Systems 23

Chapter 1 Review 30

Chapter 1 Practice Test 32

Chapter 2 Surface Area and Volume 35

Opening Matter and Charts 35

Chapter Opener 39

2.1 Units of Area and Volume 41

2.2 Surface Area 51

2.3 Volume 63

Chapter 2 Review 71

Chapter 2 Practice Test 72

Chapter 3 Right Triangle Trigonometry 75

Opening Matter and Charts 75

Chapter Opener 79

3.1 The Tangent Ratio 81

3.2 The Sine and Cosine Ratios 88

3.3 Solving Right Triangles 94

Chapter 3 Review 100

Chapter 3 Practice Test 101

Unit 1 Project 103

Unit 1 Review and Test 105

Unit 2 Algebra and Number 107

Unit 2 Opener 107

Unit 2 Project Opener 108

Chapter 4 Exponents and Radicals 109

Opening Matter and Charts 109

Chapter Opener 113

4.1 Square Roots and Cube Roots 115

4.2 Integral Exponents 123

4.3 Rational Exponents 134

4.4 Irrational Numbers 141

Chapter 4 Review 151

Chapter 4 Practice Test 152

Chapter 5 Polynomials 155

Opening Matter and Charts 155

Chapter Opener 159

5.1 Multiplying Polynomials 161

5.2 Common Factors 170

5.3 Factoring Trinomials 178

5.4 Factoring Special Trinomials 187

Chapter 5 Review 198

Chapter 5 Practice Test 200

Unit 2 Project 202

Unit 2 Review and Test 204

Unit 3 Relations and Functions 205

Unit 3 Opener 205

Unit 3 Project Opener 206

Chapter 6 Linear Relations 207

Opening Matter and Charts 207

Chapter Opener 211

6.1 Graphs of Relations 213

6.2 Relations 221

6.3 Domain and Range 228

6.4 Functions 235

6.5 Slope 242

Chapter 6 Review 250

Chapter 6 Practice Test 251

Chapter 7 Linear Equations and Graphs 253

Opening Matter and Charts 253

Chapter Opener 257

7.1 Slope-Intercept Form 259

7.2 General Form 268

7.3 Slope-Point Form 276

7.4 Parallel and Perpendicular Lines 282

Chapter 7 Review 288

Chapter 7 Practice Test 290

Unit 3 Project 292

Unit 3 Review and Test 296

Unit 4 Systems of Equations 297

Unit 4 Opener 297

Unit 4 Project Opener 298

Chapter 8 Solving Systems of Linear Equations Graphically 299

Opening Matter and Charts 299

Chapter Opener 303

8.1 Systems of Linear Equations and Graphs 306

8.2 Modelling and Solving Linear Systems 316

8.3 Number of Solutions for Systems of Linear Equations 324

Chapter 8 Review 331

Chapter 8 Practice Test 333

Chapter 9 Solving Systems of Linear Equations Algebraically 335

Opening Matter and Charts 335

Chapter Opener 339

9.1 Solving Systems of Linear Equations by Substitution 341

9.2 Solving Systems of Linear Equations by Elimination 347

9.3 Solving Problems Using Systems of Linear Equations 354

Chapter 9 Review 360

Chapter 9 Practice Test 362

Unit 4 Project 363

Unit 4 Review and Test 365

Blackline Masters

The blackline masters for *Mathematics 10* are available on the *Mathematics 10* Teacher's Resource CD-ROM.

These include generic masters, chapter-specific masters, and a series of Tech Masters.

TIME LINES FOR *MATHEMATICS 10*

The chart below shows estimated times, in minutes, for covering the material in *Mathematics 10*. Please note that times will vary depending on your particular class and its individual students.

Also note that there are alternative ways to cover and assess many outcomes. For example, student achievement of unit outcomes can be checked by having students do the **unit review** and **unit test**, *or*, more holistically, by having students complete the **unit project**, *or* by doing a combination of these things.

Mathematics 10 Time Frames

	1	2	3	4	5	6	7	8	9
Unit Opener	30–40			30–40		30–40		30–40	
Chapter Opener	50–60	30–40	30–40	30–40	45–60	30–40	30–40	30–40	20–30
Section 1	100–120	100–120	120–140	80–100	100–120	100–120	100–140	100–120	60–75
Section 2	100–120	100–120	100–120	80–100	100–150	100–120	80–120	100–120	60–75
Section 3	180–240	100–120	100–120	80–100	120–180	120–180	80–120	100–120	60–75
Section 4				100–120	100–120	180–240	90–130		
Section 5						180–240			
Chapter Review	100–120	60–90	60–90	60–90	60–90	60–90	60–90	60–90	60–75
Practice Test	50–60	40–50	40–50	40–50	45–60	50–60	45–60	60–90	50–60
Unit Project			60–90		60–90		80–100		90–120
Unit Review **Unit Test**			60–90		60–90		60–90		60–90

AN INTRODUCTION TO *MATHEMATICS 10* TEACHER'S RESOURCE

Unit Material

Each unit begins with **unit opener** notes that list the General and Specific Outcomes for the unit and provide suggestions for teaching the unit opener material, which includes an introduction to the **unit project**.

Chapter Material

Each Chapter tarts with a four-page **foldout** that provides
- an overview of the chapter outcomes and concepts, skills, and processes
- assessment suggestions for the use of the **Foldable**™ and the section **Warm-Ups**
- suggested timing for the numbered sections, chapter review, and practice test
- a list of prerequisite skills for each section
- suggested assignments for most students
- a list of related blackline masters available on the CD-ROM
- a list of materials and technology tools needed for each lesson
- the location of Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning opportunities in the chapter

The **Chapter Opener** includes
- a description of the math that will be covered in the chapter
- suggestions for introducing students to the chapter's topics
- suggestions for using the chapter's unit project questions

The opening page of each **numbered section** expands on the information provided in the foldout.

The **teaching notes** include the following:
- Answers for the **Investigate** and **Your Turn** questions
- **Planning Notes**
- **Assessment** boxes that provide a variety of short assessment strategies and related supported learning
- Each chapter ends with a **chapter review** and a **practice test**.
- **Unit reviews** reinforce the chapters in the unit.

CHARACTERISTICS OF MCGRAW-HILL RYERSON'S *MATHEMATICS 10* PROGRAM

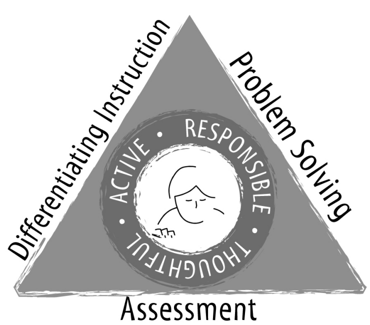
The *Mathematics 10* program was designed for students and educators using the outcomes and achievement indicators published in *The WNCP Common Curriculum Framework for Grades 10–12 Mathematics, January 2008.* This resource package will support educators and all grade 10 students enrolled in Foundations of Mathematics and Pre-Calculus Grade 10 in Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon.

The *Mathematics 10* design is based on current educational philosophy and pedagogy. The instructional design adheres to the principles set out in the WNCP Common Curriculum Framework that include beliefs about students and mathematics learning, the affective domain, mathematical processes, the nature of mathematics learning, and instructional focus. Other considerations include chapter sequence, the role of technology, and layout.

Because concern for teachers and students was paramount, the program was developed around two central questions:

1. How would the instructional design benefit/support students?

2. What would teachers require to support their implementation of the new curriculum?



Pedagogical Approach

The program is based on the philosophy that the focus of student learning is to develop a deeper understanding of mathematics and its connection to student lives, careers, and interests. For that reason, the instructional design is based on the premise that students can, and will, take responsibility for their learning, and that they are active and thoughtful learners. With these beliefs in mind, the resource supports a wide range of student interests and learning styles.

Mathematics: Making Links

Throughout the *Mathematics 10* student resource, students are given the opportunity to see the links between real life and mathematics.

- Every unit includes a **unit project** that models mathematics in the real world, engages students' interest, and gives students a meaningful purpose for learning the mathematics presented in the unit. The project is designed to engage students by making links between the mathematics in the unit and students' personal experiences and interests, as well as between mathematics and the real world.
- Most concepts or procedures are introduced in a real-life context.
- The **Big Ideas** at the beginning of each chapter list in student-friendly language the outcomes of the mathematics curriculum that are covered in the chapter. These outcomes may be from different strands that naturally fit together and further illustrate how the program makes important links among concepts within the discipline and with the real world.

- A **career connection** at the beginning of each chapter allows students to see how the math they are learning applies to a career in the real world. The visuals in many chapter openers show people performing work related to the math skills in the chapter. These jobs and careers require varying amounts of education, thus connecting more students to how mathematics may be used in their future lives.

Procedural Fluency and Conceptual Understanding

The three-part lesson structure in McGraw-Hill Ryerson's *Mathematics 10* program is designed to engage students in learning that develops their conceptual understanding and procedural fluency. The three parts are described below.

1. Investigate

- Each investigation is designed to help students build their own understanding of the new concept by working individually and in groups to explore a mathematical concept or procedure.
- The investigations emphasize personal strategies and alternative methods for solving problems.
- **Reflect and Respond** questions at the end of the Investigate help students to generalize learning about the key concept or methods being investigated.

2. Link the Ideas

- Explanations at the beginning of this section help students connect what they did in the Investigate to the **examples** that follow.
- Examples and **solutions** demonstrate how to use the concept explored in the Investigate. Many examples demonstrate how to use commonly available concrete materials and manipulatives. Many include multiple approaches to a solution.
- **Your Turn** questions after each example allow students to check their understanding of concepts.
- **Key Ideas** summarize the key concepts of the lesson.

3. Check Your Understanding

- **Practise** allows for the practice of new skills through approximately five questions.
- **Apply** introduces five to ten problems in a range of real-world contexts.
- **Extend** includes challenging questions that require higher-order thinking. Some may require additional research or connect to one or more other topics within the curriculum.
- **Create Connections** introduces up to three questions that require explanation of students' strategies or reasoning. Each question involves the use of communication related to the mathematics of the section.
- Most chapters have a **Mini Lab**. This provides a hands-on activity that encourages students to further explore the concepts they are learning.

Problem Solving

Problem solving is central to the McGraw-Hill Ryerson *Mathematics 10* program. Significant emphasis has been placed on incorporating problems that:

- have a range of contexts
- can be solved using different problem solving strategies
- may have multiple solutions

A variety of problem solving experiences are provided throughout the units:

- Each unit is based around a **unit project**. Unit project questions throughout the unit ask students to solve a number of problems as they connect the mathematics they are learning to a real-life context. The unit project is wrapped up at the end of the unit as a performance task.
- Students are frequently asked to discuss their methods for solving problems. Doing so reinforces thinking and helps students realize that there may be multiple methods for solving a problem.
- A problem provides the focus for learning in the **Investigate** at the beginning of each section, often making use of concrete materials.
- Students are challenged to higher levels of thinking and to extend their thinking in the **Extend** and **Create Connections** sections of the exercises and the **Extended Response** section in the practice test.

Differentiating Instruction

Differentiating instruction provides educators with the tools needed to create a learning environment where students are actively involved and working together. Hands-on activities engage students and help to meet their diverse needs. Significant emphasis has been placed on:

- variety — provides opportunities for students to be thoughtful about what and how they learn
- choice — encourages students to develop responsibility by making good personal decisions
- balance — is essential in having students actively involved in their learning. Students' needs are best met when they experience a variety of ways to develop and understand concepts.

Care has been taken in the McGraw-Hill Ryerson *Mathematics 10* program to ensure that all students—including special needs students (with learning disabilities or gifted), students at risk, English language learners, or students from different cultures—can access the mathematics and experience success.

- Visuals that illustrate how to carry out investigations accompany the instructions, where appropriate.
- Visuals and graphics are paired with questions and content in other strategic locations in the student resource.
- An **organizer** at the beginning of each chapter provides tips to help students organize and interpret the unit content.
- A **Foldable** at the beginning of each chapter provides a useful tool students can use to organize what they learn and keep track of what they still need to work on.
- **Key Terms** are listed in the chapter opener. When they are first used, they are highlighted in purple and defined in a marginal box, with the aid of visuals where appropriate.
- Key Terms, as well as other useful words, are defined in the **Glossary**.

FOLDABLES® Study Tool

- The Teacher's Resource provides strategies and blackline master support for accommodating different learning styles, special needs, English language learners, First Nations, Métis, Inuit, francophone, and at-risk students.
- The Assessment *for* Learning suggestions on the back page of each chapter foldout serve to activate student knowledge and concepts related to the topics in the upcoming chapter.
- The open-ended nature of many of the problems and tasks accommodate the needs of all students by allowing for multiple entry points.
- **Did You Know?** boxes present interesting information related to the math or context of the lesson. Some provide literacy information or connections to other subject areas.

Ten Needs of the Learner

Anna Sfard (2003) has identified ten needs of the intermediate learner. She claims these needs are the driving force behind learning and must be fulfilled if the learning is to be successful. The needs are universal, but may be expressed differently in different individuals and at different ages.

1. **The Need for Meaning**

   Learners look for order, logic, and causal dependencies behind things, events, and experiences. This approach requires students to actively engage in generating the meaning for themselves. It also directs students' thinking so no time is lost investigating incorrect paths. While students are developing meaning for new concepts, they are guided to develop patience, persistence, and tolerance in the face of insufficient clarity.

2. **The Need for Structure**

   This need follows from the need for meaning. Meaning involves relationships among concepts. The connections among concepts already learned and new concepts being introduced should be an integral part of instruction. Such connections must include not only real-world applications and relevance, but also assistance in building mathematical abstractions, so students can see how the results can be transferred from one context to another (Wu, 1997). The more connections that exist among facts, ideas, and procedures, the better students' conceptual understanding.

3. **The Need for Repetitive Action**

   A person who has created meaning and structure for a mathematical concept is aware of a repetitive, constant structure in certain actions. A reasonable level of mastery of basic skills is an important element in constructing mathematical knowledge (Fuson and Briars, 1990; Fuson and Kwon, 1992; Hiebert and Wearne, 1996; Siegler, 2003; Stevenson and Stigler, 1992). If students are to reflect on the processes of mathematics, they must first master those processes to a sufficient degree. This does not mean a focus on rote repetition. Rather, it should be a process of reflective practice, where mastery of the action leads to reflection on the meaning of that action, which leads to further understanding and learning.

4. **The Need for Difficulty**

   True learning implies coping with difficulties. The goal of learning is to advance students from abilities they now possess to those they have not yet developed. The best way to accomplish this goal is to present students with tasks that are demanding but still within their reach.

5. **The Need for Significance and Relevance**

   Significance means linking new knowledge to existing knowledge, so this need also stresses the importance of helping students build connections. Significance and relevance do not come from only the concrete and the real; they also come from problems that are more abstract. Focusing only on real-life applications would lead to a fragmented, incomplete picture of mathematics.

6. **The Need for Social Interaction**

   There is an inherent social nature to learning and making meaning. Jerome Bruner states that the fundamental vehicle of education is social interaction not "solo performance" (Bruner, 1985). The most obvious forms are student–teacher or student–student exchanges, but even interaction with a textbook is a form of social interaction (Sfard, 2003). Cooperative learning is another form of learning interaction in which the teacher does not have the central role.

7. **The Need for Verbal–Symbolic Interaction**

   Interaction in learning means communication, and communication means using language (speech) and symbols (written language as well as mathematical symbols) to convey thoughts. If mathematical learning is to take place in an interactive setting, students must be encouraged to "talk" mathematics.

8. **The Need for a Well-Defined Discourse**

   Discourse goes beyond the idea of a conversation. It refers to all communication practices of the classroom, both written and verbal. Discourse implies that the resulting communication follows specific rules. Researchers now recommend that rules be adjusted to the needs and potential of the learning child (Siegler, 2003). This does not mean giving up the need for rigour, but it does mean carefully choosing which rules we use and which rules we modify, and making these rules clear to students.

9. **The Need for Belonging**

   The desire to belong and be counted as a member of a particular social group is a powerful force behind our actions. Learning by participation requires us to be a part of a learning community. Students need to feel respected and free to speak their mind in the classroom. However, the extent to which students value mathematics is influenced by the value given to mathematics by the wider community (Comiti and Ball, 1996). Thus, it may be difficult to instill a desire to embrace mathematics in an environment where mathematics is not valued. The most promising directions for improvement seem to be those that incorporate historical context in the mathematics content, portray mathematics as something unique in our world, and present it as something to be valued for its own sake (Sfard, 2003).

10. **The Need for Balance**

    To meet learners' wide range of needs, the pedagogy must be variegated and rich in possibilities. The need for balance suggests an advantage in searching for the good in all theories. It does not imply that old and new are mutually exclusive. The reality is that there must be a bit of everything in the classroom: problem solving as well as skills practice, teamwork as well as individual learning and teacher exposition, real-life problems as well as abstract problems, and learning by talking as well as silent learning.

ASSESSMENT

Teachers are encouraged to assess students on an ongoing basis, using Assessment *as* Learning, Assessment *for* Learning, and Assessment *of* Learning. Through the use of a chapter Foldable, a self-assessment master, unit project checklists, and reflection, students are encouraged to assess their own progress, to identify their own strengths and weaknesses, and then to consider what they need to do in order to progress. Teachers are encouraged to coach students through this process.

Many opportunities for Assessment *as* Learning and Assessment *for* Learning assist teachers in identifying ways they can facilitate student progress to a higher level of conceptual and procedural understanding and skill development. Assessment *of* Learning further contributes to growth as teachers and students begin to use this summative assessment as a time for communication and reflections about future goals and strategies for improving.

Assessment *as* Learning (Diagnostic)

These assessment tools include student reflection. They are provided throughout the *Mathematics 10* student resource and Teacher's Resource to assist the teacher in programming by identifying student weaknesses and gaps.

- The **Foldables** activity in each chapter gives students a way to organize their learning and provides them with opportunities to express their understanding in their own words. A unique part of each Foldable asks students to keep track of what they need to work on, allowing them to be self-directed learners.
- **Reflect and Respond** questions at the end of each **Investigate** provide early opportunities for students to construct knowledge about the section content.
- The **Create Connections** questions allow students to explore their initial understandings of a concept.
- The **Warm-Up** exercises, and **chapter self-assessment** and **prerequisite skills** BLMs in the Teacher's Resource provide additional support in identifying and facilitating student learning.
- The suggested assignments, questions, and activities in the **Meeting Student Needs** boxes in the Teacher's Resource address a variety of learner needs, including those of English language learners and gifted and enrichment students.
- Diagnostic support in the form of introductory questions designed to open discussion in the classroom and in the form of exploration activities are provided in the Teacher's Resource, where appropriate.

Assessment *for* Learning (Formative)

Formative assessment tools are provided throughout the *Mathematics 10* student resource and the Teacher's Resource.

- The **unit opener** information and related discussion, **chapter opener** and related discussion, and the **prerequisite skills** BLM in the Teacher's Resource activate learning necessary for students' success in the upcoming unit and chapter.
- The **chapter self- assessment** BLM and the Assessment *for* Learning box on the back of the chapter foldout are designed to provide teachers with an opportunity to activate student knowledge and assess the understanding that students should have to begin the chapter.

- The **Reflect and Respond** questions provide an opportunity to determine students' understanding of concepts through conversations and/or written work.
- The **Your Turn** questions target key skills of a section.
- Students can use the **Practise** assignments in each section to check their understanding.
- The **chapter reviews** and **unit reviews** provide opportunities to assess knowledge/understanding, applications, communications, mental math, and problem solving.

Assessment *of* Learning (Summative)

Summative assessment is provided in the following ways:

- **Practice tests** and **unit tests** are provided at the end of the chapters in the student resource, and **chapter tests** are provided as blackline masters in the Teacher's Resource.
- The **unit project** section of the **Unit Connections** at the end of each unit provides teachers with an opportunity to check whether students have synthesized the concepts and procedures. A **unit project checklist** BLM helps students assess whether or not they have completed the unit project, and a **unit project final report** BLM helps students identify where they have shown their understanding of each concept, skill, and procedure. A rubric for each unit project is included in the Teacher's Resource.

Teachers are encouraged to use alternative assessments beyond formal testing. For example, student work on the unit project displays how well a student understands mathematical concepts and processes.

Portfolio Assessment

Student-selected portfolios provide a powerful platform for assessing students' mathematical thinking. Portfolios provide the following benefits:

- help teachers assess students' growth and mathematical understanding
- give insight into students' self-awareness about their own progress
- help parents/guardians understand their child's growth

Mathematics 10 has many components that provide ideal portfolio items. Including any or all of the following chapter items is a non-threatening, formative way to gain insight into students' progress

- student responses to the chapter and unit openers
- answers to the **Reflect and Respond** questions, which give students early opportunities to construct knowledge about the section content
- answers to the **Create Connections** questions, which allow students to explore their initial understanding of concepts
- journal responses, which show student understanding of the chapter skills and processes
- student responses to the **unit project** questions

Master 1 Project Rubric

Master 1 Project Rubric is a generic rubric developed for assessing student work. It highlights the level of development of conceptual and procedural understanding within a particular topic, and provides consistent assessment strategies for multiple approaches and/or for multiple solutions to problems and problem solving. This unique rubric includes

- a Score/Level grade ranging from 1 to 5 (Beginning to Standard of Excellence) **Note:** The Teacher Centre on the Online Learning Centre provides a four-level rubric.
- a Holistic Descriptor for each grade range, describing the level of understanding and communication skills
- Specific Notes, which provide descriptions of each grade range. These notes are meant to represent what the majority of students display. They are by no means exhaustive of all possible solutions. Teachers are encouraged to continually refer to both the specific and holistic pieces of the rubric.

Teachers are encouraged to share the rubric with students early in each project. This will help students become active participants in their own assessment and program planning. Discussing and building the Specific Notes with students allows them to engage actively in their learning.

CONCRETE MATERIALS

The McGraw-Hill Ryerson *Mathematics 10* program engages students in a variety of worthwhile mathematical tasks that span the continuum from concrete to abstract.

Where appropriate, concept development in the program begins with students working with concrete materials. Most **Investigates** have students using commonplace materials and conventional mathematical manipulatives in a hands-on approach. After an appropriate number of hands-on opportunities, students move from the pictorial to the symbolic in the **examples**, **Your Turn**, and **Check Your Understanding** exercises.

TECHNOLOGY

Where appropriate, lessons are designed to provide students with the opportunity to develop their skills in the use of various technologies, but not to rely on this technology to think mathematically. Students are also asked to use the Internet to research information related to problems they are required to solve.

The student resource provides technology learning that matches technology requirements for curriculum expectations and that deepens students' conceptual understanding.

Blackline Masters of technology activities are included in the Teacher's Resource when grade-specific outcomes suggest these are needed. The masters include directions for using different softwares and graphing calculators common in many classrooms. These worksheets can easily be used in a computer laboratory.

CAPITALIZING ON DIVERSITY AND REAL LIFE

Throughout the student resource, students are given opportunities to see how mathematics connects to real life by engaging in meaningful problem solving situations. Chapters are introduced with problems that model real life. Visual images used to introduce lessons, as well as those in the exercise sets, depict the cultural diversity within classrooms. Examples of mathematics from various cultures are evident throughout the text. Names used in the lessons and exercises reflect the diversity of Canadian society.

ALTERNATIVE LEARNING ENVIRONMENTS AND HOME CONNECTIONS

The design of the McGraw-Hill Ryerson *Mathematics 10* program recognizes that students' learning in mathematics may take place in a variety of learning environments outside of the traditional classroom. For example, students learn mathematics as they complete their homework, work with parents/guardians, and employ their mathematical skills in everyday life. The following features support learning outside of the traditional classroom:

- **Key Ideas** provide summaries to serve as references for students and parents/guardians when doing homework.
- Visuals and **Key Ideas** allow investigations to be easily followed independently.
- Opportunities for bringing mathematics activities home are provided through **Practise**, **Apply**, **Extend**, and **Create Connections** questions.
- Additional activities are available on the McGraw-Hill Ryerson **Online Learning Centre**.
- Suggestions for alternative and independent learning are provided as support notes in the Teacher's Resource.

Online LearningCentre

COOPERATIVE LEARNING

There are multiple opportunities throughout the program for teachers to use different types of classroom learning environments and groupings. The investigations at the beginning of each section lend themselves to being completed in groups, but teachers are free to choose class groupings that meet the needs of their students, whether or not they are in a traditional classroom setting. Additional suggestions are also provided in this Teacher's Resource.

Students learn effectively when they are actively engaged in the process of learning. Most sections of *Mathematics 10* begin with a hands-on activity that fosters this approach. These activities are best done through cooperative learning during which students work together—either with a partner or in a small group of three or four—to complete the activity and develop generalizations about the topic or process. For students who are distance or distributed learners, these investigations can be explored independently, allowing for valuable student-based learning.

Group learning such as this is an important aspect of a constructivist educational approach. It encourages interactions and increases chances for students to communicate and learn from each other.[1]

[1] Sternberg, R.J., and W.M. Williams, *Educational Psychology* (Boston, MA: Allyn & Bacon, 2002).

Teachers' Role—In classrooms or alternative learning environments where students are adept at cooperative learning, the teacher becomes the facilitator, guide, and progress monitor. Until students have reached that level of group cooperation, however, the teacher will need to coach them in how to learn cooperatively. This may include

- making sure that the materials are at hand and directions perfectly clear, so that students know what they are doing before starting group work
- carefully structuring activities so that students can work together
- coaching how to provide peer feedback in a way that allows the listener to hear and attend
- constantly monitoring student progress and providing assistance to groups having problems with either group cooperation or the math at hand
- using a discussion board or other medium to facilitate partner or group work for distance or distributed learners

Group Composition—The size of group may vary from activity to activity. Small-group settings allow students to take risks that they might not take in a whole class.[2] Research suggests that small groups are fertile environments for developing mathematical reasoning.[3]

Results of international studies suggest that groups of mixed ability work well in mathematics classrooms.[4] If your class is new to cooperative learning, you may wish to assign students to groups according to the specific skills of each individual. For example, pair a student who is talkative but weak in number sense and numeration with a quiet student who is strong in those areas. Pair a student who is weak in many parts of mathematics but has excellent spatial sense with a stronger mathematics student who has poor spatial sense. In this way, student strengths and weaknesses complement each other, and peers have a better chance of recognizing the value of working together.

Cooperative Learning Skills—When coaching students about cooperative learning, consider task skills and working relationship skills.

Task Skills	Working Relationship Skills
• following directions • communicating information and ideas • seeking clarification • ensuring that others understand • actively listening to others • staying on task	• encouraging others to contribute • acknowledging and responding to the contributions of others • checking for agreement • disagreeing in an agreeable way • mediating disagreements within the group • sharing • showing appreciation for the efforts of others

[2]Van De Walle, J., *Elementary and Middle School Mathematics: Teaching Developmentally*, 4th ed. (Boston, MA: Addison Wesley Longman, 2000).

[3]Artzt, A.F., and S. Yaloz-Femia, "Mathematical Reasoning During Small-Group Problem Solving," in L. Stiff and F. Curcio (eds.), *Developing Mathematical Reasoning in Grades K–12* (Reston, VA: National Council of Teachers of Mathematics, 1999), 115–26.

[4]Kilpatrick, J., J. Swafford, and B. Findell, *Adding It Up: Helping Children Learn Mathematics* (Washington, DC: National Academy Press, 2001).

Discuss common group roles and how group members can use them. Make sure students understand that the same person can play more than one role.

Role	Job	Sample Comment
Leader	• makes sure the group is on task and everyone is participating • pushes group to come to a decision	Let's do this. Can we decide ... ? This is what I think we should do ...
Recorder	• manages materials • writes down data collected or measurements made	This is what I wrote down. Is that what you mean?
Presenter	• presents the group's results and conclusions	This is what the group thinks ...
Organizer	• watches time • keeps on topic • encourages getting the job done	Let's get started. Where should we start? So far we've done the following ... Are we on topic? What else do we need to do?
Clarifier	• checks that members understand and agree	Does everyone understand? So, what I hear you saying is ... Do you mean that ... ?

Types of Groups

Three group types are commonly used in the mathematics classroom.

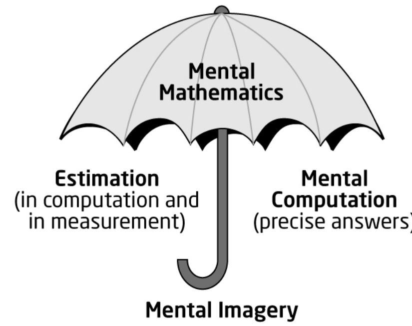
Think/Pair/Share—This consists of having students individually think about a concept and then pick a partner to share their ideas. For example, students might work on the **Create Connections** questions and then choose a partner to discuss the concepts with. Working together, the partners could expand on what they understood individually. In this way, they learn from each other, learn to respect each other's ideas, and learn to listen.

Cooperative Task Group—Task groups of two to four students can work on activities in the investigations at the beginning of each section. As a group, students can share their understanding of what is happening during the activity and how that relates to the mathematics topic, at the same time as they develop group cooperation skills.

Jigsaw—Another common cooperative learning group is called a jigsaw. In this technique, individual group members are responsible for researching and understanding a specific area of information for a project. Individual students then share what they have learned so that the entire group gets information about all areas being studied.

MENTAL MATHEMATICS

A major goal of mathematics instruction for the twenty-first century is for students to make sense of the mathematics in their lives. The development of all areas of mental mathematics is a major contributor to this comfort and understanding. Mental mathematics is the mental manipulation of knowledge dealing with numbers, shapes, and patterns to solve problems.



Estimation

Estimation refers to the approximate answers for calculations, a very practica skill in today's world. The development of estimation skills helps refine mental computation skills, enhances number sense, and fosters confidence in math abilities, all of which are key in problem solving. Over 80% of out-of-school problem solving situations involve mental computation and estimation.[5]

Estimation does not mean guessing at answers. Rather, it involves a host of computational strategies that are selected to suit the numbers involved. The goal is to refine these strategies over time with regular practice, so that estimates become more precise. The ultimate goal is for students to estimate automatically and quickly when faced with a calculation. These estimations allow for recognition of errors on calculator displays, provide learners with a strategy for checking the reasonableness of their calculations, and give students a strategy for finding an answer when only an approximation is necessary.

Mental Imagery

Mental imagery in mathematics refers to the images in the mind when one is doing mathematics. It is this mental representation, or conceptual knowledge, that needs to be developed in all areas of mathematics. Capable math students "see" the math and are able to perform mental manoeuvres in order to make connections and solve problems. These images are formed when students manipulate objects, explore numbers and their meanings, and talk about their learning. Students must be encouraged to look into their mind's eye and "think about their thinking."

Mental Computation

Mental computation refers to an operation used to obtain the precise answer for a calculation. Unlike traditional algorithms, which involve one method of calculation for each operation, mental computations include a number of strategies—often in combination with each other—for finding the exact answer. As with estimation, strategies for mental computation develop in quantity and quality over time. A thorough understanding of, and facility with, mental computation allows students to solve complicated multi-step problems without spending needless time figuring out calculations and is a valuable prerequisite for proficiency with algebra. Students need regular practice in these strategies.

---

[5]Reys, B. J., and R.E. Reys, "One Point of View: Mental Computation and Computational Estimation—Their Time Has Come," *Arithmetic Teacher* (Vol. 33, No. 7, 1986), 4–5.

Some Points Regarding Mental Mathematics

- The various estimation and mental calculation strategies must be taught and are best developed in context; opportunities must be provided for regular practice of these strategies. Having students share their various strategies is vital, as it provides possible options for classmates to add to their repertoire.
- Key to the development of skills in mental math is the understanding of place value (number sense) and the number operations. This understanding is enhanced when students make mental math a focus as they calculate.
- Mental math strategies are flexible; the student needs to select one that is appropriate for the numbers in the computation. Practice should be in the form of practising the strategy itself, selecting appropriate strategies for a variety of computation examples, and using the strategies in problem solving situations.
- Sometimes mental math strategies are used in conjunction with paper-and-pencil tasks. The questions are rewritten to make the calculation easier.
- The ultimate goal of mental mathematics is for students to estimate for reasonableness and to look for opportunities to calculate mentally.
- Students need to identify why particular procedures work; they should not be taught computation "tricks" without understanding

Keep in Mind

Practice in classrooms has traditionally been in the form of asking students to write the answers to questions presented orally. This is particularly challenging for students who are primarily visual learners. Although we are sometimes faced with computations of numbers we cannot see, most often the numbers are written down. This makes it easier to select a strategy. In daily life, we see the numbers when solving written problems (e.g., when checking calculations on a bill or invoice, when determining what to leave for tips, when calculating discounted prices from a price tag). Provide students with mental math practice that is sometimes oral and sometimes visual.

CHAPTER CORRELATION

Unit 1 Measurement

Chapter 1: Measurement Systems

Strand/Outcome	Chapter/Section	Pages
Strand: Measurement		
General Outcome *Develop spatial sense and proportional reasoning.*		
Specific Outcomes		
1. Solve problems that involve linear measurement, using: • SI and imperial units of measure • estimation strategies • measurement strategies. [ME, PS, V]	Chapter 1: 1.1–1.3 Unit 1 Project	pp. 8–53, 150–151, 154–155 pp. 19, 33, 36–37, 46, 150
2. Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS]	Chapter 1: 1.3 Unit 1 Project	pp. 36–47, 50–53, 150–151, 154–155 pp. 36–37, 46, 150

Chapter 2: Surface Area and Volume

Strand/Outcome	Chapter/Section	Pages
Strand: Measurement		
General Outcome *Develop spatial sense and proportional reasoning.*		
Specific Outcomes		
1. Solve problems that involve linear measurement, using: • SI and imperial units of measure • estimation strategies • measurement strategies. [ME, PS, V]	Chapter 2: 2.1 Unit 1 Project	pp. 56–65, 92, 95–97, 151–152, 154–155 pp. 56–57, 150
3. Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including: • right cones • right cylinders • right prisms • right pyramids • spheres. [CN, PS, R, V]	Chapter 2: 2.2–2.3 Unit 1 Project	pp. 66–97, 151–152, 154–155 pp. 76, 90–91, 150
Strand: Algebra and Number		
General Outcome *Develop algebraic reasoning and number sense.*		
Specific Outcomes		
3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]	Chapter 2: 2.2–2.3 Unit 1 Project	pp. 66–97, 151–152, 154–155 pp. 76, 90–91, 159

Chapter 3: Right Triangle Trigonometry

Strand/Outcome	Chapter/Section	Pages
Strand: Measurement		
General Outcome *Develop spatial sense and proportional reasoning.*		
Specific Outcomes		
4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	Chapter 3: 3.1–3.4 Unit 1 Project	pp. 100–149, 152–155 pp. 106–108, 122, 144, 150

Unit 2 Algebra and Number

Chapter 4: Exponents

Strand/Outcome	Chapter/Section	Pages
Strand: Algebra and Number		
General Outcome *Develop algebraic reasoning and number sense.*		
Specific Outcomes		
1. Demonstrate an understanding of factors of whole numbers by determining the: • prime factors • greatest common factor • least common multiple • square root • cube root. [CN, ME, R]	Chapter 4: 4.1 Unit 2 Project	pp. 162–171, 206, 209–210, 266–267, 270–271 pp. 169–170
2. Demonstrate an understanding of irrational numbers by: • representing, identifying and simplifying irrational numbers • ordering irrational numbers. [CN, ME, R, V]	Chapter 4 4.4 Unit 2 Project	pp. 194–205, 208–211, 266–267, 270–271 pp. 195, 204–205
3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]	Chapter 4: 4.2–4.4 Unit 2 Project	pp. 172–211, 266–267, 270–271 pp. 195, 205

Chapter 5: Polynomials

Strand/Outcome	Chapter/Section	Pages
Strand: Algebra and Number		
General Outcome *Develop algebraic reasoning and number sense.*		
Specific Outcomes		
1. Demonstrate an understanding of factors of whole numbers by determining the: • prime factors • greatest common factor • least common multiple • square root • cube root. [CN, ME, R]	Chapter 5: 5.2	pp. 224–233, 262–265, 268
4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]	Chapter 5: 5.1 Unit 2 Project	pp. 214–223, 262, 264–265, 267–271 pp. 220–221
5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]	Chapter 5: 5.2–5.4 Unit 2 Project	pp. 224–265, 268–269, 271 pp. 246–247, 257–258

Unit 3 Relations and Functions

Chapter 6: Linear Relations and Functions

Strand/Outcome	Chapter/Section	Pages
Strand: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
1. Interpret and explain the relationships among data, graphs and situations. [C, CN, R, T, V]	Chapter 6: 6.1, 6.3	pp. 268–278, 292–304
2. Demonstrate an understanding of relations and functions. [C, R, V]	Chapter 6: 6.2 Unit 3 Project	pp. 279–291 pp. 402–405
3. Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines • rate of change • parallel lines • perpendicular lines. [PS, R, V]	Chapter 6: 6.4–6.5 Unit 3 Project	pp. 305–329 pp. 264–265, 312, 314
4. Describe and represent linear relations, using: • words • ordered pairs • tables of values • graphs • equations. [C, CN, R, V]	Chapter 6: 6.1 Unit 3 Project	p. 268–278 pp. 402–405
8. Represent a linear function, using function notation. [CN, ME, V]	Chapter 6: 6.2 Unit 3 Project	pp. 279–291 p. 402–405

Chapter 7: Linear Equations and Graphs

Strand/Outcome	Chapter/Section	Pages
Strand: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
1. Interpret and explain the relationships among data, graphs, and situations.	Chapter 7: 7.1–7.2 Unit 3 Project	pp. 340–369 p. 355
3. Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines • rate of change • parallel lines • perpendicular lines. [PS, R, V]	Chapter 7: 7.1, 7.4 Unit 3 Project	pp. 340–356, 383–395 pp. 355, 402–405
5. Determine the characteristics of the graphs of linear relations, including the: • intercepts • slope • domain • range. [CN, PS, R, V]	Chapter 7: 7.1–7.2 Unit 3 Project	pp. 340–369 pp. 355, 382, 402–405
6. Relate linear relations expressed in: • slope–intercept form ($y = mx + b$) • general form ($Ax + By + C = 0$) • slope–point form ($y – y1 = m(x – x1)$) to their graphs. [CN, R, T, V]	Chapter 7: 7.1–7.3 Unit 3 Project	pp. 340–382 pp. 355, 382, 402–405
7. Determine the equation of a linear relation, given: • a graph • a point and the slope • two points • a point and the equation of a parallel or perpendicular line to solve problems. [CN, PS, R, V]	Chapter 7: 7.1–7.4 Unit 3 Project	pp. 340–395 pp. 355, 382, 402–405

Unit 4 Systems of Equations

Chapter 8: Solving Systems of Linear Equations Graphically

Strand/Outcome	Chapter/Section	Pages
Strand: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
1. Interpret and explain the relationships among data, graphs, and situations. [C, CN, R, T, V]	Chapter 8: 8.1–8.3 Unit 4 Project	pp. 416–459 pp. 430, 442, 506
3. Demonstrate an understanding of slope with respect to: • rise and run • line segments and lines • rate of change • parallel lines • perpendicular lines. [PS, R, V]	Chapter 8: 8.3	pp. 446–459
7. Determine the equation of a linear relation, given: • a graph • a point and the slope • two points • a point and the equation of a parallel or perpendicular line to solve problems. [CN, PS, R, V]	Chapter 8: 8.2–8.3 Unit 4 Project	pp. 432–459 pp. 442, 506
9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]	Chapter 8: 8.1–8.3 Unit 4 Project	pp. 416–459 pp. 430, 442, 506

Chapter 9: Solving Systems of Linear Equations Algebraically

Strand/Outcome	Chapter/Section	Pages
Strand: Relations and Functions		
General Outcome *Develop algebraic and graphical reasoning through the study of relations.*		
Specific Outcomes		
9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]	Chapter 9: 9.1–9.3 Unit 4 Project	pp. 468–501 pp. 477, 490, 500, 506

UNIT 1

Measurement

General Outcome

Develop spatial sense and proportional reasoning.

Specific Outcomes

M1 Solve problems that involve linear measurement, using:
- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:
- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

***Mathematics 10*, pages 2–5**

Suggested Timing

30–40 min

Blackline Masters

BLM U1–1 Unit 1 Project
BLM U1–2 Unit 1 Project Checklist

What's Ahead

In Unit 1, students investigate and apply their knowledge of linear measurement, surface area, and volume using units in the SI and imperial measurement systems. Students examine the SI and imperial measurement systems and learn how to convert within and between the two systems. They learn how to estimate linear measurements using personal referents; solve problems using measuring instruments; calculate the surface area and volume of 3-D objects, including right prisms, right cylinders, right cones, right pyramids, spheres, and composite 3-D objects; and explore right-triangle trigonometry, including the Pythagorean theorem and the primary trigonometric ratios.

Planning Notes

Introduce Unit 1 by pointing out the measurement organizer on page 2 of the student resource. This concept map shows how the topics in this measurement unit—SI and imperial systems of measurement, surface area and volume, and trigonometry—are related. The concept map is repeated at the beginning of each chapter and is shaded to show which topics are covered in that particular chapter.

The Looking Ahead box at the bottom of page 3 identifies the types of problems students will solve throughout the unit. You may wish to reactivate students' knowledge of these topics.

Unit 1 Project

The Unit 1 project focuses on the real-world application of mathematics in music. The project is continuous in nature and is explicitly divided by chapters.

Introduce the Unit 1 project by reading and discussing the introductory notes on page 4 of the student resource as a class. Consider distributing **BLM U1–1 Unit 1 Project** to inform students about how the project develops throughout the unit. This master provides an overview of the project as well as the requirements for completing the Unit 1 project.

You may wish to point out the questions related to the Unit 1 project. These are identified throughout Chapters 1 to 3 with a project logo. Note that these questions are not mandatory but are recommended because they provide some of the background and research needed to complete the Unit 1 project. The questions are also available on masters, one for each chapter. You may decide to use these masters to create a student booklet and have students record their finalized answers in the booklet either after they have completed their in-class work, during assigned project work time, or in conjunction with chapter assignments. Alternatively, you may wish to provide students with **BLM U1–2 Unit 1 Project Checklist**, which lists *all* of the related questions for each chapter. Students can use the checklist to monitor their progress and prepare their presentation and report. Have students collect all their work for the Unit 1 project in a portfolio.

For additional information on the Unit 1 Project, see the Unit 1 Connections on page 140 in the student resource or TR pages 103–104.

Career Connection

Use the collage of photographs to direct a discussion about career opportunities within the digital music world and how each career might use mathematics. Students may mention that the individuals operating the equipment for recording and editing music are audio technicians, often referred to as audio engineering professionals or audio engineering producers. Another aspect of the digital music industry is broadcasting. Broadcast technicians operate the equipment to route radio and television broadcasts through transmitters and networks. Ask students what they know about how each of these careers involves math.

WWW Web Link

For information about careers within the digital music industry and where to get training, go to www.mhrmath10.ca and follow the links.

Measurement Systems

1

General Outcome

Develop spatial sense and proportional reasoning.

Specific Outcomes

M1 Solve problems that involve linear measurement, using:
- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
1.1	✓ justify the choice of units used for determining a measurement
	✓ solve problems that involve linear measurement
	✓ explain the process used to estimate a linear measurement
1.2	✓ provide referents for linear measurements
	✓ describe a personal strategy used to make a linear measurement
	✓ solve problems that involve linear measurement using instruments
	✓ estimate a linear measurement using a referent
1.3	✓ compare SI and imperial units using referents
	✓ solve problems that involve conversion of linear measurements between SI and imperial systems
	✓ use mental mathematics to confirm the reasonableness of a solution to a conversion problem

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 1–1 Chapter 1 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 6 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal to explain what they personally know about measuring in SI units. You might provide the following prompts: • Where have you encountered SI units for measuring distance? • What item(s) have you measured the length of in SI units? • What did you use to make your measurement? • Do you know the approximate length of any SI units?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 1–2 Chapter 1 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 1 Foldable As students work on each section in Chapter 1, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter. • Encourage students to write examples of their own for each section. They could imitate questions from the student resource if necessary.
Assessment *for* Learning	
BLM 1–3 Chapter 1 Warm-Up This master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mathematics calculations.

Chapter 1 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 50–60 min (TR page 7)			BLM 1–1 Chapter 1 Self-Assessment BLM 1–2 Chapter 1 Prerequisite Skills BLM 1–4 Chapter 1 Unit 1 Project BLM U1–2 Unit 1 Project Checklist		TR page 6 Chapter 1 Foldable,TR page 6	TR page 6	
1.1 SI Measurement • 100–120 min (TR page 9)	Students should be familiar with • SI units of length • scale • plotting ordered pairs • ratios • multiplying and dividing powers of ten • the relationships between diameter, radius, and circumference • the relationships between velocity, distance, and time	• three items that are non-standard measuring units (e.g., coin, paperclip) • grid paper • SI measuring instruments e.g., ruler, tape measure, caliper, metre stick • CD case • watch • outdoor measuring device	BLM 1–3 Chapter 1 Warm-Up BLM 1–4 Chapter 1 Unit 1 Project BLM 1–5 Section 1.1 Extra Practice	**Essential:** #1–3, 5, 7, 9, 10, 12, 14, 18–19 **Typical:** #1–7, 9–14, 18–19, 21 **Extension/Enrichment:** #7, 8, 12–14, 16, 17–21	TR pages 11, 15 Chapter 1 Foldable, TR page 6	TR pages 13, 15	
1.2 Imperial Measurement • 100–120 min (TR page 16)	Students should be familiar with • converting between units • scale • fraction operations • the relationships between diameter, radius, and circumference • the Pythagorean relationship • estimating using a referent • area of a rectangle, circle, and triangle • rounding values • surface area • properties of similar triangles	• imperial measuring instrument, e.g., imperial ruler, caliper, measuring tape • compact disc (CD) • cassette tape case • MP3 player • envelopes of different sizes • scissors	BLM 1–3 Chapter 1 Warm-Up BLM 1–4 Chapter 1 Unit 1 Project BLM 1–6 Section 1.2 Extra Practice	**Essential:** #1, 4–8, 12, 14, 17, 18 **Typical:** #1–8, 10, 12, 13, 15, 17, 18 **Extension/Enrichment:** 9, 12–19	TR pages 18, 22 Chapter 1 Foldable, TR page 6	TR pages 20, 22	
1.3 Converting Between SI and Imperial Systems • 180–240 min (TR page 23)	Students should be familiar with • estimating using a referent • area of a circle • proportion • the relationships between velocity, distance, and time • scale factors	• compact disc (CD) • SI measuring instrument • imperial measuring instrument	BLM 1–3 Chapter 1 Warm-Up BLM 1–4 Chapter 1 Unit 1 Project BLM 1–7 Section 1.3 Extra Practice	**Essential:** #1–4, 6, 7, 9, 10, 14, 17, 18 **Typical:** #1–8, 11, 12, 14 **Extension/Enrichment:** #5, 14–18	TR pages 25, 29 Chapter 1 Foldable, TR page 6	TR pages 27, 29	
Chapter 1 Review • 100–120 min (TR page 30)		• an object with a curved surface, such as a can • SI ruler or measuring tape • imperial ruler or measuring tape	BLM 1–5 Section 1.1 Extra Practice BLM 1–6 Section 1.2 Extra Practice BLM 1–7 Section 1.3 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 1 Foldable, TR page 6	TR page 31	
Chapter 1 Practice Test • 50–60 min (TR page 32)		• SI ruler or measuring tape • imperial ruler or measuring tape	BLM 1–8 Chapter 1 Test BLM 1–9 Chapter 1 BLM Answers	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1, 6, 8, 10	TR page 33		TR page 33 BLM 1–8 Chapter 1 Test

Measurement Systems

Opener

Mathematics 10, pages 6–7

Suggested Timing

50–60 min

Blackline Masters

BLM 1–1 Chapter 1 Self-Assessment
BLM 1–2 Chapter 1 Prerequisite Skills
BLM 1–4 Chapter 1 Unit 1 Project
BLM U1–2 Unit 1 Project Checklist

Key Terms

referent
imperial system

What's Ahead

In this chapter, students develop referents for linear measurements, in both imperial and SI (Système International d'Unités) units, and they measure in standard units using measuring instruments, such as a ruler and caliper. Students learn the similarities and differences between the SI system and the imperial system, and they estimate linear measurements in both imperial and SI units. Students convert linear measurements within and between the imperial and SI systems using a variety of techniques, and they develop personal strategies for measuring objects. You may wish to point out that SI measurement is derived from the metric system.

Planning Notes

Begin Chapter 1 by inviting students to discuss what they already know about measurement. Many students may have experience with measurement in part-time jobs, hobbies, or in working in and around their homes. It may be particularly instructive to elicit any experiences with the results of measurement errors or miscommunications.

Referring to the collage in the opener, you may want to ask students to think of uses for measurement in the scenarios depicted. Students may also have relevant experience with some of the measuring tools depicted. Ask students to list the careers of the people shown in the images.

To address the Big Ideas, you could choose to have the class begin a K-W-L chart, in which they specify what they "Know" and "Want to know." At the conclusion of the unit, students can complete the "what we Learned" column. It is possible that some students will have some experience with the Key Terms, and in your discussion, you could have these students relate their experience and knowledge.

Unit Project

You might take the opportunity to discuss the Unit 1 project described in the Unit 1 opener. See TR page 2. Throughout the chapter, there are individual questions for the unit project. These questions are not mandatory but are recommended because they provide some of the work needed for the final report for the Unit 1 project assignment. You will find questions related to the project throughout the Check Your Understanding and in the section 1.3 Investigate.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs. Consider asking students the following questions:

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 7 and how it might be used to summarize Chapter 1. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase students' ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

Point out to students that each lesson has its own tab, as well as space for them to write in or cut out and paste in various conversions. Encourage students to write conversions that are new and may require referencing in the future. Point out that Unit 1 is a measurement unit and that the conversions used in Chapter 1 may be used again in subsequent chapters. Encourage students to write in examples of their own that model the concepts learned in each section.

As students progress through the chapter, provide time for them to keep track of what they need to work on, which is one of the opening tabs in the Foldable. This will assist students in identifying and solving any difficulties with concepts, skills, and processes. Have students check off each item as they deal with it. Some students may benefit from stapling **BLM U1–2 Unit 1 Project Checklist** to the back of the Foldable to help them keep track of the questions they have completed toward the project.

Meeting Student Needs

- Hand out to students **BLM U1–2 Unit 1 Project Checklist**, which lists *all* of the questions related to the Unit 1 project.
- Some students may benefit from completing all unit project questions.
- **BLM 1–4 Chapter 1 Unit 1 Project** includes all of the unit project questions for this chapter. These questions provide a beginning for the Unit 1 project.
- Consider having students complete the questions on **BLM 1–2 Chapter 1 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Ask students to brainstorm ways they use measurement in their lives or ways that people have historically used measurement. Some suggestions are designing clothes; building a komatik (sled), a shed, or a cabin; choosing duffle, fur, and fabric for parkas, duffle socks, or mitts; calculating the length of strips required for the webbing of snowshoes; and talking about distances to cabins, fishing spots, or hunting grounds.
- Post student learning outcomes. Get students involved in rewriting the outcomes so that they understand what they will accomplish by the end of the chapter. As you work through the chapter, refer to the outcomes and encourage students to make connections to the current topic. The posted outcomes may be helpful when reviewing the entire chapter because students can identify the outcomes that they still need to review in order to successfully complete the material in the chapter.

ELL

- Have students record definitions for the Key Terms and write examples into their Foldable that model the various approaches used in the chapter.

Enrichment

- Ask students to develop referents for very small measurements, such as the measurement for which the diameter of a human hair could be a referent.
- Ask students to visualize one half of a cup and two thirds of a metre. Many imperial measurements are based on fractions. Encourage students to conjecture as to why fractional references were once so common and why SI measurements have come to replace them in the scientific world.

Gifted

- Suggest students explore the effect of temperature on measurement. For example, if an engineer is designing a bridge, how can he or she compensate for changes in length of steel beams as they heat up in the sun and expand?
- Encourage students to explore the sources of measurements in other cultures.

Career Connection

You may wish to have students who are interested in learning more about photogrammetrists research the career and report to the class how mathematics relates to the career.

WWW Web Link

For more information about photogrammetry, go to www.mhrmath10.ca and follow the links.

SI Measurement

1.1

Mathematics 10, **pages 8–21**

Suggested Timing

100–120 min

Materials

- three items that are non-standard measuring units (e.g. coin, paperclip, and so on)
- grid paper
- SI measuring instruments, e.g., ruler, measuring tape, caliper, metre stick
- CD case
- watch
- outdoor measuring device

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–5 Section 1.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 7, 9, 10, 12, 14, 18–19
Typical	#1–7, 9–14, 18–19, 21
Extension/Enrichment	#7, 8, 12–14, 16, 17–21

Unit Project Note that #14 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

Before you begin this section, you may wish to have a brief class discussion on the topic of linear measurement. In particular, students could discuss the need to measure and the units used to measure. Linear measurements are among the most likely to be made in both imperial and SI systems, so it may be beneficial to establish how familiar students are with each system and in what situations they have used each system.

One key aspect of the section is the use of personal referents, so it may be useful to discover what, if any, personal referents students have already established for themselves. Students likely have some sense of the need for standard units, so it may be productive to discuss this need and what experience students have with non-standard units, as well as any difficulties they have experienced or can imagine.

Investigate Dimensions of a Rectangle

Have students work in pairs. You may wish to have a set of non-standard items available. The student resource suggests coins, the width of a finger, and paper clips. Some other suggestions are the cap from a pen, a battery, an eraser, a pencil sharpener, a bingo chip, a USB key, and so on. To emphasize the challenges of measuring with non-standard units, you could have each pair of students use a different item to make their measurements, thus making comparing and communicating their answers more difficult.

This investigation gives students an opportunity to practise estimating. Ensure that students make the estimates required, and in their team, they should discuss their estimates and come to an agreement on them. Having students estimate their measurements in standard SI units links to the development of personal referents later in the section, and so is an important part of the investigation. At the conclusion of this lesson, students should be more comfortable with the common SI units and how to estimate with them.

For step 5 in the investigation, you may wish to emphasize that length is to be plotted on the x-axis while width is to be plotted on the y-axis. While this convention is not necessary for students to acquire the concepts in the investigation, it will help reduce confusion when discussing in groups and as a class. For this reason, it may be helpful to agree as a class which dimension will be the length and which will be the width when beginning the investigation.

For step 6, you may wish to have students compare the graphs for the different units of measurement that they used. Encourage students to compare their results with other teams so that they can see that a constant ratio was achieved in each case. You may wish to discuss the results as a class and ask students to consider why standard measuring units are important, given the fact that the ratios are unchanged.

For step 7, this discussion serves to follow up any discussion on measurement that took place before beginning the investigation. This provides students with an opportunity to quantify what they have learned in the investigation and could be written about in their math journals. In your discussion, you may wish to compare the use of different standard measurements as a link to the work that is to come in the chapter and unit. This may be a good time to ask students to describe situations where measuring in a particular unit may make more sense than in another unit. This may lead to a discussion on how to determine the "best" unit for a particular measurement and the need for making conversions within and between measurement systems.

Meeting Student Needs

- You may wish to invite a representative from one of the cell phone companies to give a brief presentation about how cell phone systems operate, the range of towers, and how the system is controlled, as well as the various occupations available at a cell phone company.
- Students could create posters to illustrate the various SI units and some common items that would be accurately measured with each unit.
- Students may need to be reminded about construction of rectangles. Various methods could be explored to ensure that there are four right angles and that the opposite sides are congruent and parallel.
- Some students would benefit from a chart for #2b) being posted on the whiteboard to help them organize their data.

ELL

- Some students may not be familiar with compact discs (CDs) because many students download music directly to music devices.
- You may wish to bring in a map of the school area to help reactivate the concept of map scales. Measure in SI units the distance from school to various neighbourhood landmarks and determine the actual distance to give meaning to the map scale.

Enrichment

- Suggest students create referents that define SI measurements using healthy activities, such as running, biking, and swimming. An example might be to challenge students to establish referents for the time taken to run 100 m, bike 1 km, or swim 50 m.

Gifted

- Ask students to ponder the question of SI time. What would happen if the world decided to create 10-h days, 10-day weeks, etc.? Would it be an improvement? What might go wrong?

Common Errors

- Some students may make arithmetic errors when determining the length-to-width ratio if they use fractions to express distances.

R_x Have a brief discussion of operations with fractions.

- Some students may forget which side of the rectangle they designated as the length and which as width.

R_x Suggest that students label the length and width of their rectangle on their drawing.

Answers

Investigate Dimensions of a Rectangle

1. Accept various rectangles.

2. Examples: pencil widths, eraser widths or lengths, thumb widths, pinkie finger widths. and so on. Various estimates are acceptable and depend on the size of the rectangle. Examples include 6 pencil widths wide by 3.5 pencil widths long, 3 thumb widths wide by $1\frac{1}{2}$ thumb widths long, 4.5 eraser widths wide by 2.5 eraser widths long.

Answers

3. Examples: 2.5 paper clips wide by 1.75 paper clips long, 7 pencil widths wide by $3\frac{1}{2}$ pencil widths long. 3 thumb widths wide by 1.5 thumb widths long.

4. Estimates include 50 mm wide by 30 mm long and 5 cm wide by 3 cm long. Actual measurements could be 47 mm by 26 mm and 4.7 cm by 2.6 cm.

5. Graphs will vary, but points should appear roughly on the same line. Points plotted could be (6, 3.5) for pencil widths, (3, 1.5) for thumb widths, (4.5, 2.5) for eraser widths, and (4.7, 2.6) for centimetres.

6. a) The points should fall roughly on the same line.

b) No, the length will change at the same rate as the width. So, the ratio will be constant.

7. Examples: One advantage of using standard units is that the measurements can be compared easily anywhere because the units are standard worldwide. One disadvantage is that not everyone has a measuring tape or ruler available when they need to measure something. So, using non-standard measuring instruments can still be useful.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss their graphs. Encourage them to generalize and reach a conclusion about any patterns they see in their graph or about the length-to-width ratio of their rectangle. For #6, consider having students respond in their journal to the following prompts: – What would change in your home if there were no standard measurement units for distance? – What would change at school if there were no standard measurement units for distance?	• Have students compare their graphs with those of classmates and discuss their results. • Encourage students to share their journal responses to the prompts. • Encourage discussion about the measuring process to provide struggling learners with an opportunity to verbalize the patterns that they see. • Discuss the benefits of having a standard measurement instrument and a standard unit of measurement. Ask students why they believe it is important that we have a standard unit to measure with instead of using their referent.

Link the Ideas

In this section, students are introduced to the SI system for linear measurements. Students are presented with two ways of measuring distances: a referent and a caliper. Students are talked through a strategy for estimating using a personal referent and they are given the steps involved in reading an SI caliper. A comparison is made involving the level of accuracy allowed by different measuring instruments.

You may wish to discuss the chart showing SI units and make sure that students understand how to use the multiplying factor to convert between SI units.

Example 1

This example allows students to choose referents for given lengths. Encourage students to develop their own referents. Discuss how different parts of the hand might be used for the same referent. This discussion will tie in to section 1.2 and the discussion involving the development of the imperial system. Encourage students to use referents that are different from each other's. As students choose referents, you may wish to ask them to consider the practicality of their referent by asking questions such as the following:

- Is your referent readily available?
- How many copies of your referent are needed to measure that object?
- Can you use the same referent for more than one object?
- Would you use that referent in a real-life situation?
- In what sort of situation have you needed or used a referent in the past?

You could also have students compare their choice of referents, and see if a consensus develops as to which ones are most useful or practical.

Example 2

The solutions for this example show two methods for converting between SI units. The methods involve unit analysis, discussed in the Did You Know? on page 12, and proportional reasoning. Help students discuss the two methods by asking questions such as the following:

- Do you prefer one method to another?

- Would your choice of method depend on the situation or units? Explain why.
- Is there another method that you would prefer to the ones shown? Explain why you prefer it.

Emphasize that although students may prefer one method to another, they need to be able to use both methods. Prompt students to look for opportunities to use each method in the unit. This could lead to a metacognitive goal for students: as they progress, they can decide for which types of questions they prefer each method, and why.

Help students understand the solutions by drawing their attention to the relationships between SI units. Ask students the following questions:
- How do you convert from millimetres to centimetres? from centimetres to millimetres?
- How do you convert from kilometres to metres? from metres to kilometres?

It is important that students carefully consider the units when converting. Point out to students that in the second solution, both numerators have the same units and both denominators have the same units. You may wish to emphasize to students that keeping track of units is as important in mathematics as it is in other areas, such as science.

Example 3

In this example, students consider the length of band needed to hold together a planter made from a wooden half-barrel. Encourage students to sketch and label the planter and each band. You may wish to review the formula $C = 2\pi r$. Challenge students to consider how to solve part d) using this formula.

Discuss with the class the generalization at the end of part d). You may wish to have them test the validity of this generalization using the measurements of other cylinders before and after increasing the circumference by 1 m.

Have students do the Your Turn question and then discuss their solution with a classmate and explain how they determined the diameter of the hub of a Red River cart.

Key Ideas

The Key Ideas summarize the SI system of measurement and some of the common units used for measuring distances, as well as the process of using a referent to estimate a distance. You might have students use index cards to prepare their own summary of the Key Ideas, including a linear example for each SI unit along with a referent for this unit.

If students have not been asked to work through the solutions to Example 2 using the other method, you may wish to develop the unit analysis solution for the proportional reasoning example provided on the board. Then, develop the proportional reasoning solution for the unit analysis example provided.

Meeting Student Needs

- You may wish to have students create a mnemonic to remember the basic units of the SI system.
- Encourage students to brainstorm occupations where using a referent would be helpful, such as a real estate agent estimating the length of a room.
- To help students who may not be familiar with kilometres, create a personal referent for 1 km; suggest that they ask someone with a car to drive 1 km from a point outside the school. Have them identify a landmark at the other end and then walk the distance back.
- The concept of a standard measurement system may be new to students who have not seen or used measuring tools, because standard tools were not traditionally used. Allow students the opportunity to explore the units on an SI ruler and to practise making measurements and reading the values off the ruler.
- Have a class discussion about current examples of the use of the SI system.
- For the Example 2 Your Turn, if students are having difficulty determining an appropriate unit, suggest that they think of the conversion in one of two ways: What unit would enable the given measurement to be a more reasonable number to work with? What unit would seem appropriate to measure the distance required?

Common Errors

- When converting between SI units, students may confuse multiplying and dividing. For example, 3.5 m may become 0.035 cm.

R_x Remind students of the importance of mental mathematics and considering whether an answer is reasonable. You may wish to suggest that students estimate a measurement first, using a referent, and then perform any calculations. This will allow them an extra check to see whether their solution seems reasonable.

Answers

Example 1: Your Turn

Example for the marker tray on a whiteboard: 5 hand lengths (wrist to fingertips) with an actual measurement of 80 cm

Example 2: Your Turn

a) 20.5 cm. The centimetre is a larger unit than the millimetre and the centimetre is used more commonly.

b) 6 cm. The centimetre is a smaller unit than the metre, and then no decimals are needed.

c) 590 km. Using the kilometre gives a smaller value; this unit is more commonly used for road distances than the metre.

d) 211 cm or 2.11 m. The millimetre is not commonly used when referring to buildings.

Example 3: Your Turn

0.26 m or 26 cm

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Encourage students to verbalize their thinking. • Assist students having difficulty deciding on a personal referent by suggesting a readily available tool. Have them measure their desk and demonstrate their ability to use the referent effectively before assigning any further questions. • Some students might benefit from a discussion about the usefulness of their referent.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • Encourage students to verbalize their thinking. • Encourage visual learners to physically measure each distance if they are having difficulty determining an appropriate unit. Alternatively, you may suggest that students convert the given measurement to several other SI units until they determine one that seems most appropriate. • Although students can choose which method they prefer, they need to be able to solve questions using both methods. Have students identify their method of choice and solve several questions with their preferred process. • Have students use the other method and solve some of the same questions. Have students verbally describe how the methods differ, how they are similar, and why students chose the method they did.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Provide students with a similar problem to solve. • Have students draw a diagram of a Red River cart and label the measurements they know and any that they can determine using circle properties.

Check Your Understanding

Practise

Question #1 provides an opportunity for students to select a referent to use to estimate the perimeter of a shape.

For #2, students practise drawing objects of specified sizes, in specified units.

For #3, students read measurements from rulers and calipers. Students often read calipers incorrectly, so it may be necessary to discuss this question and reinforce that students must choose the divisions that line up most closely with the sliding (vernier) scale.

Emphasize that readings must be made by looking directly at the caliper and that reading it from an angle will cause errors.

Question #4 gives students an opportunity to work with ratios as they use the photograph.

Question #5 has students identify the most appropriate unit for a measurement and make a conversion. This question may provoke some discussion about whether there is only one correct answer for the most appropriate unit. Encourage students to contribute their experiences in life or in other classes to this discussion.

Apply

For #6, students relate the circumference of a circle to its radius. They make a conversion from centimetres to millimetres. This question requires students to use their estimation skills.

Question #8 may be challenging for some students. In order to make the question more accessible, you may want to have several small wheels available so that students can model the problem.

Question #9 relates to the investigation for this section. You may want to suggest to students having difficulty that they revisit that investigation. You can ask questions such as the following:

- How is the task in this question related to the investigation?
- What property of rectangles did you discover in the investigation?

Did You Know?

For #10, point out that in Inuktitut, the word *Inukshuk* means likeness of a person. The pronunciation of *Inukshuk* varies slightly in different communities. In Nunavik, it is pronounced i-nuk-suk. In other communities, it is pronounced inn-uuk-shook.

For #11, students need to measure and determine the ratios among the sizes of Canadian coins. This allows for several solution methods and could provide an opportunity for students to explain the method they have chosen and why they chose it.

For #13, students extend the idea of radius and circumference to the situation of a geostationary satellite. This may be a good opportunity for students to work with one or two partners.

Extend

Question #15 relates to photography. While students are working with scale, in this case the units are different, so students are required to make unit conversions as part of their solution. This question also links to the career of photogrammetrist as profiled in the chapter opener.

For #16, students may model the situation and add up the distances travelled, or they may find a pattern in the distances formed as the lawn is mowed. Whatever the method they use, students need to make unit conversions and decide which units to convert and when in the process to make the conversions.

Create Connections

For #17, you may wish to refer students to Example 3.

Students will need to work in teams of two for #18. This problem refers to the technique of wrapping an ankle and foot in a figure 8 pattern. Some students may not be familiar with this technique and may need help visualizing that the 8 is formed by 'circling' the ankle and then 'circling' the foot.

Question #19 is another real-life application. Students need to work within a context and make unit conversions as they solve a problem. In this question, a variety of answers are possible, so students will need to be prepared to justify their answers.

Question #20 requires students to make a judgment about the accuracy of the formula and to justify their response.

Question #21 provides an opportunity for students to work outside the classroom and develop a referent for a distance that most students are unlikely to have a personal referent for.

Unit Project

The Unit 1 project question, #14, provides students with an opportunity to use a personal referent to estimate the dimensions of a cassette case. Students can choose which instrument to use for their measurements.

Meeting Student Needs

- Provide **BLM 1–5 Section 1.1 Extra Practice** to students who would benefit from more practice.
- The last question could be linked to an "in-motion" walk for your class. You could actually measure out a route prior to class of either 1 km or 2 km. Then, have students walk the route. Each student should stop at the point where they believe they have walked 1 km. You could then indicate the 1-km marker you put out the day before.
- Allow students to use manipulatives as necessary to work through the problems. For example, in #9, students may need to physically cut out rectangles that represent the photograph and the given rectangle.
- When completing #11, allow students to work with a partner to make sure they have access to the required coins and in case there are not enough calipers for each student.
- Concrete and kinesthetic learners may benefit from using items to help them visualize and work through some of the problems. For example, students may use a tennis ball to model Earth in #13, drinking straws for #19, or a frying pan for #20.

- For #16, you may wish to allow students to use a wide highlighter or paintbrush to model the lawn mower's path around a scale diagram of the lawn.

ELL

- Remind students of the meaning of reduction factor (and enlargement factor), so that they can apply their knowledge as they work through the scale problems in the Check Your Understanding.
- Explain the meaning of a trundle wheel and what it is used for.

Common Errors

- Some students may forget how to read a caliper.

R_x Refer students to the Link the Ideas section on measuring instruments and have them review the steps involved in reading an SI caliper. Alternatively, you may wish to have students watch a video showing how to read a caliper by going to www.mhrmath10.ca and following the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, 7, 9, 10, 12, 14, and 18. Students who are able to complete these questions could move on to the other Apply and Extend questions.	• For #1, make sure that students have a good understanding of the use of their referent before beginning. Have them measure an item on their desk with their referent before proceeding. Refer students who have difficulty converting between SI units to the relevant worked example. Then, provide a similar problem to solve by changing the dimensions. • Encourage estimation and have students verbalize why they feel their estimates may be reasonable. • Students experiencing difficulty with #3 should be prompted in the use of the measuring device and provided with additional items to measure to demonstrate their learning. • You may wish to suggest to students completing #5 and 7 that they use their solutions as samples in their Foldable. • Question #10 provides students an opportunity to use a variety of measuring tools. Students may benefit from working with a partner for the activity. • Question #12 allows students to estimate distances. Students could write their response to the comparison between their estimate and determined distance, providing work for the problem. This could be used as an Assessment as Learning grade if needed.
Unit 1 Project If students complete #14, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers. • Students may benefit from handling and viewing an actual cassette case to help them visualize the perimeters required. • Make sure students can identify which record in the photo is the LP and which record is the 45. • Bringing in actual records for students to see and hear may further engage them in the project.
Assessment *as* Learning	
Create Connections Have all students complete #18 and #19	• Encourage students to verbalize their thinking with a partner. • The hands-on activity in #18 provides students a link between their estimation skills and actual product use. When they have completed that activity, have them write responses to questions such as the following: – Why is estimating important? – Where in your daily routine do you estimate? – Can you think of any careers that would use estimation? The responses could be entered into their Foldable or used as an Assessment as Learning piece.

1.2 Imperial Measurement

Mathematics 10, pages 22–35

Suggested Timing

100–120 min

Materials

- imperial measuring instrument, e.g., imperial ruler, caliper, measuring tape
- compact disc (CD)
- cassette tape case
- MP3 player
- envelopes of different sizes
- scissors

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–6 Section 1.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 4–8, 12, 14, 17, 18
Typical	#1–8, 10, 12, 13, 15, 17, 18
Extension/Enrichment	#9, 12–19

Unit Project Note that #12 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

As you introduce this section, you may wish to activate students' prior knowledge of some fraction concepts. In particular, the use of mixed numbers is common in imperial measurement, as well as multiplication and addition of fractions.

Depending on your class, you may wish to discuss the intervals on an imperial ruler. Ask students the following questions:

- What measurement unit is shown on an imperial ruler?
- What do the markings represent?
- Why are the markings different lengths?
- How can you use the different markings to help you read an imperial ruler?

Investigate Referents for Imperial Measurement

Have students work in pairs, as indicated in the student resource. After each group has made a list of referents for an inch, foot, and yard, you could have each group add its referent to a master list for the class. Questions you could discuss include the following:

- Are all of these referents close (in size) to the unit they represent?
- Are all of these referents equally accessible?
- Are all of these referents equally convenient?
- Are there specific careers or situations in which some of these referents would be particularly useful?
- Which of these referents do you already use? in what context?

Meeting Student Needs

- You may wish to discuss the origin of imperial units for distance and then have students research why Canada chose to make the transition to the SI system of measurement.

- Students could be encouraged to discuss the imperial system of measurement with a parent or grandparent. They could determine whether that particular person has been able to make the transition. In the interview, students could also discuss some of the key areas where the imperial system may still be used today and determine why not all professions have changed to SI.
- Students could create posters illustrating the various units of imperial measurement and some common items that could be accurately measured using that unit. Common conversions could also be included.
- You may wish to arrange for a carpenter or other contractor to make a short presentation to the class. During the presentation, the common units of measurement could be discussed as well as the reason this trade did not switch to SI units. Alternatively, you may wish to invite an architect, mechanic, or person from another appropriate occupation to talk to the class about the tools they use to measure and whether any use both imperial and SI units.
- Some students may find it useful to keep a taped or oral summary of what they are learning. Others may benefit from having conversion charts posted in the classroom for quick reference, particularly in the imperial system, which will be new to most students.
- You may wish to have students measure objects in the classroom to determine whether they were likely built using the SI measurement system or the imperial system.
- Using tangible materials, such as measuring cups, tools, and so on, have students devise ways in which they can use the imperial measurement system to generate problems for their classmates to solve.
- Some students may benefit from conducting a survey of their friends and relatives to see who uses SI units and who uses imperial units. Do some people use both? Do more elders use imperial units? Which units do students feel most comfortable using?

ELL

- The imperial system can be better understood through constant exposure to imperial measuring devices. Post signs on windows or teacher boards or bulletin boards that label the sides using various imperial measurements. For example, one board could show measurements in feet and inches. Another could show yards.
- To help students recall the concept of scale factors, you may wish to demonstrate a reduction. Drawing it to scale using an imperial ruler on the board would tie the new knowledge to previous knowledge. Discuss how a reduction can also be seen as an enlargement and how to state a scale factor. Discuss strategies for using mental mathematics to determine whether a given scale indicates a reduction or an enlargement.

Enrichment

- Have students ask someone familiar with construction to explain why imperial measurement is commonly used in that industry and have students try to find out why a *two-by-four* does not measure 2 in. by 4 in.

Gifted

- Have students investigate the way in which imperial measurements have been improved in terms of consistency using science. For example, how is the measurement of a yard maintained as the temperature of the surroundings changes?

Common Errors

- Some students may try to perform fraction operations with mixed numbers.

R_x Remind students to write a mixed number as an improper fraction before performing operations.

Answers

Investigate Referents for Imperial Measurement

1. a) 12 in. **b)** 3 ft **c)** 36 in.

2. inch: eraser, foot: ruler or your own foot (two, if your feet are small), yard: one stride or big step or a metre stick.

3. 12 city blocks, or the distance travelled in 1 min by car or in 15 min by walking

4. a) Example: A referent might be 1 pencil length is approximately 6 in. long. Measure the table or desk top as 6 pencil lengths wide by 4 pencil lengths long. Then, multiply these measurements by the number of inches in 1 pencil length. An estimate for the dimensions of a desktop might be 36 in. by 24 in.

b) Examples: the referent is smaller than the object being measured; the referent is readily available and easy to use.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the investigation. Encourage them to generalize and develop a method of estimating distances.	• Some students may benefit from referring to their work on estimating in SI units in section 1.1. • As the imperial system may be very new to most students, provide ample opportunities for measuring with a referent and a measuring device. Provide objects to measure that have great variation in sizes; for example, the height of a door knob versus the length of a counter. • For part b), students may find it helpful to explain why other items may not have been appropriate referents, and this might help them verbalize why they chose their particular referent.

Link the Ideas

The Link the Ideas section shows two different methods for converting between imperial units: one uses proportions and the other substitutes inches for yards. Students should be comfortable with a variety of approaches to unit conversions, whether working with imperial or SI units.

You may wish to emphasize the conversions, because students are unlikely to be familiar with conversions among miles, yards, and feet. Draw students' attention to the symbols for feet (') and inches ("). You may want to have students examine their measuring tools to discern the differences from SI devices. In particular, you could ask the following questions:

- How many divisions is 1 in. broken into on your device?
- Does this make it practical to write fractional measurements using decimals?
- What form will measurements including parts of inches take?
- What is the smallest division on your device?
- Consider the measurement $\frac{1}{2}$ in. Using the divisions on your device, how many equivalent measurements are possible?

Use the Measuring Instruments section starting on page 23 to discuss the use of a ruler, measuring tape, and caliper. Make sure students understand how to read the particular type of caliper that you have available in the classroom. The calipers shown in the student resource measure to the nearest thousandth of an inch and give readings in decimal form. To read this type of caliper,

- Read the whole number and tenth from the fixed scale.
- Determine what fraction of a tenth is indicated by the zero on the moving scale (e.g., 0.025, 0.05, 0.075).
- Find the subdivision on the moving scale that best lines up with the subdivision on the fixed scale. Read this number as a thousandth.
- Now, add the numbers from each step.

Example 1

There is likely to be some discussion about measuring the height of the bear in the example. The fact that the bear is walking adds complexity to this discussion. Most experts agree that the best way to measure the height of a bear is to measure at the front shoulder. Similarly, biologists measure the length of a polar bear from the nose to where the tail joins the rump.

You may wish to use the Think-Pair-Share strategy to discuss this example. In this technique, each student works individually. When finished, each person works with a partner. They compare and contrast techniques, and correct any errors or misconceptions. Then, one person from each pair shares their work with the whole group. Try to discuss several different solutions

You may need to refer students to the investigation as they rewrite a number of inches as a measurement in feet and inches:

- Look back at the investigation. How many inches are in 1 ft?
- Can we find a pattern to convert inches to feet and inches? For example, how would you rewrite 18 in.? 24 in.? 32 in.? How does this apply to our example?
- How would you describe a method for rewriting inches in feet and inches?

Example 2

This situation involving the size of a television is one in which inches are the standard unit of measurement. As such, it highlights the need for fluency in both measurement systems.

To begin the discussion, you could ask students the following questions:

- What does it mean when the screen size of a TV is stated as 50 in.?
- If a TV screen is 50 in., can we determine the actual dimensions of the screen? What other information, if any, would be required?

This discussion may help students see the need to make the comparisons in the example.

This example provides a natural place for students to recall the Pythagorean relationship, although it requires a more algebraic approach than students are likely used to. Some questions to help them include

- In which situations does the Pythagorean relationship apply?
- Looking at the use of the Pythagorean relationship in this solution, how is the order of operations important?
- What process allows us to determine the value of x when we know the value of x^2?

Example 3

Ask students what referent they would use for the drum frame. Then, discuss how large they think the actual drum might be. Have them talk throught the solution.

To help students express their answers in yards and inches, ask them the following questions:

- Do you recall how many inches are in 1 yd?
- How is writing an answer in yards and inches similar to what we did in Example 1? How is it different?

Did You Know?

Point out that the *qilaut* is pronounced kee-la-oot. This drum is also called a wind drum. It is divided into two sections: the *isik*, which is the surface that is struck and the *pablu*, which is the handle of the drum. It is used for drum dances.

Key Ideas

This may be a natural place to compare aspects of the two measurement systems. For example, in SI there is a big difference between the units 1 cm and 1 m, while in imperial there is an intermediate unit: the foot exists between the inch and the yard. Students could discuss whether this poses any practical problems, and if so, how this makes each system more or less convenient in different situations.

Meeting Student Needs

- Place posters in your classroom illustrating the connections among inches, feet, yards, and miles.
- You may need to spend extra time showing students how to convert from inches to feet. Place emphasis on the idea that $3\frac{1}{2}$ ft is the same as 3 ft 6 in., or other similar conversions where students must change fractional (or decimal) values to accurate measurements.
- Spend adequate time examining Example 3; students generally have an interest in the technology of LCD televisions. You may wish to have some students investigate other aspect ratios of televisions, including 720 versus 1080 pixels, and so on. Allow students to make a presentation to the class, including the comparison of the standard 4:3 aspect ratio and the 16:9 widescreen aspect ratio.
- Students may wish to confirm that the diameter of the drum in Example 2 is just over 1″. Allow students to verify this estimate using a ruler.

Gifted

- Photographs taken with a digital camera generally use the 4:3 ratio; however, when photographs are printed, they traditionally use the ratio 3:2, as in the most common print size, 4" × 6". Have students investigate the maximum possible area that can be preserved from a 4:3 photo file if a 3:2 print is to be made.

Common Errors

- Students may incorrectly determine the power of a quotient and only determine the power of one of the factors. Students may write $\left(\frac{16}{x}\right)^2 = \frac{16}{x^2}$ or $\left(\frac{16}{x}\right)^2 = \frac{256}{x}$.

R_x Remind students that when calculating the power of a quotient, such as $(ab)^m$, they need to rewrite each factor in the quotient with the same exponent and then determine the powers.

Answers

Example 1: Your Turn

5 ft, 2 ft 6 in.

Example 2: Your Turn

The standard television has a greater viewing area by about 112 in.^{2}.

Example 3: Your Turn

$13\frac{3}{4}$ ft

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • Some students may need prompting and reminding of the meaning of scale. Coach students through the questions and provide an additional question from the problem set for them to demonstrate their learning. • Provide students with a similar problem to solve.
Example 2 Have students do the Your Turn related to Example 2.	• Provide students with a problem similar to Example 2 before they work on the Your Turn. Allow them to work with a partner and talk through their thinking. • Allow students to select one method for determining the scale factor and to use only this method when solving the problem. • Encourage students to describe the steps they will follow as they work through this problem. • Some students may need coaching and review work on the Pythagorean relationship. Provide students with several sample questions to determine the hypotenuse to one decimal place.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Have students draw and label a diagram to visualize the problem. • Some students may benefit from cutting one piece of string to represent the perimeter of the frame and another piece of string to represent the length of sinew. Then, have students explain the relationship between intervals on each piece of string.

Check Your Understanding

Practise

For #1 and 3, students read measurements from rulers and calipers. It may benefit some students to review the Link the Ideas starting on page 23.

Question #2 requires students to round imperial measurements. Many students may benefit from some discussion of their results and methods with a partner, a small group, or the class.

For #4, students will need access to imperial rulers, calipers, and/or micrometers. You may wish to talk through measuring an item to the nearest sixteenth of an inch before having students complete this question.

For #5, you may wish to have students assess the accuracy of their referent compared with the actual measurement.

For #6, it may be beneficial for students to work with a partner. Encourage students to justify their choice of a personal referent.

For #7, in order to give students a sense of the significance of the feat, you could invite them to compare Billy Loutit's achievement to that of a contemporary marathon or long-distance runner.

Apply

The window in #8 is a composite figure, and allows students to activate prior knowledge of perimeter and circumference.

When students have completed #9, you might ask them to consider these questions:

- What ratio do you get between the drive wheels and caster wheels if you use the circumferences?
- How does this compare to the ratios between the diameters? Why?

Have students discuss the strategies they used to solve #10. They may use the idea of a constant length-to-width ratio or they may approach it by using proportional reasoning.

Question #11 addresses an error often made with imperial measurements. While students are quite used to equating the fraction $\frac{1}{2}$ to the decimal value 0.5, this is not as straight-forward when measuring in feet and inches.

In question #13, discuss with students the difference between distances according to a GPS reading and what they might find if they walked the same distance.

Extend

Consider having students work in pairs to complete the Extend questions.

When working on #14, encourage students to discuss the advantages and disadvantages of using each length of material. They may extend this discussion to consider how the different lengths of material could affect both the amount of waste and the circumference of the retaining wall around the pool, and how the distance between the retaining wall and the pool wall might affect the pool's ability to retain heat. Remind students that they need to justify their ideas for part c); there is no one correct answer.

In #15, students use proportional reasoning to make conversions between miles and astronomical units, a unit that is likely new to most students. This is a more abstract problem.

Create Connections

For #17, ask students to consider, “What would be some implications of purchasing the bed without first doing the mathematical analysis?”

Question #18 refers to a unit that students might not be very familiar with, the megawatt (MW). You may wish to mention that mega is an SI prefix meaning million. Some students may recognize that the watt is a unit for measuring electricity as indicated on light bulbs.

You may wish to have students work in pairs for #19, then compare and discuss their solutions with another group.

Unit Project

You might have students use **BLM 1–4 Chapter 1 Unit 1 Project** and finalize their answers to #12 on the master.

You may wish to have a class discussion, before students work on #12, on how the sizes of music devices have changed over time and whether students think that music devices could be getting too small and what some disadvantages or advantages might be.

Students measure the dimensions of a CD, cassette case, and MP3 player using imperial units.

Meeting Student Needs

- Provide **BLM 1–6 Section 1.2 Extra Practice** to students who would benefit from more practice.
- Ensure that students spend an adequate amount of time measuring various objects in the classroom. Encourage students to record their results and then have a classmate check the measurement. If they do not have the same measurement recorded, they will need to discuss which answer is correct and why.
- Students may benefit from using linking cubes when working on #12.
- When working on #9, concrete and kinesthetic learners may benefit from cutting out a circle to represent each wheel. This may help students visualize the required ratio as they solve the problem.

ELL

- Explain to students that a Norman window is a composite figure with a semicircle above a rectangle, as illustrated in #8.
- Some students may not be familiar with the term *geocache* referred to in #13. Explain to students that it is a hiding place (or cache) in the ground that can be used for storage.

Enrichment

- You may wish to have students use imperial calipers that measure in 32nds or 64ths of an inch. Students will be required to add the fractions from each step of the reading. For most students, this is a more difficult task and will require additional time and practice.
- You may wish to ask students to estimate the length of magnetic tape in a 90-min audio cassette tape and then research to find out the actual measurement.

Common Errors

- Some students may confuse the markings on an imperial ruler.

R_x Remind students to identify the whole numbers that indicate inches on the ruler and then look at the markings that represent fractions of an inch.

- In #2, students may not give their answer to the required fraction of a unit.

Rx Make sure students understand what units the question is asking for; that is, for part a), students need to understand that their answer will be given in either whole inches or whole inches plus one half of an inch. You may wish to suggest that students write down some sample measurements that indicate the possible fractions of a unit that their measurement could be written in.

- In #11, students may think that 37.5 ft is the same as 37 ft 5 in.

Rx Remind students that 0.5 is a decimal which equals $\frac{1}{2}$. So, 37.5 ft = $37\frac{1}{2}$ ft, which is the same as 37 ft 6 in., not 37 ft 5 in.

WWW Web Link

Some students may find a visual representation helpful when working on #7. For an illustration of the journey completed by William Loutit, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–8. Students who have no problems with these questions can go on to the remaining questions.	• Have students draw part of an imperial ruler and highlight the smallest subdivision. • Some students may benefit from a review and coaching on how to identify the fraction of a segment being represented on the ruler. • Provide additional coaching to students who have difficulty reading imperial measuring instruments. Talk them through corrections and clarify any misunderstandings before allowing them to proceed. • Questions #1 to 6 are directly related to measuring. You may wish to provide additional classroom examples for students to practise their measurements. Some students may require additional coaching to work with the scale values given. Reviewing Example 1 may assist students in their attempts to begin. • Some students may need prompting for #7. To assist them in setting up their ratio, ask them what fraction of an hour 15 min represents. • Students having difficulty with #8 should be asked to verbalize their thinking. Some students may attempt to measure the trim on the window, including the semi-circle at the top. Ask these students what formulas they have used in the past that allowed them to find the distance around a circle. Prompt them to explain how they could use this to solve the problem. • If students are having difficulty using scale factors, you may wish to use an overhead and measure or estimate the dimensions of a picture, paragraph, or something “projectable.” Enlarging and measuring it may benefit the entire class.

Unit 1 Project
If students complete #12, which is related to the Unit 1 project, take the opportunity to access how their understanding of the chapter outcomes is progressing.
	• You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers.
	• You may wish to suggest that students store their work on #12 in their project portfolio.
	• Encourage students to compare units used with a partner. Some students may require prompting relating to the unit of size in the imperial system. Ask questions such as the following:
	  – Which is the largest unit?
	  – What units can we use to measure small distances?
	• If students have placed the conversion charts in their Foldable, encourage students to use these throughout their work. The emphasis is not on how much each device stores, but on the comparison of size by calculating diameter and perimeter.
	• Where possible, bring in samples that the students can handle and accurately measure.

Assessment *as* Learning	
Create Connections Have all students complete #17 and 18.	• Encourage students to work with the dimensions they know and to cut out 2-D scale diagrams to represent the furniture, so that they can move each piece around to see how things might fit in the bedroom. • You may wish to have students share and compare their answers to #18 with a partner.

Converting Between SI and Imperial Systems

1.3

Mathematics 10, pages 36–47

Suggested Timing

180–240 min

Materials

- compact disc (CD)
- SI measuring instrument
- imperial measuring instrument

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–7 Section 1.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 7, 9, 10, 14, 17, 18
Typical	#1–8, 11, 12, 14
Extension/Enrichment	#5, 14–18

Unit Project Note that #14 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

This section in the student resource begins with a suggestion to think of songs that include measurements in the title or lyrics. You may wish to prepare for the discussion by finding a few examples yourself before the class starts. In your classroom discussion, you could consider the ratio of songs that include imperial units to those that include SI units. You could classify songs that mention measurements according to genre or decade to see if there are trends. You could see which student has the highest number or highest proportion of songs mentioning measurement on his or her personal listening device. As most students identify strongly with music, this is likely to be a lively discussion, one that will take as much time as you are willing to devote to it.

The student resource suggests that students determine SI measurements comparable to the imperial measurements in the songs. This is an excellent time to practise estimation and mental mathematics, as well as to revisit the personal referents students established in the prior two sections. In addition, rewriting song titles or lyrics by replacing measurements with your students' personal referents is certain to create some humour in your classroom.

Investigate Relationships Between SI and Imperial Measurements

Unit Project

Have students complete this investigation individually or in groups of two to three students. During this investigation, which is part of the Unit 1 project, students will need to use measuring instruments, such as rulers, to measure the diameter of a CD. Some students may find it challenging measuring the diameter without knowing where the centre of the CD is located. You may wish to discuss some strategies for finding the diameter of a circle in this situation.

Students are asked to determine the diameter of several sizes of vinyl records by converting from imperial units to SI units. Explain to students the difference between proportional reasoning and unit analysis in unit conversions. Remind students that in proportional reasoning, an equation is used to solve for an unknown. In unit analysis, an expression is created to eliminate the initial unit and work toward the desired new unit.

Students also determine the circumferences and compare the sizes of the four recording devices. After students have determined the circumference of a CD, encourage them to check their calculation by measuring the circumference, to the nearest millimetre. Students may choose to use a measuring tape or a piece of string.

Students compare the amount of music stored on a CD and a vinyl record to the size of the device. Some questions to consider include the following:

- How many songs are on a typical CD? vinyl record?
- How many minutes of music can a typical CD or vinyl record play?
- What measurements could you compare on a CD and vinyl record?
- What are some advantages to having small-sized music storage devices? What are some disadvantages?

Meeting Student Needs

- Some students may benefit from tracing the outline of a CD onto paper and then measuring the diameter.
- You may wish to have students design and measure a pattern for a dance outfit using both imperial and SI measurement systems. Have students explain which system they prefer to work with and why. Can they think of a situation where they might prefer to use the other measurement system?
- Some students may benefit from asking their grandparents or elders which system they prefer to use and why. How did they feel in the 1970s when Canada transitioned to the SI system? You may wish to invite a Community Elder or one student's grandparents to share their experience.

ELL

- Prior to CDs, long-play records (LPs) were used for storing music.

Enrichment

- Have students develop methods of converting (approximately) between SI and imperial using mnemonics.

Gifted

- Ask students to research the connection between 300 000 000 m/s and 186 000 mi/s. What travels that quickly?

Answers

Investigate Relationships Between SI and Imperial Measurements

1. a) 12 cm. One small fingernail is approximately equal to 1 cm. Count the number of fingernails along the radius of a CD to be 6. Then, multiply by two. The diameter measures approximately 12 fingernails, which is approximately 12 cm.

b) The diameter of a CD measures approximately $4\frac{1}{4}$ paper clips, which is approximately $4\frac{1}{4}$ in.

2. 120 mm

3. 30.48 cm, 25.4 cm, 17.78 cm

4. a) CD: 37.7 cm, vinyl records: 95.76 cm, 79.80 cm, 55.86 cm

b) Example: The CD has the smallest circumference of the four recording devices. The LP has a circumference that is about double the circumference of the 45 record.

5. a) Example: CD

b) Example: Determine the ratio of the number of minutes of music stored on a device to the area of one side of the device. Example: a CD might store 0.6 min per cm^2 or 3.9 min per $in.^2$.

c) Example: Laser technology enables tremendous amounts of music to be stored on very small devices and to be played with high quality sound.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 1 Project Have students complete the investigation.	• Consider having students work in pairs. • Help students recall what they learned in earlier courses about determining diameter and circumference and making conversions between SI and imperial units for length. Provide coaching for students who need it. • You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers.
Assessment *as* Learning	
Reflect and Respond Listen as students discuss the amount of music held on a CD versus a vinyl record.	• Encourage students to suggest and consider many methods for comparing the amount of music and size of recording devices. • This reflection allows students to bring together the measurements they have completed in sections 1.1 and 1.2 and to draw the conclusion that bigger is not necessarily better. Some students will need to have their attention drawn to the number of songs stored on each device. Encourage them to write ratios for each to help them visualize the size of the device versus the capacity.

Link the Ideas

This section provides an opportunity to discuss the difference between exact and approximate values. The conversion given, 1 yd = 0.9144 m, is an exact value. Students can compare this, for example, with the times in this unit when they have made calculations using pi and have had to give approximate answers. You may want to have the class brainstorm some reasons that an answer would be approximate and instances where exact values exist. You could also discuss situations in which an exact value is required and those in which an approximation is expected.

This section also provides a reference for exact and approximate conversions between SI and imperial measurements.

Example 1

Encourage students to use mental mathematics to estimate the answer to this question before beginning the calculations or looking at the solution. You may want to allow different students to show their method and defend their choice of unit.

For access to more information about the research project referred to in this example, see the Web Links at the end of this section.

Example 2

Visualization is an integral part of this example. As students determine the dimensions of the mats or read the solution in the student resource, ask them, "Why is the factor 30.5 being used in the calculations for each measurement system?" This will direct their attention to the scale being used and may help explain why the length-to-width ratio is the same for the mats regardless of the measurement units used. This relationship was explored previously in the section 1.1 Investigate.

To determine the number of mats needed, the relevant information is not the area of the mats or the area of the gymnasium floor, but rather the (linear) dimensions of the gym floor and the mats. It is quite common for students to solve problems like these by comparing the area of the gym to the area of one mat and obtaining the answer by dividing. A common situation that models this problem is the tiling of a floor. Encourage students who may have encountered a similar situation to share their experiences.

To illustrate this, you could present a similar but smaller problem. Once the size of the mats has been determined to be 8' by 4', suppose that a floor measuring 20' by 10' needs to be covered. Ask students to find the relevant areas and determine the number of mats needed according to the area calculation. Then, have the students make a diagram of how they could physically lay out the mats, and count them this way. This experiment should help them see that it is the length and width of the mats and the room dimensions that matter in this problem, not the areas.

The Your Turn questions allow students to explore this concept in a different context. You may want to remind students that drawing a diagram is an important part of the problem solving process.

Example 3

Many students in grade 10 are learning to drive. A short discussion of reaction time would be appropriate. Have students brainstorm factors that affect reaction time, as well as braking time, before reading those listed in the question. You may wish to have students predict whether reaction time or braking time is affected more as speed increases and whether students expect total braking time to be directly proportional to speed. For example, do students think that braking from 100 km/h takes twice as long as braking from 50 km/h.

It will take some practice for students to learn to read the graph given in this example. Before working through the example, you may wish to have students work with a partner and determine the answers to questions like the following:

- What does the 16.7 m label indicate?
- The dot farthest to the left is labelled with 135.6 m and 6.76 s. What do each of these values represent?
- How could you determine the reaction time for a given speed?
- Why is the graph curved?

The SI calculations are quite straightforward in this example. The imperial calculations allow for students to make conversions by their preferred method.

This example also provides an opportunity to revisit personal referents. Given that driving is (or will be) a reality for much of the class, students should recognize that this is a situation in which it is important to have a way to estimate distances in the 50–150 m range.

When students have completed the Your Turn section, you may want to complete your discussion of the non-linearity of this relationship. That is, they should note that the increase in stopping distance from 110 km/h to 120 km/h is proportionally larger than the increase in speed.

Key Ideas

As you discuss the Key Ideas, you may wish to ask students to describe the consequences of using standard units in different measurement systems when solving a problem. Some real-life examples include a 767 aircraft that made an emergency landing in Gimli, MB, when it ran out of fuel, and the Mars Climate Orbiter lost by NASA because one team working on it measured in SI units while another measured in imperial units.

This may be a good time to further discuss situations in which estimating values is acceptable (for example, theoretical braking distance) compared to situations where estimating is not appropriate (installing baseboards).

Meeting Student Needs

- You may wish to have students create bookmarks from recipe cards. The bookmarks could summarize the various units of measurement in both the SI and imperial systems. This would enable students to have an easy reference chart when working through problems in this section. You would need to determine whether students will be allowed to use this bookmark for the summative assessment for the chapter.
- Example 2 will be of interest to students currently participating in a driver training program. You may wish to have an accident investigator visit your classroom to discuss some of the methods used to determine the speed at which an accident might have taken place. The presentation should include pictures of skid marks and a discussion on how skid marks are used to analyse and determine which person is at fault.
- Allow students to use a manipulative to represent the mats in Example 3 and lay them out in various patterns to fill a designated space.

ELL

- For Example 2, review the labels on the chart and make sure that students understand what is being indicated. Encourage students to make the connection between the description of the label and the measurement units in the chart.
- In Example 3, the terms *rows* and *columns* may be new to some students, although most students should be familiar with these terms from using tables and spreadsheets. Explain to students that columns run vertically side by side and rows run horizontally above and below each other. Students may find it helpful to use a diagram with labelled columns and rows.

Answers

Example 1: Your Turn

400 m = 0.4 km and 1 mi ≈ 1.609 km. So $\frac{1}{4}$ mi ≈ 0.40225 km.

Therefore, the two measurements are not equivalent. Example: The SI measurement is likely more accurate because the SI system is used internationally and might be the preferred system at an international competition, such as the Olympics.

Example 2: Your Turn

a) 16 paving stones **b)** 88 tiles

Example 3: Your Turn

a) 6 ft **b)** 55.923 mph, about 77 yd

WWW Web Link

For more information about the research done on "frozen light," go to www.mhrmath10.ca and follow the links.

To learn more about the first SI aircraft in Canada making an emergency landing in Gimli, MB, including specific details about the measurements and subsequent calculations, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Suggest that students write the conversions presented in section 1.3 into their Foldable for future reference. • Multiple approaches are possible in solving the problem. Ask students to verbalize what they see as being reasonable units to compare for miles and metres. Encourage students to predict which is likely more accurate and why. • Allow students to write the units in their ratios to help visualize the conversion process.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • Allow students who are having difficulty to measure and cut out a representative tile that they can then use their referent on to make predictions. They may also wish to mark off an area of 1 square yard to help them visualize the placement. Although it would not be practical to do this for every real-life scenario, it is important that students be provided opportunities initially to make these comparisons and engage in these hands-on approaches. They are converting between two systems of linear measurement, one of which is new to them. Measuring and cutting out the two areas allows students to strengthen their visual images of what is being compared. • Allow students to work through the example by laying the mats in the other direction, i.e., horizontally. Encourage students to discuss their results with a classmate.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • You may wish to work through Example 3 as a group and then have individuals complete the Your Turn. • Encourage students to write out all the values with the units from the diagram provided in stopping distances before beginning their conversions. • You may wish to provide the class with a scenario such as, "You are travelling at 70 mph and a deer crosses the road. You have 4 feet to stop. Will you be able to stop in time?" This would provide students an additional opportunity to demonstrate their understanding of the chart as well as conversions.

Check Your Understanding

Practise

Students must use an SI ruler to complete #4.

Question #6 involves the use of calipers. Some students may benefit from a quick discussion of this measuring device.

Apply

For #7, students are given data in imperial units and required to solve the problem and give final answers in SI units.

Did You Know?

Point out that in Iniktitut, *komatik* means dog sled. It is pronounced ko-'ma-tik. A Web Link with audio of several Inuit terms is referenced at the end of this section.

For #8, you may wish to have students work in pairs, with one student converting the distances to imperial units and the other student converting to SI units. Then, have students compare and discuss their answers.

Question #9 has students discuss their methods of conversion with a classmate. Students should justify their preferences. Also, they should identify whether they prefer the same method for each given conversion or if their preferences vary. This may also be a good time to discuss exact versus approximate values again.

For #11 students need to choose a unit in order to make a comparison.

In #13, students need to determine the perimeter of a composite object with curved sides and discuss their steps with a classmate. Then, students identify and describe what they believe to be the easiest way to solve a similar problem.

Extend

Question #15 gives students additional experience in determining the number of small objects needed to complete a particular task.

For #16, students work with an unfamiliar formula and the measuring and estimating skills they have acquired in this chapter to find the area under a curved arch. You may wish to have students discuss the pros and cons of using the formula for imperial units

Create Connections

Question #17 provides an opportunity for students to look back and summarize their learning.

You may want to allow classmates to compare their answers to #18 to see which units were chosen and whether the factors are consistent.

Unit Project

For #14, students compare the storage on an MP3 player to that on an LP. The size of this ratio is likely to make students question the answers they obtain.

You may wish to have a class discussion about why it might be practical to have small devices with great storage capacities.

Meeting Student Needs

- Provide **BLM 1–7 Section 1.3 Extra Practice** to students who would benefit from more practice.
- For #4, you may wish to have floor plans of cottages, homes, bedrooms, etc., available. Students could be encouraged to visualize the placement of various pieces of furniture, and they could discuss whether the rooms would be adequate in size. This could serve as a project for those students who prefer to work with a partner.
- Students sometimes have difficulty attempting to use unit analysis when converting between or among measurement systems. Allow students to convert units in intermediate steps to get them closer to the required conversion. You may wish to have students review the examples in the student resource that explain unit analysis.

Common Errors

- Some students may use diameter or circumference rather than radius when solving #12 because the question is about Earth.

R_x Emphasize to students the importance of reading a problem carefully and making sure they understand what they are being asked to find.

- Some students may convert the harness length in #7 to an inappropriate unit in SI.

R_x Once they have completed their conversion, ask students to explain their rationale for the unit they used. Provide students with a similar scenario, such as a tug of war rope, the length of their classroom, and ask them what unit would be most appropriate.

WWW Web Link

For audio files that provide pronunciation of common Inuit words, including *quimmiq*, go to www.mhrmath10.ca and follow the links.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1–4, 6, 7, 9, and 10. Students who have no problems with these questions can go on to the remaining questions.</td><td>• Questions #1 to 4 provide students the opportunities to convert between measurement systems.
• Prompt students to determine which of the measurements given in the floor plan for #4 will assist them in determining the scale. Encourage students to write the units in their ratios when completing the conversion so it is clear what they are solving for.
• For #6, ask students to identify whether the ruler represents SI or imperial units before reading the values. Some students may need prompting to help them determine the fractional divisions.
• It is important that all students be provided with ample opportunities to visualize equivalent linear measurements in SI and imperial systems to assist students in making mental connections while learning the imperial system.
• Question #10 provides students a real-world calculation of distances. Prompt students through the conversion. You may wish to provide an additional question to check for understanding such as 15 km versus 9 miles. If students are struggling with number calculations, keep the values smaller because you are checking for understanding in conversions and not for the students’ ability to manipulate larger numbers.</td></tr>
<tr><td>Unit 1 Project
If students complete #14, which is related to the Unit 1 project, take the opportunity to assess how students' understanding of the chapter outcomes is progressing.</td><td>• You may wish to provide students with BLM 1–4 Chapter 1 Unit 1 Project and have them finalize their answers.
• Question #14 provides students with a link to the measurements they have been making and the reality of how storage devices for music have changed over time.
• Assist struggling learners by asking them to verbalize and demonstrate how they will determine the number of LPs needed to store 20 000 songs. Prompt students through how this number will be used to find the height of the stack.
• Ask the class, in general, whether they expect a smaller number for the number of required LPs or a larger number. To assist in putting their answers in perspective, challenge the class to generate some examples of easily visible heights that would model their LP stack.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have students complete #17 and 18.</td><td>• Both questions #17 and 18 allow students to generate their own examples and explain their thinking related to measurement and conversion. You may wish to suggest that students write their responses into their Foldable. Encourage them to use examples in #17 that are common or often used measurements as they will likely be using these types of measurements in subsequent chapters. Encourage students to write out as many examples as they feel are necessary to help prompt their own thinking in other chapters.</td></tr>
</table>

1 Chapter 1 Review

***Mathematics 10*, pages 48–50**

Suggested Timing

100–120 min

Materials

- an object with a curved surface, such as a can
- SI ruler or measuring tape
- imperial ruler or measuring tape

Blackline Masters

BLM 1–5 Section 1.1 Extra Practice
BLM 1–6 Section 1.2 Extra Practice
BLM 1–7 Section 1.3 Extra Practice

Planning Notes

The review section should serve mainly as a means for students to assess their understanding of the concepts and Key Ideas in the chapter. Before beginning the review, students could benefit from revisiting their discussion in the chapter opener. If you began a K-W-L chart as a class, students should review the "Want to know" column and complete the "what did you Learn" column. As well, you could have them revisit the Key Terms to make sure that students understand them.

Another possible source of review before students begin the review exercises is the chapter Foldable. Students can use it to make sure that they are familiar with the terms, concepts, and tools that they have used in this chapter.

The review exercises are grouped according to the sections in the chapter. As students work independently through the review, this gives them direction about where to look if they encounter difficulty. The skills and concepts in the review are the same as in the lessons and practice problems, so that students may look back to a similar problem if they have difficulty.

If students make a note of their difficulties, then these difficulties can be addressed when the review is complete. Consider having students engage in peer teaching, to assist each other with review questions, for example.

Meeting Student Needs

- Have students refer to the posted student learning outcomes as they work through the questions in the review. They will be able to self-assess and make knowledgeable decisions about the sections that will require more review time.
- At the end of the review, have each student rate their progress for each objective (self-assess their progress). You could create a scale of 1 to 5 or you might wish to use pictures that students can circle to indicate what they believe they know for each outcome. For example, a fist with thumb up indicates "I understand," a fist showing no thumb could indicate "I am almost there," and a fist with thumb down could indicate "I do not understand." Encourage students to spend extra time reviewing those areas in which they do not have a "thumbs up" indication.
- Map reading is an important skill. Allow students enough time to work on #13. Some students may benefit from cutting a piece of string that is the distance on the map that represents 57 km and then placing that string along the highway.
- Students who require more practice on a particular topic may refer to **BLM 1–5 Section 1.1 Extra Practice**, **BLM 1–6 Section 1.2 Extra Practice**, and **BLM 1–7 Section 1.3 Extra Practice**.
- After answering #7, some students may have difficulty finding *another* way to determine the answer. Encourage students to think of what the readings on the ruler mean and what all of the markings indicate. You may wish to pair students up so that they can talk through this question together.

ELL

- Students may need to be reminded of how to write a fraction in lowest terms.
- When working on #8 you may need to explain to students what *cross-section* means. The strips of lumber are quarter round trim. Some students may have seen or worked with wood trim.

Enrichment

- Have students create a list of ten extreme measurements (such as the speed of light) in SI and imperial.

Gifted

- Instruct students to research the theory of relativity and especially the way in which speed affects the dimensions of objects, particularly length contraction.

Web Link

Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 1 Review The Chapter 1 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

1 Chapter 1 Practice Test

Mathematics 10, pages 51–53

Suggested Timing

50–60 min

Materials

- SI ruler or measuring tape
- imperial ruler or measuring tape

Blackline Masters

BLM 1–8 Chapter 1 Test

Planning Notes

Before students begin the Multiple Choice questions, you may want to discuss some test-taking strategies. For example, students should try to highlight key words in the stem, answer the questions without looking at the choices, and eliminate any multiple choice answers that are unreasonable. They should also try to estimate answers to the questions where possible, as they have in many situations in this chapter.

For the Short Answer section, encourage students to show all of their reasoning and calculations. In particular, students should show the conversion factors they are using, as well as the steps in the method they choose to make conversions.

In the Extended Response section, students need to show their work and give explanations. In #9, there are many possible correct procedures. Students should be aware that the explanation of the procedure they choose is crucial to the answer. You may want them to answer as if they were explaining the solution to a friend that was absent from class that day. If time allows, you may wish to have students demonstrate or show their solutions to the class, to emphasize the breadth of possible approaches.

The practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1, 6, 8, and 10.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	1.2	Investigate, Example 3	✓ use a referent to estimate a distance in imperial units
#2	1.3	Example 1	✓ verify a conversion between SI and imperial units for length
#3	1.1	Example 1	✓ estimate and compare distances in SI units
#4	1.3	Example 1	✓ convert and compare distances between SI and imperial linear units
#5	1.2	Example 1	✓ measure distances to the nearest fraction of an inch
#6	1.1 1.2 1.3	Example 1 Investigate Link the Ideas	✓ provide referents for linear measurements ✓ convert within and between SI and imperial units
#7	1.1	Investigate, Example 1	✓ visualize and estimate dimensions in SI units
#8	1.3	Link the Ideas	✓ solve a problem involving the conversion of units within and between SI and imperial systems
#9	1.1 1.2	Example 3 Example 1	✓ describe and explain a strategy for determining a linear measurement
#10	1.3	Example 2	✓ solve a problem involving the conversion of units between SI and imperial systems and justify the reasonableness of the solution

Answers

Chapter 1 Practice Test

1. D **2.** B **3.** B **4.** C **5.** B

6. **a)** Example: arm span, length of stride; metre, yard, foot

b) Example: centimetre, inch, 1 in. ≈ 2.5 cm

7.

Estimate: 7 cm by 15 cm

8. 7 mm, 8 mm, 9 mm

9. Measure the height of the rectangular part of the archway. Set up a ratio comparing the horizontal and vertical measurements using the given width of 6 ft. Determine the circumference of the semicircle using the given diameter. Add the width of the archway for the bottom, two times the height for the sides, and the circumference of the semicircle. The perimeter of the archway is 40 ft 10 in.

10. **a)** 8

b) $3\frac{3}{4}$ yd

c) $403\frac{3}{4}$ yd

d) 32 yd. The answer seems reasonable because it is less than the distance run using yards and there is one less line to run to and back.

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 1 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Have students use their responses on the practice test and work they completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 1 Test After students complete the practice test, you may wish to use **BLM 1–8 Chapter 1 Test** as a summative assessment.	• Consider allowing students to use their Foldable.

Surface Area and Volume

2

General Outcome

Develop spatial sense and proportional reasoning.

Specific Outcomes

M1 Solve problems that involve linear measurement, using:
- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:
- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
2.1	✓ solve problems that involve area and volume units within SI and imperial systems
	✓ use mental math to judge the reasonableness of a solution to a problem
2.2	✓ solve problems involving the surface area of three-dimensional objects
	✓ find an unknown dimension of a three-dimensional object given its surface area
2.3	✓ solve problems involving the volume of three-dimensional objects
	✓ find an unknown dimension of a three-dimensional object given its volume

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 2–1 Chapter 2 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 54 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal to explain what they personally know about area and volume. You might provide the following prompts: • What units of area and volume do you use most often, in SI and/or imperial systems? • In what instances in your life did you need to know volume or surface area? • Where have you encountered composite objects? Why might the surface area and/or volume of a composite object be important, and to whom?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 2–2 Chapter 2 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 2 Foldable As students work on each section in Chapter 2, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter.
Assessment *for* Learning	
BLM 2–3 Chapter 2 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 2 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 30–40 min (TR page 39)	Students should be familiar with • surface area of right rectangular prisms and right cylinders • volume of right rectangular prisms and right cylinders • SI and imperial units of length, area, and volume • square roots		BLM 2–1 Chapter 2 Self-Assessment BLM 2–2 Chapter 2 Prerequisite Skills BLM 2–4 Chapter 2 Foldable BLM 2–5 Chapter 2 Unit 1 Project BLM U1–2 Unit 1 Project Checklist		TR page 38 Chapter 2 Foldable, TR page 40	TR page 38	
2.1 Units of Area and Volume • 100–120 min (TR page 41)	Students should be familiar with • SI units of length, area, and volume • imperial units of length, area, and volume • area of rectangles and circles • ratios • solving proportions • multiplying fractions	• rulers or tape measures in imperial units	BLM 2–3 Chapter 2 Warm-Up BLM 2–5 Chapter 2 Unit 1 Project BLM 2–6 Section 2.1 Extra Practice	**Essential:** #1a), b), 2a), b), 4a), 5, 6, 8, 14–16 **Typical:** #1a), b), 2a), b), 3, 4a), 5–8, 11, 12, 14–16 **Extension/Enrichment:** #6, 8, 11–16	TR pages 44, 50 Chapter 2 Foldable, TR page 38	TR pages 44, 47, 50	
2.2 Surface Area • 100–120 min (TR page 51)	Students should be familiar with • square roots • area of circles, triangles, and rectangles • relationship between radius, diameter, and circumference • Pythagorean relationship • surface area of right rectangular prisms and right cylinders • nets for 3-D objects	• right cones • spheres • right pyramids • right cylinders	BLM 2–3 Chapter 2 Warm-Up BLM 2–5 Chapter 2 Unit 1 Project BLM 2–7 Investigate Surface Area of Three-Dimensional Objects BLM 2–8 Section 2.2 Extra Practice TM 2–1 How to Do Page 79 #18 Using TI-Nspire™ TM 2–2 How to Do Page 79 #18 Using Microsoft® Excel	**Essential:** #1–4, 6, 8, 9, 19, 20 **Typical:** #1–3, 5, 6, 8–10, 12–14, 18–20 **Extension/Enrichment:** #3, 8, 10–12, 14–20	TR pages 53, 62 Chapter 2 Foldable, TR page 38	TR pages 58, 62	
2.3 Volume • 100–120 min (TR page 63)	Students should be familiar with • volume of right rectangular prisms and right cylinders	• conical cup • paper • scissors • tape or glue • sand, rice, or popcorn	BLM 2–3 Chapter 2 Warm-Up BLM 2–5 Chapter 2 Unit 1 Project BLM 2–9 Section 2.3 Extra Practice TM 2–3 How to Do Page 91 #19 Using TI-Nspire™ TM 2–4 How to Do Page 91 #19 Using Microsoft® Excel	**Essential:** #1, 2, 4–9, 15, 16, 21 **Typical:** #1–5, 7–11, 13–16, 20, 21 **Extension/Enrichment:** #8, 11–21	TR pages 64, 70 Chapter 2 Foldable, TR page 38	TR pages 68, 70	
Chapter 2 Review • 60–90 min (TR page 71)			BLM 2–6 Section 2.1 Extra Practice BLM 2–8 Section 2.2 Extra Practice BLM 2–9 Section 2.3 Extra Practice	Have students do at least one question related to each concept, skill, or process that has been giving them trouble.	Chapter 2 Foldable, TR page 38	TR page 71	
Chapter 2 Practice Test • 40–50 min (TR page 72)			BLM 2–10 Chapter 2 Test BLM 2–11 Chapter 2 BLM Answers	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1–7, 9–12	TR page 73		TR page 73 BLM 2–10 Chapter 2 Test

Surface Area and Volume

Opener

Mathematics 10, pages 54–55

Suggested Timing

30–40 min

Blackline Masters

BLM 2–2 Chapter 2 Prerequisite Skills
BLM 2–4 Chapter 2 Foldable
BLM 2–5 Chapter 2 Unit 1 Project
BLM U1–2 Unit 1 Project Checklist

Key Terms

surface area	volume
cylinder	prism
lateral area	cone
slant height	sphere
pyramid	apex

What's Ahead

In this chapter, students learn about working with SI and imperial measurements to determine surface area and volume of 3-D objects. They begin by converting within and between SI and imperial measurements before calculating areas and volumes. They follow up by learning how to determine the surface area and volume of right cones, right pyramids, spheres, right cylinders, right prisms, and composite objects, and determine square roots and cube roots of numbers. Throughout the chapter, they solve problems involving surface area and volume.

Planning Notes

Before beginning this chapter, ask students to bring in their favourite CDs, as well as cases for their PDAs, MP3 players, and cell phones. During the first class, have students share what they like about the designs on the various cases. Once students have shared their ideas, challenge them to consider how mathematics might have been used in the development of these designs. Brainstorm ideas as a class.

Students may mention using surface area calculations to determine the amount of material needed to cover each item and volume calculations to determine the amount a case might hold.

Explain that the chapter is about converting within and between SI and imperial measurement systems, and determining surface areas and volumes of 3-D objects. Tell students that they will rely on their existing knowledge and skills of linear conversions, surface area, and volume, as well as their ability to use proportional reasoning.

Direct students to the information about industrial designers and what they do. Ask them about careers in industrial design that they are familiar with (e.g., graphic designer, furniture designer, pop bottle designer). Have students discuss what they know about the work that these designers do, and how math and surface area and volume are related to the work. You might ask which measurement systems industrial designers use in their various tasks. Which tasks might involve both SI and imperial measurement systems?

Unit Project

You might take the opportunity to discuss the Unit 1 project described in the Unit 1 opener on TR page 2. Throughout the chapter, there are individual questions for the unit project. These questions are not mandatory but are recommended because they provide some of the research needed for the final report for the Unit 1 project assignment.

The Unit 1 project is integrated throughout the chapter. You will find questions related to the project in the section 2.1 Investigate and Check Your Understanding sections.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 55 and how it might be used to summarize Chapter 2. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

As students progress through the chapter, provide time for them to keep track of what they need to work on. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 2–2 Chapter 2 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Some students may find it useful to use **BLM 2–4 Chapter 2 Foldable**. You can photocopy and enlarge the first page of the master onto 11 × 17 paper and distribute copies. The second and third pages of the master do not need to be enlarged. For page 1 of the master, have students fold the master so that there is a right and a left flap and a centre panel. Have students label the outside of each flap as shown in Step 1 on page 55 in the student resource. On the centre panel, consider having students staple or tape a copy of **BLM U1–2 Unit 1 Project Checklist**, which provides a list of all the related questions for the Unit 1 project. Have students use pages 2 and 3 of the master for Step 2 and Step 3 respectively. The conversion charts and formulas are included as a reference tool for students.
- Some students may find it useful to keep a taped or oral summary of what they are learning. Others may work best with a keyboarded version in a software of their choice.
- Invite an industrial designer to talk to the class about design and how math skills related to surface area and volume are used.
- To reinforce the Key Terms, post sheets of paper around the room, each labelled with one Key Term. Have student pairs respond to the following prompts for each term: definition, example in daily life, and facts. Have student pairs move around the room and use diagrams and words to contribute to each Key Term. Once each pair has contributed, have students review all the entries. As a class, debrief each sheet to conclude the activity. Leave the sheets on display throughout the chapter.
- Throughout the chapter, encourage students to use strategies such as making models and drawing diagrams to help them move from the concrete to the abstract level. Encourage students to "say their thinking." Look and listen for unorthodox, yet mathematically correct procedures as much as you observe for correcting mathematically unsound procedures.
- Consider allowing students to work with a partner on all Unit 1 project questions.
- Some students may benefit from completing all unit project questions.
- **BLM 2–5 Chapter 2 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

ELL

- Encourage students to create their own vocabulary dictionary for the Key Terms using written descriptions and diagrams.
- Explain that a *jewel case* is a CD storage case.

Enrichment

- Challenge students to consider the construction of buildings in the past (e.g., cathedrals, pyramids, long houses) and how builders in the past must have created ways to measure accurately. Ask them to speculate how this was accomplished without the benefit of current measurement systems and measuring tools. Students may enjoy researching the history of measurement units. You might ask them to reflect on the difficulties of using body parts as referents. Have students present their findings to the class.

Career Connection

Use the photograph and the text to highlight a career in industrial design. Invite students to research training and qualifications, employment opportunities, and career outlook. You might have them address how math concepts and skills are important in what industrial designers do. They may find the related Web Link that follows helpful.

Web Link

For more information about industrial design, go to www.mhrmath10.ca and follow the links.

Units of Area and Volume

2.1

***Mathematics 10*, pages 56–65**

Suggested Timing

100–120 min

Materials

- rulers or tape measures in imperial units

Blackline Masters

BLM 2–3 Chapter 2 Warm-Up
BLM 2–5 Chapter 2 Unit 1 Project
BLM 2–6 Section 2.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- mental math strategies

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), 2a), b), 4a), 5, 6, 8, 14–16
Typical	#1a), b), 2a), b), 3, 4a), 5–8, 11, 12, 14–16
Extension/Enrichment	#6, 8, 11–16

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce prerequisite skills needed for this section.

The day before the start of this section, you may wish to ask students to bring different methods of storing music to the next class. If possible, collect some older media from the library or your own personal collection. Try to have records of different sizes, as well as different types of music tapes. Encourage students to bring the most modern storage devices they have.

Start the class with a display and discussion of different music storage devices.

- What ones do students own?
- What ones have students used?
- Which ones pre-date student memories?

Encourage students to discuss their experiences with the different types of music storage devices. If you have students with music collections, encourage them to share their enthusiasm for the media of their choice.

Alternatively, start the class by accessing the material in the Web Link on page 57 and having students watch the video on how a vinyl record is made. This video could initiate a discussion on LP records. Ask students the following questions:

- Have you ever seen a vinyl record?
- Do your parents have any vinyl records stored away?
- Do you think that anyone still listens to vinyl records?
- What other early forms of music storage do you know about?

Lead from these discussions into the Investigate.

Investigate Units of Area

Unit Project

During this investigation, which is part of the unit project, students determine

- the lateral surface area of a cylindrical disk used to store music
- the circular area of both sides of an LP
- the jacket area of a record of their choice

In this Investigate, students use their knowledge of area to explore technologies for storing recorded music. Advancements in technology have resulted in several improvements in music storage methods. For example, early wax cylinders could hold two minutes of recorded sounds; modern MP3s and cell phones have a capacity to store thousands of songs.

Have students work with a partner or in small groups, possibly with three students in each group. Initially, students will determine the area of the wax cylinders used in the 1800s and the area of the more modern LP record. In the student resource, the wax cylinder dimensions are given in imperial units; LP record dimensions are provided in both imperial and SI units. You may wish to have students convert the wax cylinder dimensions to SI units and then answer #1 and 2 in both imperial and SI units. This will allow students to compare the amount of music storage on the wax cylinder and the LP record.

The wax cylinder pictured with #1 is one of the earliest recording devices. From studying the photo, have students visualize what this cylinder might have looked like. Ask the following questions:
- What dimensions do these wax cylinders have?
- What familiar item has a similar size?

Suggest that students make a net for the cylinder and then put together a scale model. This will give them a better idea of the size. Alternatively, they could draw a labelled sketch. Ask the following questions:
- On what part of this cylinder was the music recorded?
- What information do you need to determine the area of this part of the cylinder?
- How can you use that information to determine the area of this part of the cylinder?

Once students have determined the outside surface area of the cylinder, have them compare this area to the number of minutes of recorded music the cylinder holds.

For #2 students determine the circular area of both sides of an LP. Ask the following questions:
- How can you describe the recorded area using familiar shapes?
- What dimensions do you need to determine this area?
- How can your prior knowledge of areas help you determine the recorded area?

Students then determine the ratio of the circular area to the number of minutes of music. There are several things to consider and discuss when working on this part of the question. You may wish to discuss these things ahead of time or let students make assumptions and later discuss what assumptions groups made when answering #2b). Consider the following discussion points:
- Do both sides of the LP hold the same amount of music?
- Is the entire surface area of the record used to store music?
  - The visuals with #3 indicate that the centre of a record is not used to store music. There is a hole for placing the record on the record player and place for a label. Labels have a diameter of approximately 10 cm.
  - Students who are familiar with LP records may point out that the outside area of each record has no grooves for storing music, nor does the inside area closest to the label. Will they consider this area as part of the recording area or not? If not, what approximate diameter will they assess for this part of the record?
  - In their calculations, how will they deal with any parts of the record surface that do not hold recorded music?)

For #3, have students refer to the diagram of the different records and their diameters. Discuss the meaning of rpm, which refers to the number of revolutions per minute. There are different ways to organize answering this part of the Investigate.
- You may wish to have each group choose one size and do the related calculations. If you do this, have students provide the answers in both imperial and SI units. Ask students what the relationship is between the linear measurement they have used in SI and imperial systems.
- If you have groups of three, you may wish to have students in each group divide out the record sizes so that the group handles each size. Ask how the size of the record affects the jacket design.

Have students complete the Reflect and Respond questions within their groups and then have the groups report to the whole class. You may wish to provide some or all of the following information during class discussion.
- Some music enthusiasts prefer the sound from a vinyl record to that of a digital recording.
- There may also be some genres of music that are better suited to the visual allure of a vinyl record.
- Record companies are concerned with the amount of downloaded music taken from the internet. LP records are one way of curbing this revenue loss.
- LP record jackets have a larger surface area than the outside of a CD jewel case. This allows more room for advertising such as artwork or graphics and information about the music group.
- One advantage of the wax cylinder was that the owner could have it wiped clean and a new recording placed on it. When the sound of the recording diminished, they simply made a new recording.

Technology has played a huge part in the change of storage devices. The drive to have music storage technology as small as possible has made this a big business. In the brainstorming for #4b), encourage students to include discussions of the latest music devices, the size of each device, how difficult or easy each is to use, and the number of songs that can be stored.

Meeting Student Needs

- Help students recall how to determine the area of a cylinder.
- Help students recall the formula for the area of a circle.

ELL

- Have actual sample CDs and records available. Use samples of each record to clarify the meaning of LP, 78, and 45. Explain that the designations LP ($33\frac{1}{3}$), 78, and 45 refer to a record's rotational speed in revolutions per minute (rpm).

Enrichment

- Challenge students to compare the density of storage among the 45, 78, and LP. In particular, you can invite them to consider how the speed of rotation of each type of vinyl record might affect the amount of storage.
- Have interested students research the type of storage and playback devices currently being developed and report to the class. They may wish to report on the minutes of recording per surface area and compare their findings to the ratios for the wax cylinder and LP record.
- Machines have been developed by Dr. Ian Foulds of the University of Victoria that are smaller than the diameter of a human hair and have the potential of being inserted into the human body via a needle. Ask students to research the ways in which very small objects can be measured accurately and have them report on the units used to work with such objects.

Common Errors

- Students may confuse linear measurement units with area units.

R_x Emphasize that area is always measured in square units, such as square millimetres, square centimetres, square metres, and square kilometres that can cover a surface. Some students may need to see this visually. Provide them with centimetre grid paper. Have them draw a rectangle four centimetres by eight centimetres. Ask them to count the number of squares in the rectangle. When they count 32, model what they just did:

$$\begin{aligned}\text{Area of rectangle} &= (4\text{ cm})(8\text{ cm})\\ &= 32\text{ cm}^2\end{aligned}$$

Discuss how the 32 cm^2 refers to the number of squares within the rectangle. This is math terminology for the amount of space the rectangle covers, or area. Have students use a think aloud to explain this idea using a rectangle with different dimensions.

- Students may struggle with conversions within and between SI and imperial systems of measurement.

R_x Encourage students to make a chart showing common linear and area conversions within SI and imperial systems. For example,

$$\begin{aligned}12\text{ in.} = 1\text{ ft so } 1\text{ ft}^2 &= (12\text{ in.})(12\text{ in.})\\ &= 144\text{ in.}^2\end{aligned}$$

Include square inches to square feet to square yards and vice versa. Suggest that they use visuals to help make each conversion clear.

Answers

Investigate Units of Area

1. a) SA = 28.27 in.^{2} or 182.4 cm^2

b) 0.071 min of music/in.^{2} or 0.011 min of music/cm^2

2. a) Example: The imperial diameter of the LP was picked up from the visual with #3 in the student resource. All area answers are rounded to hundredths. The circular area of an LP would be 1413.72 cm^2 or 226.19 in.^{2}. The diameter of a label would be approximately 10 cm or 3.94 in. The circular area of the label is 157.08 cm^2 or 24.38 in.^{2}. Therefore, the recording area of an LP would be 1256.64 cm^2 or 201.81 in.^{2}.

b) 0.036 min of music/cm^2 or 0.223 min of music/in.^{2}

3. a) Example: Assuming a square with side lengths the same size as the record's diameter

Speed (RPM)	Diameter (in.)	Jacket Area (in.^{2})	Jacket Area (cm^2)
33-1/3	12	144	929.0
78	10	100	645.2
45	7	49	316.2

b) Look for a design that reflects the artist(s) and songs, including visuals, graphics, and text.

Answers

4. a) Example:

Type	Advantages	Disadvantages
Wax Cylinder	• re-recordable	• poor sound reproduction • sensitive to ambient temperature changes • loss of song detail over time • difficult to store
Vinyl Records	• excellent sound reproduction • tolerant to ambient temperature changes • easy to store	• scratch easily • can warp in extreme heat • over time, develop random popping noises • non re-recordable

c) Look for the name of modern advances and information such as the recording time, the number of songs, and the size of the storage device.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 1 Project Have students complete the Investigate.	• Consider having students work in pairs. • Listen as students discuss how to solve the problems. As you circulate, clarify any misunderstandings. • Help students recall what they learned in earlier courses about calculating area and making conversions between linear SI and imperial units. Provide coaching to students who need it. • You may wish to provide students with **BLM 2–5 Chapter 2 Unit 1 Project**, and have them finalize their answers. • Remind students to store all project-related materials in a portfolio for that purpose.
Assessment *as* Learning	
Reflect and Respond Listen as students discuss the advantages and disadvantages of vinyl records and wax cylinders, and brainstorm advances in music storage. Encourage them to consider the size of modern storage devices compared to the amount of music they hold.	• Some students may benefit from making a model of a wax cylinder and seeing an actual LP record to help them assess the advantages and disadvantages of each. Also have them consider the characteristics of the material. What happens, for example, when wax gets warm? • Some students may benefit from comparing the size of modern storage devices to the number of minutes of recorded music they hold.

Link the Ideas

You might have students recall what they know about linear conversions between SI and imperial units of measure. How can this knowledge help them with area and volume calculations? Have students discuss their thoughts in pairs.

Ask each pair to join up with a second pair to make a group of four. Have the pairs share what they discussed in their original groups. After time for discussion, ask the following questions:

- What did you learn by sharing your ideas with another pair?
- How did this sharing allow you to modify your ideas?
- What did you learn?
- What, if any, misconceptions did you correct?

Example 1

As a class, work through both methods for calculating the area. Have students compare the two methods for solving the problem. Ask the following questions:

- Which method do you find easier? Why?
- Which method leads more naturally to a mental math approach? Explain.

- How can you use mental math to determine measurements that are in square centimetres, in square metres, and vice versa?

Have students visualize areas. Ask the following questions:
- How many 1 cm by 1 cm squares are inside a square that is 1 m by 1 m?
- How many 1 mm by 1 mm squares are inside a square that is 1 m by 1 m?
- How can you use this information to help you if you want to figure out how many square centimetres are in a square metre? square millimetres in a square metre?

Have students do the Your Turn questions. Afterward, have students work with a partner and explain how they determined the areas.

Example 2

Consider introducing the problem by having students recall the linear conversion factors that they already know. The mental math box with the solution to a) shows the relationship between a square foot and an approximation in square centimetres. It may be useful to recall and discuss other linear conversions and approximations at this time. How can students use these to determine the area of various objects?

Direct students' attention to the abbreviations for imperial units of measure in the margin. Have them note the period in *in.*, the abbreviation for inches. The period helps avoid confusion with the word *in*.

For part b), have students outline an area of the classroom that is eight feet by four feet. Discuss what area this might compare to in their home or apartment. For example, it is the same size as a sheet of panelling. Some students may have a galley kitchen or a hallway this size.

Before they see the solution, discuss with students how to solve this question. Then, have them work through the solution. As you do, either have students model to each other what they understand by talking through what they are thinking or provide guiding questions such as the following:
- How can you convert eight feet to centimetres?
- How can you convert four feet to centimetres?
- Is there another way to do this?
- If you converted four feet to centimetres first, how could you determine the length of eight feet in centimetres?
- How can you determine the area of the tile layer in centimetres?
- How can you determine the area of the tile layer in metres?

To provide a referent for a square metre, you may wish to have students measure and tape a one-metre square on the floor or wall. Have them measure the area in centimetres and post the linear length in both metres and centimetres. For example:

$$\begin{aligned}\text{One square metre} &= (1\text{ m})(1\text{ m})\\ &= (100\text{ cm})(100\text{ cm})\\ &= 10\ 000\text{ cm}^2.\end{aligned}$$

Have students model for each other how they can use this information to determine 29 728.97 cm^2 in square metres.

For Your Turn part a), prompt students to recognize that the number will be smaller going from square centimetres to square feet.

Have students use strategies of their choice to solve the Your Turn questions, then, have them discuss their work with a classmate and compare their procedures and preferences. Which procedures are more efficient?

Example 3

Introduce the problem by inviting students to consider the conversion 1 $m^3 \approx 35$ ft^3. This is likely to be surprising to them, and you may want to discuss or consider a model in order to help them integrate this fact. You might also appeal to their knowledge of linear conversions and volumes to consider a cube of length 1 m and convert each length to feet.

Before having students consider the solution, you may wish to have them use their own strategies to solve the problem. Ask the following questions:
- How did you determine the volume?
- How else could you determine the volume?
- When did you convert from SI to imperial? Why did you do it at this point?

Have students do the Your Turn questions using the method of their choice and then, explain their method to a classmate.

Key Ideas

You may wish to use the following prompts to promote a discussion about what students learned:
- How were your skills with conversions between linear units important in this section?

- What other methods do you use for conversions?
- How can you use mental math when making unit conversions?

Have student pairs talk through the sample conversions within the SI and imperial systems. Have them verbalize their understanding of these conversions. How do they work?

Have students use index cards to prepare their own summary of the Key Ideas. Encourage them to record an example of how they would convert

- an area in metres squared to square centimetres
- an area in SI linear units to imperial units

Make sure that students use linear conversions only when converting from one measurement system to another. They need to do these conversions *before* calculating area or volume.

Meeting Student Needs

- The worked examples assume that students are quite familiar with unit conversions. Some students may benefit from a flash card activity to review SI and imperial unit conversions. Pair students and have them use index cards to make up four conversion questions each: two with SI units and two with imperial units. Have them write the correct answer on the back of each card. Then, have students flash each other.
- For Example 1, help students recall the area formula for a square and how answers are expressed in square units.
- For Example 3, help students recall the volume formula for a right rectangular prism and how answers are expressed in cubic units.
- Some may have difficulty visualizing the comparison between SI and imperial units. Provide them with a tape measure, which has both SI and imperial units, to visually compare centimetres and inches, centimetres and feet, and metres and yards. You may wish to display coloured-ribbon representations of each of these lengths. Use one colour for SI and another for imperial. Label each length with its respective unit. Consider including the conversion to the alternative unit.

ELL

- Explain that a *mural mosaic* is a design constructed from smaller pieces and is displayed on a wall. You may wish to provide a visual of an example, such as one of the murals in Chemainus, BC. Visuals of these are readily available online or through BC Tourism.

Common Errors

- Students may confuse units of area and volume.

R_x Remind students that area is always measured in square units. Volume is always measured in cubic units since volume involves three dimensions.

To help students visualize the difference between area and volume, you may wish to display a model of a large cube made from 1000 centimetre cubes. Provide the following label on the top of the cube:

Area = (10 units)(10 units)
= 100 units squared

Provide the following label beside the cube:

Volume = (10 units)(10 units)(10 units)
= 1000 units cubed

Use a think aloud to show why the model might be useful. Have students do their own think alouds to explain their understanding of the difference between area and volume.

WWW Web Link

To view enlarged details of each block on the mural mosaic for Example 1, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) 42 500 cm^2 **b)** 0.00125 m^2

Example 2: Your Turn

a) 1.076 ft^2 **b)** 15 480 mm^2

Example 3: Your Turn

7.32 $in.^3$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Help students recall that going from a larger unit of measure to a smaller unit results in a larger number and vice versa. • Have students summarize how to express metres in centimetres and millimetres in metres. Ask the following questions: – How is expressing millimetres in metres related to expressing centimetres in metres? (Encourage students to verbalize their understanding of the base 10 used for the SI system. It may be helpful for them to use a mnemonic such as that there are 100 cents in a dollar, so there are 100 centimetres in a metre. *Cent* refers to 100. Similarly, *milli* refers to 1000. There are 1000 mm in 1 m.) – How might expressing square millimetres in square metres relate to expressing square centimetres in square metres? (Example: They square the unit being used.) – Which do you find easier? Why? • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.
Example 2 Have students do the Your Turn related to Example 2.	• It may benefit some students to cut a piece of cardboard measuring 1 ft^2, and then to label it with the following information: 1 ft^2 = 144 $in.^2$ = 929.03 cm^2 Post this as a handy referent. You may wish to have students display other referents as the need arises, for example, for 1 ft^2 and 1 m^2. • Give students a problem similar to Example 2 to solve before trying the Your Turn. Allow them to work with a partner and talk through their thinking. • Coach students who get confused when expressing measurements in different SI units by having them record units along with numbers in the proportion.
Example 3 Have students do the Your Turn related to Example 3.	• Give students a problem similar to Example 3 to solve before trying the Your Turn. Allow them to work with a partner and talk through their thinking. • Allow students to use the method they are most comfortable with. • It may benefit some students to build a metre cube, and then to label this with the following information: 1 m^3 = 10 000 cm^3 = (3.281 ft)(3.281 ft)(3.281 ft) ≈ 35 ft^3 Display this as a handy referent. You may wish to have students build and display other referents as the need arises, for example, for 1 ft^3 and 1 yd^3.

Check Your Understanding

Practise

These questions should be accessible to most students. For #1 and 2, remind students to note the units asked for.

Discuss how to use the conversion chart on page 61. Ask students how they can use the chart to convert 5 cm to in. and 5 in. to cm. Have them explain the difference between these two conversions. Then, ask which results in a larger number: going from in. to cm, or going from cm to in.

For #2, have the class brainstorm the different dimensions that could make an area 30 feet square. Divide the class into groups of four and have individual members of each group use a different set of dimensions for their work. Have students make the necessary conversions and show the area of their shapes in metres squared. Ask them what they notice about their answers and what this suggests about their shapes.

You may then wish to have students choose a partner and do parts b) and c) using a similar technique. Students discuss what dimensions could make the given area, choose a different set of dimensions, and use that set to determine the area in the units given.

For #3, refer students to the Did You Know? on the *Festival du Voyageur*.

For #3 to 5, discuss with students what conversions need to be made and when to make them.

Apply

It may not be necessary to assign all questions to all students. Allow students choice in determining the questions they need to do. In several questions, it would be easier for students to convert linear measurements before calculating area or volume. You may wish to discuss the best time for making these conversions, or encourage students to try the question both ways and decide which way is more efficient. Remind students to read the problem carefully and note the given units of measure and the units of measure that are asked for.

The photo with #6 was taken in the Museum of Civilization in Hull, Quebec. As of 2008, the Quilt of Belonging did not have a permanent residence because of its large size and the fact that it is still a work in progress.

There are a variety of ways to solve #6 and 7. You might have students compare their method with that of a classmate who solved the problem in a different way.

For #8, students determine the area of a circle with a radius of 2.5 mi in SI units that are not specified. This requires students to consider the most appropriate SI units (i.e., square kilometres) before proceeding.

For #9, have students sketch a diagram to help visualize what the question is asking. Explain that this is how a tile layer would approach this problem. Discuss the need for purchasing 10% extra material (e.g., breakage).

For #10, consider having students work in pairs and compare their answers. Does the timing of conversions make a difference? Why or why not?

For #11, you may wish to use a Think/Pair/Share. Have one student of each pair consider how to determine the volume for a) and the second how to determine the volume for b). After an agreed amount of time, have them share their thoughts with each other. Encourage students to listen to and reflect back what they hear. Also ask them to consider how well the method that is being explained works. Once students are satisfied with each other's explanations, have them separately do c) and then share their method and answer. Students may wish to discuss how their methods were the same/different, and how they used mental math.

Extend

The Extend questions require students to use their problem solving skills. In #12, students solve a problem involving acres and hectares, units that may be unfamiliar. These terms are likely more familiar to students who live in rural areas.

For #13, encourage students to work in pairs that each choose the same area. Have them do their calculations separately, and then compare their answers and methods.

Ask students when they might need to use such a skill. You may wish to discuss the difference in measurements systems between the United States and Canada. Many Americans visiting Canada do not understand the SI system. They need measurements expressed in the imperial system so that they can visualize the size. Similarly, many Canadians need the measurement in SI units. Being able to communicate in both systems aids in international communication.

Create Connections

These questions allow students to communicate their understanding about SI and imperial measurements.

For #14, consider having students work in pairs or small groups to brainstorm occupations that use both imperial and SI measurements. Some examples include architects, carpenters, civil engineers, landscape designers, metal fabricators, structural engineers, and tile layers. Have them explain how SI and imperial measurements are used in each occupation. Then, have students present their own summary orally or in written form. Alternatively, you might have students interview family members or friends involved in these occupations and ask about the conversions they use. Have students report back to the class.

For #15, students describe their preferred method for convering between SI and imperial measurements. Encourage students to use an example to help with their explanation.

For #16, students explain for which departments in a grocery store they would use measurement conversions. Consider having a class discussion and then have students use the ideas as a springboard to develop their own response. Students may also wish to interview their grandparents and neighbours who were adults when Canada used the imperial system. These people may have additional ideas that students who have been brought up in the SI system may not consider.

You might consider an alternative scenario to the grocery store such as a home improvement store or a woodworking or metal shop class. Students could then interview tradespeople who grew up using the imperial system.

Meeting Student Needs

- Allow students to work in pairs.
- Encourage students to use the conversion charts in their Foldable as a reference.
- For #7, be prepared to help students who struggle with determining whether to divide the difference in area by the area of the current bedroom or the area of the new bedroom.
- For #8, some students may benefit from coaching. You might use the following prompts:
  - What is the radius of the area the cell-phone tower covers?
  - What SI units might you convert this to? Explain your thinking.
  - How would you do this conversion?
  - How do you determine the area of a circle if you know the radius of the circle?
- Some students may approach #9 in a different way. Have students discuss their strategies in a small group or class discussion.
  (Example: 5'2" is (5)(12) + 2 = 62 in., so
  $\frac{62}{4}$ = 15.5 or 16 tiles are needed for the length;
  3'6' is (3)(12) + 6 = 42 in., so $\frac{42}{4}$ = 10.5 or
  11 tiles are needed for the width;
  (16)(11) = 176 tiles

  To get the 10% extra material, students could multiply the partial tiles and then add 10% to that total.
  (15.5)(10.5) = 162.75
  Add 10% = 162.75 + 16.3
  = 179.05
  Purchase 179 tiles.

  Alternatively, they could add the 10% to their rounded total.
  (176)(1.1) = 193.6
  Purchase 194 tiles.

  You may wish to discuss with students how the timing of calculating the 10% extra material makes a difference to the amount needed.
- For #11, consider having students complete a placemat activity. Group students into groups of four and place a large sheet of paper in the centre (covering about two thirds of all four desks put together). Divide the sheet into four quadrants. Give the groups a volume unit conversion to do. Have each student copy the question and show their method for solving in one quadrant. Have students rotate the sheet around to see their neighbour's method for solving the same question; and then rotate one more time. Ask students to make up a volume unit conversion of their own and to show their method. Have them rotate and see their neighbours' approaches.
  As an extension, you might have students discuss the different methods and use the centre of the sheet to create a pros and cons list of the methods used.
- Provide **BLM 2–6 Section 2.1 Extra Practice** to students who would benefit from more practice.

Enrichment

- For #14, have students research occupations that use both SI and imperial measurement systems. They may find the Web Link about trades at the end of this section useful.

Gifted

- One hectare is approximately 2.56 acres. Challenge students to determine the answer to #12c) by going directly from acres to hectares. What methods might they try? Why? How can proportional reasoning help them solve this question?
- Challenge students to consider the mathematical claim that a circle has more area within its perimeter than any other figure with the same perimeter. Is there a way this claim can be investigated using their knowledge of area and/or by using grid paper? Have them draw a conclusion about the truth of the statement and describe their findings.

Common Errors

- Students may substitute dissimilar units when using a formula.

R*x* Insist that students write the appropriate formula as step 1 in their solution, followed by the substitution of the correct units from the problem. Ensure that students do not mix units. Emphasize that any conversion to a common unit should occur prior to substitution. Consider having students write down the facts known in each problem, circle or highlight the units used, decide what to convert to, and then make the conversions before using any necessary formula. You may wish to model this process for them as you go through an example; then, have them talk through their thinking as they do an example.

Web Link

The Canadian Interprovincial Standards Red Seal program provides information about trades that use both measurement systems. Go to www.mhrmath10.ca and follow the links.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1a) and b), 2a) and b), 4a), 5, and 6. Students who have no problems with these questions can go on to the remaining questions.</td><td>• Provide additional coaching with Example 1 to students who need coaching with #1a). Work with them to correct their errors and then have them try #2a).
• Provide additional coaching with Example 2 to students who need coaching with #1b) and c). Work with them to correct their errors and then have them try #2b) and c).
• Encourage students to use the method they are most comfortable with to make conversions.
• Provide additional coaching with Example 3 to students who need support with #4a). Work with them to correct their errors and then have them try #4b) on their own.
• For #4a), have students identify the steps needed to solve the problem (convert inches to centimetres; determine the volume). Work with them to correct their errors and then have them try part b) on their own.
• For #5 and 6, invite students to explain their answers to a small group, or the whole class, with emphasis on the problem solving process.
• For #6, encourage students to make a diagram of a quilt section.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #14 to 16.</td><td>• Encourage students to verbalize their thinking.
• Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form.
• For #14, it may be beneficial to brainstorm some occupations that use both SI and imperial measurement systems. Have students use these as a springboard to generate their own response.
• For #15, help students identify different methods used to make conversions (e.g., using a proportion, using unit analysis). Help them explain their thinking about which method they find easier. Students could record their response in their Foldable. Encourage them to include a diagram and a worked example of their own that they can use as a study tool.
• For #16, encourage students to walk through a grocery store and identify departments that use measurement conversions. Have students provide a sample conversion from each department.</td></tr>
</table>

Surface Area

2.2

Mathematics 10, **pages 66–79**

Suggested Timing

100–120 min

Materials

- right cones
- spheres
- right pyramids
- right cylinders

Blackline Masters

BLM 2–3 Chapter 2 Warm-Up
BLM 2–5 Chapter 2 Unit 1 Project
BLM 2–7 Investigate Surface Area of Three-Dimensional Objects
BLM 2–8 Section 2.2 Extra Practice
TM 2–1 How to Do Page 79 #18 Using TI-Nspire™
TM 2–2 How to Do Page 79 #18 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:

- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 8, 9, 19, 20
Typical	#1–3, 5, 6, 8–10, 12–14, 18–20
Extension/Enrichment	#3, 8, 10–12, 14–20

Unit Project Note that #8 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, read and discuss the introductory text and photograph of the Blackfoot Crossing building. The first bullet in Enrichment on TR page 55 suggests how you might incorporate information about this site into classroom work.

You might discuss that architects work with other professionals including structural, mechanical, and electrical engineers to apply knowledge of surface area for calculating the amount of materials needed to construct the design. Ask how each professional applies knowledge of surface area.

You might ask what types of 3-D objects are components of the building design (e.g., truncated cones, cylinders, curved walls). Consider having students brainstorm other careers that require knowledge of surface area (e.g., furniture design, clothing design). Have students describe an application of surface area for each career.

Investigate Surface Area of Three-Dimensional Objects

In this Investigate, students work in groups and use their knowledge of surface area to develop and share strategies for determining the surface area of a right cylinder or a right rectangular prism.

When forming groups of three to four students, ensure that each group represents students with a range of abilities and learning styles. Ensure that cylinders and prisms are represented among the groups.

For #1, you might have groups use **BLM 2–7 Investigate Surface Area of Three-Dimensional Objects** to record their work. For Quadrant 3, consider having students estimate the surface area before doing the calculation.

Much of the learning for this section should take place within the Investigate; therefore, allow sufficient time for groups to circulate and complete their work.

While students are working in their groups, circulate and ask questions to help them focus on the key ideas. Consider asking the following questions:

- What do you recall about surface area?
- What ways do you know for determining the surface area of right prisms and right cylinders?
- What methods do you have for representing 3-D objects in 2-D form?
- How can representing a 3-D object in 2-D form help you determine the surface area?
- Does the diagram provide all the information you need to determine the surface area? Explain.

While groups review the work of other groups, ask the following questions:

- How do you know whether this work is correct?
- How can you differentiate between a method that is different than yours and one that is incorrect?

Debrief after the activity by having students discuss their findings. You might have students summarize the strategies used to determine surface area.

Give each group sufficient time to complete the Reflect and Respond questions. For #6, prompt students by asking the following questions:

- How are cones and pyramids similar to prisms and cylinders? How are they different?
- Some objects contain right triangles. What do you know about right triangles? How can you use what you know about right triangles to help determine surface area?

As a class, compare and contrast the group responses. Find out if there is a consensus among students about which (correct) methods are preferred. (Students might say for a cylinder: use a net to identify each surface; determine the area of one circular base ($SA = \pi r^2$); add the areas of the two circular bases; determine the area of the rectangular area (using $C = \pi d$ to find the length); and add the areas together). Honour the preferences, even if some choices are unconventional.

In your debriefing, help students make connections to their prior knowledge. In earlier math courses, they worked with 2-D nets of 3-D objects and learned how to determine the surface area of rectangular prisms and cylinders. The discussion is important for students to understand the development of the surface area formulas, rather than memorize the formulas.

Have students connect what they discussed in #1 to 5 in their answers to #6. Consider starting with the right pyramid. Ask the following questions:

- How can you adapt techniques you used in calculating the surface area of a right prism or right cylinder to calculating the surface area of a right pyramid?
- What is the shape of each surface of a pyramid?
- How can this help you with calculating the surface area?

Have students do a hands-on activity to help them consider how to determine the surface area of a cone. Have students cut out a large circle. Use the following prompts:

- How can you make a cone using this circle? (Cut along one radius to the centre and overlap part of the circle.)
- What is the area of your circle?
- Look at the cone you have made. Is it the same area? Explain. (The cone will have a smaller area than the original circle because, to make it, you overlap parts of the circle.)

Have students experiment with making different cones from their circle. Ask the following questions:

- What do you notice about the area of the circle the cone uses? (Some cones use almost the complete circle area; others can use half or even less.)
- How can you take this into consideration when calculating the surface area of a cone?
- What else do you need to consider when calculating the surface area of a cone? (The surface area of a cone includes the surface area of the cone shape plus the surface area of the circle at the base of the cone.)

Finally, have students think about what they have done with cylinders, pyramids, and cones to consider how to determine the surface area of a sphere. Encourage them to consider what might be entailed. The purpose is to get students thinking about how such problems might be solved.

Meeting Student Needs

- Make models of right prisms, right cylinders, right cones, spheres, and right pyramids available to help students visualize the 3-D objects. The models may help them translate information between 2-D and 3-D.

Enrichment

- Before the beginning of the chapter, ask one or more students to visit the Blackfoot Crossing Historical Park web site and prepare an explanation of the importance of this site to the tribes of the Blackfoot Confederacy. Ask them to present this information when the class discusses the Blackfoot Crossing Exhibit Hall photo on page 66. Also ask them to research the cultural significance of the eagle feather for discussion with the eagle feather fan visual on page 67.
- Challenge students to answer the following questions:
  - If two objects have different shapes, but the same surface area, will they have the same volume? Justify your answer.
  - Is it possible to make a cone that uses the entire area of a circle? (Such a cone would be flat, which would not be a cone at all.)

WWW Web Link

For information about Blackfoot Crossing Historical Park, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Surface Area of Three-Dimensional Objects

1., **2.** Quadrant 1: Example: The sum of the surface area of all faces of a 3-D object.
Quadrant 2: Example: Find the area of each face, and then add all the areas.
Quadrant 3: The surface area of the cylinder is 374.7 cm^2. The surface area of the prism is 348 $in.^2$.
Quadrant 4: Students should show a different method than in Quadrant 3.

6. Sample answers might be derived using formulas for the following: circle, square, sector of circle, and triangle.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss the strategies for finding surface area of 3-D objects. Consider having students use the reflection/math journal and respond to one or more of the following prompts: – How has your understanding of surface area changed? – How could formulas you already know help you with determining the surface area of a right cone, a sphere, and a right pyramid?	• For the benefit of students who may hesitate to ask questions, have other students share the journal responses to the prompts. Listen carefully and clarify any misunderstandings. • Challenge students to develop nets for a right pyramid and a sphere. Ask them how a net might help them to develop a formula. • Have students consider how to determine the surface area of a composite object. How might these ideas help them with the surface area of a right pyramid, right cone, or sphere? • Encourage students to suggest and consider many strategies. Discussing the strategies may benefit students who can use them to enhance their own understanding and possibly springboard from them to choose their personal strategy.

Link the Ideas

In this section, students are introduced to how the formulas for the surface area of a right cone and a sphere work. It is likely that students will have difficulty visualizing the cone (and the sphere) in 3-D from the 2-D presentation in the student resource. Have at least one model of a cone and a sphere available when you discuss this section.

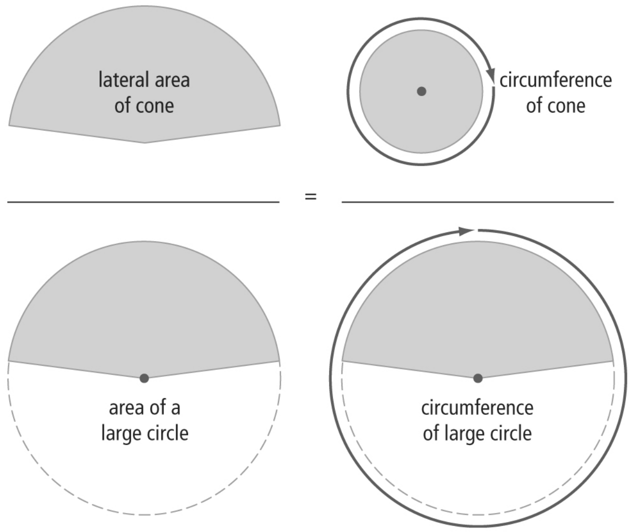
Have students make a net of a cone of their choice. To help them clarify the distance between the slant height (s) of the lateral area and the radius (r) of the base, have them label all parts on their net. Have them use this net as they work through the explanation of the surface area of a right cone.

Help them visualize that the lateral area is part of a circle by putting their net onto a piece of paper and drawing a circle with radius s. Discuss what fraction of this circle the lateral area takes up. Example: less than half in the sample on page 68.

Have them manipulate their lateral area around the base to clarify that the arc formed by the lateral area is the same length as the circumference of the base. Have them verbalize the formula for this length.

Encourage students to manipulate their net and the related circle as the class discusses the proportion developed to determine the lateral area of the cone. Use models, pieces of nets, or other visuals to make each step in this proportion concrete. It might be useful to have students sketch the visual needed for each part to clarify their understanding of the proportion.

Example:



Have students talk about how the proportion works. Ask the following questions:

- How can you make a large circle from the lateral area of the net?
- In this large circle, what is the centre? What is the radius?
- What part of this large circle does the lateral area of the cone form?
- What is the circumference of the base?
- How is the circumference of the base of the cone related to the lateral area?
- How can you determine the circumference of the large circle?
- How do you use the slant height to determine the circumference of the large circle?
- How do you use the slant height to determine the area of the large circle?

Use a model of a sphere and a strip of paper (cut to the same length as the circumference of the sphere and the same width as the diameter of the sphere) to walk through the explanation for the surface area formula of a sphere. Point out the definition of the term *sphere*. Have students consider the following questions:

- How is the diameter of the sphere related to the right cylinder?
- How is the circumference of the sphere related to the cylinder?
- Use the model and show me how the surface area of the sphere is related to the lateral area of the cylinder with similar dimensions.

Example 1

This Example is a relatively straightforward application of the formula for the surface area of a right cone. Use a model of a cone or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the right cone. Ask the following questions:

- What shapes make up the net of a right cone?
- What measurements do you need to determine the surface area of the base? the curved lateral surface?
- How are the diameter and the radius of a circle related?
- How would you find the total surface area?

Consider having students estimate the surface area of the cone.

$SA_{\text{cone}} = (3)(15)(15) + (3)(15)(40)$
$SA_{\text{cone}} = (3)(225) + (45)(40)$
$SA_{\text{cone}} = 675 + 1800$
$SA_{\text{cone}} = 2475\text{ cm}^2$

Help students develop their algebra and number skills by asking why the symbol for pi is present in all lines of the solution except the last one. Ask at what point to replace pi with a numerical value. You may need to remind students to use the pi button on the calculator (not 3.14), and to round in the final step of the calculation.

Have students complete the Your Turn and then compare their solution with that of a classmate who used a different sketch. Ask the following questions:

- How did drawing a diagram help you solve the problem?
- How did drawing a net help you solve the problem?
- Which sketch did you find most helpful? Why?
- How would you find the total surface area of any right cone?

Example 2

This Example introduces the formula for the surface area of a right pyramid, which is a new concept. Direct students to the definition of *pyramid* on page 70 in the student resource. Use a model of a right pyramid or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the right pyramid. Ask the following questions:

- What would the net of the right pyramid look like?

- What shapes make up the pyramid?
- How can your knowledge of 2-D area help you break down the surface area problem into manageable steps?
- What measurements do you need to determine the surface area of the base? the lateral surface?
- What is the area of the base? the lateral surface?
- How do you find the total surface area?
- Why do two of the terms in the formula for surface area involve multiplying by 2?
- How would the formula change for pyramids with a square base?

Ask students for an alternative way to determine the surface area of a pyramid. For example, find the surface area by drawing the five shapes that make up the pyramid, determine the area of each shape separately, and then add them together.

Have students use more than one method for solving the Your Turn, and compare their answers.

Example 3

This Example introduces the formula for the surface area of a sphere, which is another new concept. Use a model of a sphere or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the sphere. Ask the following questions:

- What is the relationship between the circumference, diameter, and radius of a circle?
- Would that relationship be true for a sphere? Why or why not?
- In the calculation $\frac{95.8}{2\pi}$, what potential error could easily be made? You might have students keystroke this division to see if they arrive at the same answer. Discuss why some people got the answer in the student resource while others got 150.48. (Correct keystrokes: $95.8 \div (2 \times \pi)$. Incorrect keystrokes: $95.8 \div 2 \times \pi$.) Discuss why the last sequence is incorrect.

After students complete the Your Turn, invite them to consider questions similar to the ones above again. In particular, have students think about the relationship between radius and diameter of a sphere. You might ask students to derive a formula for surface area that is based on diameter rather than radius. If so, have them work with a partner and be prepared to discuss their findings with the class.

Example 4

This Example draws on students' prior skills with determining square roots. In this example, they need to find square roots in a problem solving context, as well as while solving an equation in which the variable is squared. To help them with the process, consider asking questions such as the following:

- How is the equation $4\pi r^2 = 459.96$ similar to the equation $4r = 459.96$? How is it different? (The equations are similar in that they require division as the first step in the solution. You can solve $4r = 459.96$ by dividing both sides by 4. Similarly, you can solve $4\pi r^2 = 459.96$ by dividing. Some students may divide by 4, and then divide the result by π, while others may divide by 4π in one step. There are several differences for $4\pi r^2 = 459.96$, including the following:
  - There are two possible divisions depending on which method is chosen.
  - To complete the solution, the square root needs to be determined.
- How do your equation solving skills come into play in this example?
- What would you do to solve the equation $4x^2 = 200$?
- What is the opposite operation of squaring a number?
- How do you determine the square root of a number that is not a perfect square?
- How do the order of operations rules help you solve an equation?

For the last line in the computation of each solution, prompt students to double the radius to get the diameter that the question asks for.

Have students complete the Your Turn. They should note that the units asked for are millimetres. Consider having students compare their solution with that of a classmate and discuss any differences. Ask the following questions:

- How do the units asked for have an impact on how you work?
- How do you prefer to handle the units in this question? Why? (Students may prefer to express $1\ \text{m}^2$ as $10\ 000\ \text{mm}^2$ before solving the problem.)

Example 5

This Example features a grain bin that is a composite of a right cylinder and a right cone. It may be useful to have a 3-D model of the grain bin; even one made from paper would be useful. It is likely that students will need support to visualize the bin and understand how to apply the concepts they have learned to the problem. You might use the following prompts to discuss the problem:

- What 3-D objects form the bin?
- Are all of the surfaces of each object part of the bin? Explain.
- How do you determine the surface area of each shape on the grain bin?
- What parts of each shape should not be included in the surface area?
- How are the radius and diameter of the cone and the cylinder related?
- What shape is formed by the height of a cone, the radius of a cone, and its slant height?
- How can you use this shape to determine the slant height of a cone if you know the height and radius of the cone?
- What if you just know the height and diameter?
- Why is it important to understand where each term in a surface area formula comes from?

You may wish to have students work in pairs or small groups to complete the Your Turn. As you circulate, you might guide students using prompts such as the following:

- What surface areas are included in the dumbbell?
- Are there parts of some shapes that should not be included in the surface area? Explain.
- How does where the bar meets the weight disk affect the total surface area of the dumbbell? How will this affect your calculations?

Meeting Student Needs

- Make 3-D models of right prisms, right cylinders, right cones, right pyramids, and spheres available to students to help them visualize the problems.
- Before introducing the worked examples, have students who would benefit revisit how to determine the surface area of right rectangular prisms and right cylinders.
- Before introducing the surface area of a right cone, help students recall how to form a ratio.
- Have students verbalize the process for calculating the area of a right cone, a right pyramid, and a sphere.
- Have students use grid paper to draw nets and then cut out the shapes.
- Prior to starting Example 5, some students may benefit from using models, such as a right cone and a right cylinder with the same circumference, to create a composite shape to help them visualize which surfaces are eliminated.
- Have students create a poster for the classroom showing a diagram and a net of each 3-D object studied. Beside the diagram, have them record the steps for finding the surface area.

ELL

- Explain that *slant height* refers to the distance along a lateral face from the base to the highest point of a 3-D object, such as a right cone or a right pyramid. Use a model of a cone and a pyramid to show the difference in the use of this term for these two objects.

Enrichment

- For the Your Turn for Example 1, ask students how they could write a formula for surface area of a cone that uses diameter instead of radius.
- For the Your Turn for Example 2, challenge students to simplify the formula. For example, $SA = (\text{side})^2 + 2(\text{side})(\text{slant height})$.
- For Example 5, you might explain that metal fabricators build grain bins. Metal fabricators must know about blueprints, be able to use SI and imperial measures for fabrication or fitting specifications, and be knowledgeable about metals and welding procedures.

Gifted

- Give students the following challenges:
  - Develop a problem for the surface area of a right cone that provides neither the radius nor the diameter of the cone, but gives enough information to solve it.
  - Develop a problem for the surface area of a right pyramid that requires applying the Pythagorean relationship to solve it.

Common Errors

- Students may use incorrect surface area formulas in their calculations.

R_x Ensure that students use the correct formula for each object. Encourage students to refer to their list of formulas in their Foldable. For each formula, encourage students to add a diagram and a worked example that demonstrates important steps. Some students may need to make a model of each object and attach the related formula to the appropriate part. Clarify that students understand how to use their models by asking them to explain how to determine the surface area of each one. In problems where the surface area of part of a particular object may not be necessary, ask the student to pick up the model and show what parts need to be determined. For example, in the case of Example 5, it is not necessary to determine the area of the base of the cone or the ends of the cylinder.

- Students may not follow the correct order of operations when finding area.

Rx Remind students that in the formula $A = \pi r^2$, they must square the radius before multiplying by π. They could think of the formula as $A = \pi(r)(r)$ rather than $A = \pi r^2$. It may be useful for these students to record the order of operations on a sticky note and keep the sticky note in the front of their student resource.

- In Example 3, students may perform the operation $\left(\frac{95}{2}\right)(\pi)$ instead of $\frac{95.8}{2\pi}$.

Rx Have students recall the rules for the order of operations. Invite students to explore how their calculators handle the order of operations, and when brackets must be input. Have them record the correct sequence for their calculator key strokes. More importantly, remind students that the denominator of the fraction is understood to be $\frac{95.8}{(2\pi)}$, but that the brackets are usually omitted.

- When solving equations of the type $4\pi r^2 = 459.96$, students may determine the square root before dividing by the coefficient.

Rx Remind students to divide first and then determine the square root when isolating the variable, r. They may be unfamiliar with equations in which the coefficient of the variable is not a single value. Coach them through solving the equation by isolating the variable. This is done by dividing first by 4, then by π.

- Students may have difficulty distinguishing height from slant height for a right cone or right pyramid.

Rx Some students will find it beneficial to use a mnemonic. The slant part of slant height means that something is on an angle. The height is always at 90°. It may be useful to demonstrate that the slant height is the hypotenuse of a triangle formed by the radius of the base, the height of the cone or pyramid, and the slant height. Have students draw this triangle inside a cone and possibly build a triangle inside a cone to reinforce this understanding. Encourage students to recognize that the slant height is always greater than the height for any cone or pyramid.

Key Ideas

The Key Ideas summarize calculating the surface area of a right cylinder and a right prism, a right pyramid and a right cone, and a sphere. You might have students use index cards to prepare their own summary of the Key Ideas, including an example for each 3-D object. In addition, have students write their own notes on solving an unknown dimension of a right cone, a right pyramid, or a sphere when given the surface area. Beside each step, have them describe the reason for the step.

Answers

Example 1: Your Turn

$SA = 502.65$ cm^2

Example 2: Your Turn

$SA = 270$ cm^2

Example 3: Your Turn

1787 cm^2

Example 4: Your Turn

282 mm

Example 5: Your Turn

1041.3 cm^2

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Have students use mental math to do a quick estimate of the surface area of the cone. They can use this quick calculation to check the reasonableness of their answer. (SA = (3)(8)(8) + (3)(8)(12) Round the 8 and 12 to numbers that are easy to work with. $SA \approx (3)(10)(10) + (3)(10)(10)$ $SA \approx 600$ cm^2) • Have students explain how to determine the surface area if they know the diameter instead of the radius. • Encourage visual learners to use grid paper and draw and label a diagram and a net of the cone. • Have students verbalize the method they prefer to solve the surface area of any right cone. • Consider having students check each others' answer. • Provide students with a similar problem to solve.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • Encourage visual learners to use grid paper and draw and label a diagram and a net of the pyramid. • Have students verbalize the method they prefer to solve the surface area of any right pyramid. • Provide students with a similar problem to solve.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Have students verbalize the method they prefer to solve the surface area of any sphere. • Provide students with a similar problem to solve.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work with a partner. • Have students draw and label a diagram to help visualize the problem. Use the following prompts: – What measurements are needed to solve the problem? – What measurements are given? – What measurement is missing? – How can you use what you know to find the missing measurement? • Provide students with practice solving for a variable to gain proficiency in the algorithm of solving for a variable and in order to master the necessary calculator keystrokes for their particular calculator. • Provide two similar problems: one that has students determine square root, and another where determining a square root is not required. Discuss how to tell the difference.
Example 5 Have students do the Your Turn related to Example 5.	• You may wish to have students work with a partner. • Use a 3-D model of the dumbbell using three cylinders that disconnect so students can see what surfaces need to be subtracted from their calculations. (Four bases need to be subtracted: two bases on the bar and a fraction of a base on one side of each disk.) Have students identify the components of the object and the surfaces of each component (three cylinders; three surfaces per cylinder), and the areas to be subtracted. Ask students how they could determine the part of this surface area (pointing at the inside disk where the bar meets it) that needs to be subtracted. • Have students use the formula and estimate the surface area of the composite object to check the reasonableness of their answer. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each part of the diagram as they determine its surface area. • Have students verbalize their thinking related to their calculations and the number of surfaces used in them. • Provide students with a 3-D composite object in the classroom and its measurements, and have them determine its surface area. • Provide students with an additional question that allows them to determine slant height using the Pythagorean relationship.

Check Your Understanding

Practise

Question 1 provides an opportunity for students to demonstrate their understanding of calculating the surface area of right prisms, right cylinders, right pyramids, right cones, and spheres. The diagrams are provided and no unit conversions are required.

For #2, students are asked to make their own sketch before solving the problem. This is intended to help students develop the habit of drawing a diagram or a net as part of the problem solving process.

For #3 and 4, students are required to find a missing dimension, given the surface area.

The sculpture in #5 is produced by Linda Tanaka, an artist from Lethbridge, Alberta. Students solve a surface area problem involving a composite object composed of a sphere and a right pyramid with a rectangular base. Before students begin working on this question, ask what assumptions they need to make to answer this question. Also ask if everyone's assumptions will be the same.

Consider pairing students with different assumptions. Have them each solve the problem and then talk through their thinking with their partner. They can check each other's answers for reasonableness and discuss how differing assumptions may result in different answers.

Apply

The Apply questions provide a variety of contexts for students to solve problems involving surface area. In some cases, the real work simulations are very realistic. You may wish to engage students in a discussion of which simulations are more accurate than others, and discuss the idea that although mathematical models are not always exact representations of real life, they are useful.

It is useful for students to assess an effort to solve a problem, as in #6. Before looking at the solution, ask students to consider the problem and decide how they would solve it. Do they agree with Austin's method and solution? Discuss why or why not.

You might have students extend #7 by having them research the number of tipis set up at Blackfoot Crossing Historical Park and the total amount of canvas needed for these structures.

Did You Know?

Some students may question the spelling of tipi. Point out that tipi is the Canadian spelling; teepee is the American spelling. You may wish to brainstorm math terms that have different spellings, such as centre (Canadian) and center (American) and metre (Canadian) and meter (American).

For #9, ask students how their answer would change if they decided to use a diameter of 60 cm.

For #10, students use the concept of percent. You may wish to mention that this problem involves proportional reasoning.

For #11 to 13, challenge students to convert their answer to the other measurement system.

In #11, ask students familiar with hand drums to share their knowledge of the cultural significance and use of these drums.

Refer students who are interested in the Muttart Conservatory in #12 to the Web Link at the end of this section.

For #14, students solve a surface area problem involving a composite object. You may wish to refer students to look at how they solved #5, if they encounter difficulty.

If students are unfamiliar with the use of a light tent such as that shown in #14, you may wish to explain that the lights shining on the outside walls of the tent provide diffused and reflected lighting for the items inside. Tents such as these are commonly used by professional photographers because the diffused and reflected lighting is ideal for capturing the beauty of small shiny objects. See the related Web Link at the end of this section.

For #15, explain that 40 mm ± 0.5 mm is a short-hand way of writing: 40 mm + 0.5 mm and 40 mm – 0.5 mm. Explain that in this question the diameters of the squash balls can fall between the values of 40.5 mm and 39.5 mm. For part b), encourage students to draw a diagram on grid paper to help understand the problem.

Extend

These questions require students to perform several steps to solve the problems.

For #16, students determine the surface area of a truncated cone. If students encounter difficulty, prompt them to identify the shape of the piece of the cone that is cut out, and determine its surface area. Encourage students to model the problem to help visualize the cones (one cut out and one whole). Assume that there is no paper overlap or rim.

For #17, students are challenged to calculate the surface area of a composite block that has holes in it. Since the dimensions are fairly small, have students draw a scale model or create a 10:1 scale using centimetre grid paper and shade parts to be painted. Prompt students to think about how the holes in the block reduce the surface area in one way, but increase it in another way. Students should also think about the cylindrical shape of the holes, and which parts of the cylinder add to and subtract from the surface area.

For #18, students need to use spreadsheet software to analyse how changing the radius of a sphere changes its surface area. You may wish to have students use **TM 2–1 How to Do Page 79 #18 Using TI-Nspire™** or **TM 2–2 How to Do Page 79 #18 Using Microsoft® Excel** to do this question.

The result is counterintuitive; caution students that seem to answer the question very quickly. If you have access to some 3-D spheres, you may wish to make them available to students to assist them in visualizing the relationship.

Have students discuss the meaning of *stretch ratio* in the given context.

Create Connections

These questions allow students to communicate their understanding about surface area and why it is expressed in square units.

For #19, consider inviting students to present their response in a class discussion.

For #20, allow students to use a communication method of their choice; i.e., a written, taped, or graphic explanation. The visual shows students using American Sign Language.

Unit Project

The Unit 1 project question, #8, provides students with an opportunity to solve problems involving the surface area of a cylindrical CD case and a rectangular CD jewel case. Students can use multiple strategies including using grid paper and drawing and labelling a net, or making a table and listing the shapes for which calculations are needed.

Meeting Student Needs

- For #6, students find the error in a solution, which is a higher-order skill. Consider having students complete their own solution to the problem before attempting to identify the error in the given solution.
- For #11, help students recall how to change $14\frac{7}{8}$ to a decimal.
- For #14, refer students who have difficulty to Example 5.
- If you wish to make the spreadsheet work in #18 accessible to all students, use the Web Link on the next page.
- Provide **BLM 2–8 Section 2.2 Extra Practice** to students who would benefit from more practice.

Enrichment

- For #12, mention that although architects designed the Muttart Conservatory, likely ironworkers and glaziers were involved to create the buildings and install the glass. Ask how these trades apply knowledge of surface area.

Gifted

- The palm of the hand is approximately 1 percent of the total surface area of a human body. This is sometimes used by medical staff to estimate the percentage of an individual's body damaged by a burn. This is useful in determining treatment.

  Ask students to trace their palm onto 1-cm grid paper and count squares to find its surface area. Have them approximate their total body surface area from this information. Ask students to predict how changing the grid paper to 0.5 cm might affect the accuracy of their findings. Have them test the prediction and look for a pattern. Then, have them test with even smaller grid paper. How does decreasing the size of the grid paper squares affect the accuracy of the approximation of surface area of irregularly shaped objects? Note that this line of thinking is related to calculus reasoning.

Common Errors

- Students may incorrectly round their answers.

Rx Review any rules for rounding. Remind students that giving an answer to the nearest tenth of a unit requires one place after the decimal point. To the nearest hundredth is the same as two places after the decimal. When using a calculator, students should maintain exact values in all computation steps, rounding only at the end.

Students who still have problems with rounding might use the following process:

1. Identify the part of the number that might be rounded. Circle that number.

2. Underline the numeral that immediately follows the numeral that might be rounded.

3. Ask the following question:
Is the underlined numeral 5 or more?

YES	NO
↓	↓
Leave the circled number as is.	Round the circled number up one.

- Students may neglect to include the area of one side of a 3-D object when finding surface area.

Rx Remind students to add the areas of all sides of the object unless the question states otherwise or it is impractical to do so. Have a model of a right prism, a right cylinder, a right cone, a right pyramid, and a sphere on display in the classroom. Encourage students to examine the models and to consciously ask themselves whether it makes sense to include the base of the solid. For example, would you include the surface area of the base for a conical drinking cup?

Some students may need to make a model of the item they are calculating the surface area for. Have them put a checkmark on each surface as they determine its surface area. You may wish to have them number the surfaces and put matching numbers on the parts of their solution. This will help some students identify what surfaces they might have missed and what surfaces they may have used that they should not have.

WWW Web Link

For information about the Muttart Conservatory, go to www.mhrmath10.ca and follow the links.

For more information on the use of light tents such as that discussed in #14, go to www.mhrmath10.ca and follow the links.

To make #18 available to more students, have students go online to www.mhrmath10.ca and download one of the prepared spreadsheets. By entering the stretch ratio and radius, students can get the calculations, determine a pattern, and make predictions.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1 to 4, 6, and 9. Use students’ responses to #1 to assess their understanding of calculating the surface area of prisms, cylinders, pyramids, cones, and spheres. Then, have students attempt #3, which asks students to solve for the missing dimension. Students who have no problems with these questions can go on to the remaining Apply questions.</td><td>• For #1, have models of all shapes available for students to examine and manipulate. Make sure that students have a good understanding before proceeding.
• Have students note any 3-D objects that they have difficulty with. Coach them through the corrections and clarify any misunderstandings. Provide a problem similar to #1 before having students move on to 3.
• Provide grid paper to students who would benefit from drawing and labelling a diagram and/or a net for each object, particularly for #2 and 9.
• Provide coaching to students who have difficulty with #3. Coach them through solving equations and calculating square roots, if needed.
• Refer students who have difficulty with recalling the formulas to their Foldable.
• It may benefit some students to verbalize the dimensions of each object to assist in linking them to the variables in a formula. Have students write the dimension that corresponds to each variable in the formula.
• Encourage students to use the method they prefer for solving surface area problems.</td></tr>
<tr><td>Unit 1 Project
If students complete #8, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• Encourage students to use the actual items, manipulate a model of the same shape, and/or draw and label a diagram and a net for each situation.
• You may wish to assign only #8a) to students who are having difficulty with surface area problems.
• You may wish to provide students with BLM 2–5 Chapter 2 Unit 1 Project, and have them finalize their answers. Have them store the work in their project portfolio.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #19 and 20.</td><td>• Encourage students to verbalize their thinking.
• You may wish to have students work with a partner and then prepare an individual response.
• Some students may wish to answer #19 on the back of their Foldable, where they can refer to it for review purposes.</td></tr>
</table>

Volume

2.3

Mathematics 10, **pages 80–91**

Suggested Timing

100–120 min

Materials

- conical cup
- paper
- scissors
- tape or glue
- sand, rice, or popcorn

Blackline Masters

BLM 2–3 Chapter 2 Warm-Up
BLM 2–5 Chapter 2 Unit 1 Project
BLM 2–9 Section 2.3 Extra Practice
TM 2–3 How to Do Page 91 #19 Using TI-Nspire™
TM 2–4 How to Do Page 91 #19 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:

- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4–9, 15, 16, 21
Typical	#1–5, 7–11, 13–16, 20, 21
Extension/Enrichment	#8, 11–21

Unit Project The Unit 1 project questions are, 15, 16, and 21.

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, read and discuss the introductory text and photograph. Reinforce that citizens need to understand mathematics in order to make decisions about Canada’s natural resources. Use the opportunity to discuss examples of people working in the natural resources sector and how they apply knowledge of volume. Have students discuss different examples of units of volume that they are familiar with.

Investigate Volume

In this Investigate, students explore the relationship between the volume of a right cylinder and the volume of a right cone with the same radius and height. They construct a right cylinder, and then test their prediction of the relationship between the volumes of these two objects.

Have students work individually to make a prediction about the relationship between the volumes of the cone and cylinder before they construct the model of the cylinder and test their prediction. Have them tape the base of the cylinder securely to its lateral surface in order to avoid spills.

As students work, circulate and consider using some of the following prompts to help them extend their thinking:

- How did you arrive at your prediction?
- Does your result depend on the type of material used to fill the cup? Explain.
- Would this relationship hold for other combinations of right cones and right cylinders (with the same height and the same base area as the cone)? Explain.
- How could your result help lead to a formula for the volume of a right cone?
- What do you predict would happen if you repeated this experiment for a right rectangular prism and a right pyramid?

Have students work with a partner to answer the Reflect and Respond question. Have the pairs discuss their response with another pair of students, and then report to the whole class. Students who did not determine the expected relationship will benefit from the class discussion. You might have students discuss the possible reasons for any discrepancies.

Meeting Student Needs

- Make 3-D models of right cylinders and right cones available to students to help them visualize volume.
- Help students orally recall what they learned about volume from earlier math courses. You might have them use a reflection/math journal and respond to one or more of the following prompts:
  - In your own words, define volume.
  - Compare volume with surface area.
  - Why is volume always expressed in cubic units?

ELL

- Explain that *potash* is the common name for potassium chloride, which is used in fertilizers and manufactured products, such as soap and glass.

Enrichment

- Challenge students to research careers in the natural resource sector and how people working in these careers apply knowledge of volume. For example, they might research careers in mining, forestry, and the oil and gas industry.

Common Errors

- Students may confuse area measurement units with volume units.

R_x Emphasize that volume is always measured in cubic units, such as cubic millimetres, cubic centimetres, cubic metres, and cubic kilometres to represent the amount of space an object occupies. Have students develop a 2D model showing area and a 3D model showing volume. Have them talk aloud about the differences between the models and what they understand about the difference between area and volume.

WWW Web Link

For information about careers in the natural resource sector, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Volume

4. b) Example: The cylinder holds three times as much as the cone.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about the volume of a right cone.	• As there will likely be variations in the dimensions of the cones and cylinders that students create, have students discuss the results of their experiment. • Have students use the class discussion to help them get a clear understanding that the volume of a right cone is one third of the volume of the related right cylinder.

Link the Ideas

This section follows up on what students did in the Investigate by introducing how the formula for the volume of a right cone is derived from the formula for the volume of a right cylinder. Walk through the explanation with students. Begin by asking students to recall the formula for the volume of a cylinder.

Students may benefit from a visual representation of the relationship between the volumes of a cone and cylinder by watching the video mentioned in the Web Link on page 81 in the student resource.

Have students consider the two formulas for the volume of a cone: $V = \frac{1}{3}Bh$ and $V = \frac{1}{3}\pi r^2 h$. Discuss how these two formulas are related. (The base of a cone is a circle; the area of a circle is πr^2.) Once students realize this, they will have to remember only the first formula. You may wish to have them put a diagram and sample formula in their Foldable.

Have students look at the visuals on page 81 and discuss how the volume of a right rectangular pyramid might compare to the volume of the related right rectangular prism. Lead this discussion toward the formula for the volume of a right pyramid. You might ask students to recall how to determine the volume of a right prism. Walk through the explanation with students. Begin by clarifying the term *apex*.

Have students use a model of a right rectangular prism (with the same base and height) and a right rectangular pyramid to help them understand the relationship between the volumes of these two objects. Have students make a prediction and then consider questions such as the following:

- How did you arrive at your prediction?
- If you fill the pyramid with material, predict how many cups of material it would take to fill the prism.
- Use the model and show how the volume of the prism is related to the volume of the pyramid.
- How could your result help lead to a formula for the volume of a pyramid?

Have students consider how the formula for the surface area of a right cone and a right pyramid are related. Again, have students consider the two different formulas for the volume of a right pyramid and discuss the similarities and differences. Once students understand the formula, they will have to remember only the fact that, to solve for the volume of a right prism, they determine the volume of the related right rectangular prism and divide by 3. Understanding is more important than memorization.

Once students have considered the relationship between the volume of a right cone and a right pyramid, have them consider the visual of the right cylinder and the sphere. Ask the following questions:

- How do you determine the volume of a right cylinder?
- What suggests that the sphere is more than one-third the volume of the related right cylinder?
- What fraction of the related cylinder does the explanation suggest it holds?
- How can you use this information to develop a formula for the volume of a sphere?

Again, make sure that students understand the steps of the calculation. Rather than memorizing a formula, they may prefer to think through the calculation each time. For example:

1. Determine the volume of a related cylinder with the radius of the circle and the height of the diameter of the circle.
2. Multiply the answer by $\frac{2}{3}$.

It is far more important for students to understand what they are doing and to use formulas they understand than to memorize formulas without understanding.

Have students use their Foldable and record how to determine the volume of a right cone, a right pyramid, and a sphere.

Example 1

This Example follows up the Investigate by calculating the volume of a right cylinder and a right cone by using a formula and volume relationships. Walk through the example. Extend students' thinking by asking the following questions:

- What is the relationship between the diameter and the radius of a right cylinder? Is this relationship the same for a right cone?
- Explain how you know the formula for the volume of a cylinder is $V = \pi r^2 h$.
- Is the value for pi squared in the formula? Explain.
- If you know the volume of a cylinder, then based on your investigation, what do you expect the volume of the related cone to be?
- Which method do you prefer for calculating the volume of a cone? Why?
- Give a situation when using a formula would be more convenient than using volume relationships. Explain.
- Give a situation when using volume relationships would be more convenient than using a formula. Explain.

Direct students to the note beside the solution for part a). Discuss the difference between writing the solution as 75π cm^3 versus 235.6 cm^3.

For the Your Turn, students determine volumes given radius and height. Ask students what is different about what they have been given compared to the problem in the Example. How can they handle this difference?

Consider having students estimate the volume of the cylinder and the cone using the respective formulas.

Have students use the method of their choice to solve each problem. Have them compare their answer with that of a classmate who used another method. Ask which method they prefer. Alternatively, encourage students to solve part a), then to solve part b) using two different methods. In this case, have students compare their solutions for part b).

Example 2

This Example determines the volume of a right pyramidal greenhouse with a square base using a formula. As a class, walk through the Example. Extend students' thinking by asking questions such as the following:

- Does the square base of the right pyramid affect the way you use the formula?
- How could you write a formula that would apply only to the volume of a square-based pyramid? What would the formula look like?
- Would such a formula be useful? Why or why not?
- Does it matter which side is labelled *l* and which is labelled *w*? For that matter, does it matter which side is labelled *h*? Explain.
- What is the volume of the pyramid in cubic feet? How did you make the conversion?
- How else could you solve this question? (Encourage students to develop strategies that they are comfortable with and understand.)

For the Your Turn, students determine the volume of the smaller greenhouses at the Muttart Conservatory. For part a), students determine the volume of a small pyramid-shaped greenhouse with a given side length and height. As there are several ways to correctly solve this problem, you may wish to have students discuss their answers in pairs or small groups, and then report to the whole class.

For part b), have students identify the shape of the greenhouse. Ask the following questions:

- Will this greenhouse be taller or shorter than a pyramid with the same volume?
- How can a visual help you understand this problem?

Students may predict the height as the reciprocal of the actual height. In other words, they may use the correct ratio, but in the wrong direction, to answer the question. To help students who have difficulty understanding what this question is asking, draw a right rectangular pyramid and a right rectangular prism (with identical-sized bases) on the board, and ask students when the two volumes would be equal.

Example 3

This Example determines an unknown dimension when given the volume of a sphere. In the solution, a cube root is determined, which is a new procedure and a new concept for students. As you discuss the example, consider asking how the rules for the order of operations help solve equations.

Many students will need help to extract the cube root using their calculator. Be prepared to help them find the correct procedure, and then practise the skill using an example.

For the Your Turn part a), students determine a cube root. Have students estimate the cube root using a Guess and Check strategy such as (7)(7)(7) = 343. They may write a note in their Foldable about how to find cube roots on their calculator.

For part b), students determine the diameter of a sphere given its volume. Ask the following questions:

- How can you use mental math to estimate the answer? (Discuss different strategies.)
- What strategies can you use to determine the answer to this question? Encourage students to develop as many different strategies as they can.
- What unit should your answer be in?

Have students who use different strategies compare answers and review each others' calculations if their solutions do not agree.

Example 4

Example 4 determines the volume of a composite object made up of a sphere and a right cone. The volume of a composite object is generally more straightforward than the surface area, as frequently the total volume is the sum of the individual volumes.

As students discuss the Example, be prepared to ask questions such as the following:

- How is the radius related to the diameter?
- At what point in the solution do you substitute for pi? Explain why.
- Do you think the total volume would change if the objects were joined in a different location? Explain your thinking.

Students are required to keystroke a third power in this example. Review how to do this on their calculators.

For the Your Turn, students determine the volume of a building composed of a cylinder and half a sphere. Ask students how the volume of the roof of the building is related to the volume of the corresponding sphere. Have them explain why.

Consider asking students to compare the method for calculating the volume of this building with the method for calculating its surface area.

Key Ideas

The Key Ideas summarize calculating the volume of a right cone, a right pyramid, and a sphere. You might have students use index cards to prepare their own summary of the Key Ideas, including an example for each 3-D object. Also, have them include notes on how to keystroke cube roots. They can refer to Example 3 for a specific example to record. Beside each step, have them describe the reason for the step and provide the keystroke sequence (where applicable).

Meeting Student Needs

- Make 3-D models of right prisms, right cylinders, right cones, right pyramids, and spheres available to students to help them visualize volume problems.
- Provide hands-on learning opportunities for each 3-D object to students who would benefit. Pairing students of similar ability will encourage them to explore together. Encourage student pairs to compare their findings with another pair of students.
- Have students create a poster that shows the relationship between the volume of a right cylinder and a right cone, the volume of a right prism and a right pyramid, and the volume of a sphere and the related right cylinder.
- Have students explain the difference between the slant height and the apex in a right pyramid.
- For the Example 3: Your Turn, coach students who try to evaluate and round 288π when solving, and lose degrees of accuracy. Explain that when given the exact value of a volume, it is necessary to maintain the exactness when solving a problem. Show the correct solution.

$$\begin{aligned} 288\pi &= \frac{4}{3}\pi r^3 \\ (3)(288\pi) &= 4\pi r^3 \\ 864\pi &= 4\pi r^3 \\ \frac{864\pi}{4\pi} &= \frac{4\pi r^3}{4\pi} \\ 216 &= r^3 \end{aligned}$$

- Have students verbalize the process for calculating the volume of right prisms, right cylinders, right pyramids, right cones, and spheres.

Common Errors

- Students may use incorrect keystrokes when entering $\dfrac{12\ 566.4}{4\pi}$.

R_x Remind students of the proper sequencing and entries for keying the expression.
Correct keystrokes: 12566.4 ÷ (4 × π)

- Students may not understand cubes and cube roots.

R_x Remind students that $4^3 = (4)(4)(4)$
$= 64$
When asked for the cube of 64, they are being asked to find a number whose cube is 64.
(___)(___)(___) = 64
Relate this calculation to a 3D cube. The volume of the cube is 64 units cubed. You are asking them to find the length of one side.
At this level, many students already know the perfect squares of the numerals from 1 to 10. Encourage them to use this information to determine and record the cubes of the same numbers. This will help them with many calculations at this level.

WWW Web Link

For a video showing the relationship between the volume of a right cone and the volume of a right cylinder, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) 6912 cm^3 **b)** 2304 cm^3

Example 2: Your Turn

a) 2281.5 m^3 **b)** 6 m

Example 3: Your Turn

a) 7 **b)** 120 mm

Example 4: Your Turn

5235.59 m^3

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Some students may require individual coaching. Using the same diagram but a different set of dimensions, provide a similar problem to solve. Have students explain how to determine the volumes.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • Some students may require individual coaching. Using the same diagram but a different set of dimensions, provide a similar problem to solve. Have students explain how to determine the volume for part a).
Example 3 Have students do the Your Turn related to Example 3.	• For part a), it may be beneficial to review key strokes on a calculator for calculating the square and cube root of a number. • For part b), students may require individual coaching. Have students talk through the process of solving for an unknown dimension. Have them verbalize how to divide by pi using the order of operations, and then how to determine the cube root. • Give students a similar problem to solve. Allow them to work with a partner and talk through their thinking.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Use a model of a right cylinder and a semi-sphere to help students identify, and sketch and label the individual components. • Using the same diagram but a different set of dimensions, provide a similar problem for students who would benefit from more practice.

Check Your Understanding

Practise

Question 1 provides an opportunity for students to demonstrate their understanding of calculating the volume of right prisms, right cylinders, right pyramids, right cones, and spheres. Check that students are able to determine the volume of each object.

For #2, note that this is the first time that students are given more information than is required to determine the volumes. They will need to identify the extraneous information. You might ask students to identify in what context the extraneous measurements would be needed.

Both #3 and 4 involve composite objects. For #4, students identify the correct solution to a problem. You may wish to have students work individually to develop their own solution to the problem before working with a partner to identify the error. Note that in part b), students may choose their own method. Ensure they can justify their method.

For #5, students determine the missing dimension, given the volume for four objects.

Apply

The Apply questions provide a variety of contexts for students to solve problems involving volume of 3-D objects. Encourage students to draw and label diagrams for word problems when diagrams are not provided.

Reinforce that drawing a sketch is an important step in solving problems involving volume, even when it is not specified. Encourage students to draw a sketch for #6, 7, 10, and 12.

For #6, you might point out that 1 cm^3 = 1 mL and 1000 cm^3 = 1 L. How many litres of oil can this section of pipe hold?

For #7, students find the cube root of a calculated volume. It may help students crystallize their thinking if you ask whether it is possible to determine the dimensions of a right rectangular prism, if only the volume is given. Some students may solve by finding the volume of the rectangular prism, and then cube root the answer. However, encourage them to solve by trial and error. Ask what number times itself, and times itself again, yields 216 in.^{3}.

Ask students to read the Did You Know? related to #8.

For #11, you might use prompts since it involves two half spheres, one inside the other, whose volumes must be subtracted to find the volume of snow. Ask students how to deal with the size of the vent.

Have students read #14, which involves a composite object. Have them explain what they are being asked to do. Then, ask how they will deal with the overlap of the sphere and the right rectangular prism.

Extend

These are multi-step problems.

For #17, students describe the relationship between a right cone and a sphere. They may benefit from exploring the interactive Web Link described at the end of this section, which allows them to enter various radii and heights, and then determine the volume of the related cone, cylinder, and sphere, in order to understand the relationship.

For #18, students solve a problem that requires using formulas for volume and the relationship $V = Bh$, where B is the area of the base. In this case, they need to use the lateral surface area of a cone as the base of an object, and its thickness to determine the volume of chocolate lining the cone.

For #19, students use technology to explore how changing the radius of a sphere changes its volume. You may wish to have students use **TM 2–3 How to Do Page 91 #19 Using TI-Nspire™** or **TM 2–4 How to Do Page 91 #19 Using Microsoft® Excel** to do this question. Because this relationship is not intuitive, students should be given an opportunity to discuss their work in pairs or small groups.

Create Connections

For #21, students have an opportunity to apply their understanding about volume by using SI and imperial measurements to estimate and determine volume of an object related to their Unit 1 project.

You will need to decide whether students will work individually or in small groups, and if they can choose an object from within the school or from home.

Students work with both SI and imperial measurements, and there is plenty of opportunity for mental mathematics. Have students present their findings in a whole class discussion, which is likely to be interesting, as a variety of objects, units, and methods will be chosen.

Unit Project

The Unit 1 project questions, #15, 16, and 21 give students opportunities to solve problems involving the volume of 3-D objects.

For #15, students estimate the volume of a cell phone.

For #16, students work with an MP3 player and a vinyl record to compare the storage capacity per cubic centimetre for an MP3 player and a vinyl record. This gives students an opportunity to apply much of what they learned during the chapter, including linear conversions, volume calculations, and volume conversions.

For #21, remind students that even though they work in small groups, they will be responsible for providing an individual response with their findings.

Meeting Student Needs

- For #4, students may benefit from practice identifying errors in solutions involving less complex formulas and fewer steps. Have them develop their own solution to the problem before identifying the error in the given solution.
- If you wish to make the spreadsheet work in #19 accessible to all students, use the Web Link below.
- Provide **BLM 2–9 Section 2.3 Extra Practice** to students who would benefit from more practice.

Enrichment

- As an extension for #8, invite students who are interested to research jewellery making and report on how knowledge of surface area and volume are important skills.
- Give students one of the following challenges:
  - The size of the star cluster Westerlund 1 is given as 6 light years in diameter. Research the distance of a light year in kilometres, and then determine the volume of Westerland 1.
  - The human brain is an amazing information storage device. Estimate the volume of the human brain. Compare the estimate of the memory capacity of the brain and the current capacity of memory devices. What are the implications for the future?

Gifted

- Building on the work done in 2.1, where students investigated the area of a circle as being the maximum area of an object with a given perimeter, ask students to speculate if a sphere contains the maximum volume within its surface area. Challenge them to support their ideas without using outside sources, but by building on their understandings of the relationships between shape and measurements.

Common Errors

- Students may forget the formulas for volume.

R_x Have students refer to the formulas in their Foldable. Encourage them to accompany each formula with a diagram and a worked example showing all the steps.

- Students may forget to use cubic units for volume.

R_x Emphasize that volume is always measured in cubic units. Demonstrate by showing the three dimensions on a solid and indicate that multiplying a unit for each dimension results in a cubic value.

- Students may give the incorrect volume when finding volumes of parts of objects.

R_x Remind students to read the question carefully and highlight (in their notebook) the part(s) of the object asked for.

@WWW Web Link

For information about a career in jewellery making or gemology, go to www.mhrmath10.ca and follow the links.

For a video that describes the cell phone development by Motorola, go to www.mhrmath10.ca and follow the links.

For an interactive activity that allows students to explore the relationship between a right cone and a sphere, go to www.mhrmath10.ca and follow the links. Students enter various radii and heights, and then determine the volume of the related right cone, right cylinder, and sphere.

To make #19 available to more students, have students go online to www.mhrmath10.ca and download one of the prepared spreadsheets. By entering the stretch ratio and radius, students can get the calculations, determine a pattern, and make predictions.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2, and 4 to 9. Students who have no problems with these questions can go on to the remaining Apply questions.	• Provide additional coaching to students who have difficulty with calculating volume for one or more of the objects in #1. Coach them through corrections and clarify any misunderstandings before allowing them to proceed. • Refer students who have difficulty calculating volume for a particular object to the relevant worked example. Then, provide a similar problem to solve by changing the dimensions on an existing diagram. • For #2, have students draw a sketch and verbalize what dimensions are needed to determine the volume of each object. They might find referring to their Foldable or to the worked examples helpful. • Have students explain how to determine the volume of a right rectangular prism, and then consider how to handle what is missing in #5a). You may wish to have them talk through that calculation, and then try b) on their own using a similar method. Next, have them talk through a solution to c), and then try d) on their own. • Encourage students to draw sketches for #6 and 7.
Unit 1 Project If students complete #15, 16, and 21, which are related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• Consider asking students to draw labelled sketches for each question. • For #16, encourage students to sketch and label the dimensions of the MP3 player and the record. They will need to convert the volume of the record into cubic centimetres. • For #21, have students brainstorm some possibilities of objects related to music. • You may wish to provide students with **BLM 2–5 Chapter 2 Unit 1 Project**, and have them finalize their answers. Have them store the work in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #21.	• Give students an opportunity to explain how they arrived at their estimates and determined the volume of their object. Ask them to discuss how they used mental math in answering the questions.

Chapter 2 Review

2

***Mathematics 10*, pages 92–94**

Suggested Timing

60–90 min

Blackline Masters

BLM 2–6 Section 2.1 Extra Practice
BLM 2–8 Section 2.2 Extra Practice
BLM 2–9 Section 2.3 Extra Practice

Planning Notes

Have students work independently on #1 and 2, and then compare their answers with a classmate. After students have made corrections, have them work independently on the remaining questions. After completing the questions for each of sections 2.2 and 2.3, you might have students check their solutions with a classmate.

If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their Foldable, the worked examples, and previously completed questions in the related sections of the student resource.

Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the practice test.

Meeting Student Needs

- Encourage students to draw and label diagrams and nets, when appropriate.
- Make grid paper available to students.
- Encourage students to use their Foldable and to add new notes if they wish.
- The intent of #3 is for students to consolidate their personal referents. Some students may need to draw areas the related size and identify what areas familiar to them have approximately the same size.
- For #7, be prepared for students who ask whether the answer should be in square millimetres or square centimetres. Have them justify their choice of unit.
- In #13, you may wish to discuss how a slump test is used. If the centre of the cone settles too much, the engineer knows that the concrete is not stiff enough. Another batch of concrete will need to be mixed.
- Students who require more practice on a particular topic may refer to **BLM 2–6 Section 2.1 Extra Practice**, **BLM 2–8 Section 2.2 Extra Practice**, and **BLM 2–9 Section 2.3 Extra Practice**.

Enrichment

- For #13, you might have students research careers in structural or civil engineering and how engineers apply their knowledge of surface area and volume. Challenge students to develop a related contextual problem involving surface area or volume.

Gifted

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 2 Review The Chapter 2 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

2 Chapter 2 Practice Test

Mathematics 10, pages 95–97

Suggested Timing

40–50 min

Blackline Masters

BLM 2–10 Chapter 2 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need a little help with, a lot of help with, or no help with. Have students first complete the questions they know they can do, followed by those they know something about. Finally, have students do their best on the questions that they are struggling with.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–7, and 9–12.

Make grid paper available to students.

For #8, point out that the dimensions of the kennels are accurate. These kennels are designed so that dogs cannot turn around easily during transport.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	2.1	Example 1	✓ solve problems that involve area and volume units within SI and imperial systems
#2, 10	2.2	Key Ideas	✓ solve problems involving the surface area of three-dimensional objects
#3, 8	2.3	Link the Ideas Example 1 Key Ideas	✓ solve problems involving the volume of three-dimensional objects
#4, 7, 13	2.2	Example 3 Key Ideas	✓ solve problems involving the surface area of three-dimensional objects
#5	2.2	Example 2 Key Ideas	✓ solve problems involving the surface area of three-dimensional objects
#6, 12	2.3	Example 3	✓ find an unknown dimension of a three-dimensional object given its volume
#7, 13	2.3	Example 4 Key Ideas	✓ solve problems involving the volume of three-dimensional objects
#9, 11	2.3	Link the Ideas	✓ solve problems involving the volume of three-dimensional objects
#14, 16	2.3	Link the Ideas Example 1 Key Ideas	✓ solve problems involving the volume of three-dimensional objects
#15a)	2.3	Example 3	✓ find an unknown dimension of a three-dimensional object given its volume
#15b)	2.2	Example 4	✓ find an unknown dimension of a three-dimensional object given its surface area

Meeting Student Needs

Enrichment

- For #11, ask students how much of the container is occupied by tennis balls.

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 2 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Make grid paper available to students. • Have students use their responses on the practice test and work they completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 2 Test After students complete the practice test, you may wish to use **BLM 2–10 Chapter 2 Test** as a summative assessment.	• Consider allowing students to use their Foldable.

Right Triangle Trigonometry

3

General Outcome

Develop spatial sense and proportional reasoning.

Specific Outcome

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
3.1	✓ explain the relationships between similar triangles and the definition of the tangent ratio
	✓ identify the hypotenuse, opposite side, and adjacent side for a given acute angle in a right triangle
	✓ develop strategies for solving right triangles
	✓ solve problems involving one or more right triangles, using the tangent ratio
3.2	✓ use the sine ratio and cosine ratio to solve problems involving right triangles
	✓ solve problems that involve direct and indirect measurement
3.3	✓ explain the relationships between similar right triangles and the definitions of the trigonometric ratios
	✓ solve right triangles, with or without technology
	✓ solve problems involving one or more right triangles

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 3–1 Chapter 3 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 98 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about right triangles, the Pythagorean relationship, and trigonometry. You might provide the following prompts: • Where have you encountered the Pythagorean relationship? How was it used? • Have you ever had any experience with trigonometry? If so, when? • How do you think the Pythagorean relationship and trigonometry could be used in daily life and in different careers?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 3–2 Chapter 3 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 3 Foldable As students work on each section in Chapter 3, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter.
Assessment *for* Learning	
BLM 3–3 Chapter 3 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 3 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 30–40 minutes (TR page 79)			BLM 3–1 Chapter 3 Self Assessment BLM 3–2 Chapter 3 Prerequisite Skills BLM U1–1 Unit 1 Project BLM U1–2 Unit 1 Project Checklist		TR page 78 Chapter 3 Foldable, TR page 78	TR page 78	
3.1 The Tangent Ratio • 120–140 minutes (TR page 81)	Students should be familiar with • right triangles • ratios • similar triangles • using their calculator	• grid paper • protractor • ruler	BLM 3–3 Chapter 3 Warm-Up BLM 3–4 Chapter 3 Unit 1 Project BLM 3–5 Section 3.1 Extra Practice BLM 3–6 Protractor	**Essential:** #1a), b), 2, 3a), 4a), b), 5, 6, 9, 17, 18, 20 **Typical:** #1, 3a)–c), 4a), b), 5–7, 9–13, 18–21 **Extension/Enrichment:** #10–21	TR pages 83, 87 Chapter 3 Foldable, TR page 78	TR pages 85, 86, 87	
3.2 The Sine and Cosine Ratios • 100–120 minutes (TR page 88)	Students should be familiar with • similar triangles • right triangles • labelling the sides of a triangle in relation to the reference angle • using their calculator to find trigonometric ratios and to find the reference angle	• protractor • ruler • calculator • 1 m of foam pipe insulation, cut lengthwise • marble or small steel ball • eight to ten thick books or bricks or a chair • masking tape • measuring tape • table	BLM 3–3 Chapter 3 Warm-Up BLM 3–7 Section 3.2 Extra Practice	**Essential:** #1a), c), e), 2a), c), 3a), b), e), f), 4a), 5, 6a)–c), 7, 8, 13 **Typical:** #1a), c), 2a), c)–e), 3a), b), e), f), 4a), 5, 6a)–c), 7, 8, 11, 13, 17 **Extension/Enrichment:** #10, 12–17	TR pages 89, 93 Chapter 3 Foldable, TR page 78	TR pages 91, 93	
3.3 Solving Right Triangles • 100–120 minutes (TR page 94)	Students should be familiar with • primary trigonometric ratios • Pythagorean relationship • problem solving strategies	• metre stick or measuring tape • masking tape • calculator	BLM 3–3 Chapter 3 Warm-Up BLM 3–4 Chapter 3 Unit 1 Project BLM 3–8 Section 3.3 Extra Practice	**Essential:** #1a), c), 2, 3, 4a), b), 5a), e), 6, 7, 11, 16 **Typical:** #1a)–c), 2, 3, 6, 7, 11, 12, 16 **Extension/Enrichment:** #9, 13, 14–16	TR pages 95, 99 Chapter 3 Foldable, TR page 78	TR pages 97, 99	
Chapter 3 Review • 60–90 minutes (TR page 100)		• ruler • protractor • calculator	BLM 3–5 Section 3.1 Extra Practice BLM 3–7 Section 3.2 Extra Practice BLM 3–8 Section 3.3 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 3 Foldable, TR page 78	TR page 100	
Chapter 3 Practice Test • 40–50 minutes (TR page 101)		• ruler • calculator	BLM 3–9 Chapter 3 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** 1–4, 6–9	TR page 102		TR page 102 BLM 3–12 Chapter 3 Test
Unit 1 Project • 60–90 min (TR page 103)			Master 1 Project Rubric BLM U1–3 Project Final Report BLM 1–4 Chapter 1 Unit 1 Project BLM 2–5 Chapter 2 Unit 1 Project BLM 3–4 Chapter 3 Unit 1 Project				TR page 104 Master 1 Project Rubric
Unit 1 Review and Test • 60–90 min (TR page 105)		• ruler	BLM 1–10 Chapter 1 BLM Answers BLM 2–11 Chapter 2 BLM Answers BLM 3–10 Chapter 3 BLM Answers	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapters 1, 2, and 3 Foldables	TR page 105	TR page 105

Right Triangle Trigonometry

Opener

***Mathematics 10*, pages 98–99**

Suggested Timing

30–40 min

Blackline Masters

BLM 3–2 Chapter 3 Prerequisite Skills
BLM 3–4 Chapter 3 Unit 1 Project
BLM U1–1 Unit 1 Project
BLM U1–2 Unit 1 Project Checklist

Key Terms

opposite side	hypotenuse
adjacent side	tangent ratio
cosine ratio	sine ratio
primary trigonometric ratios	

What's Ahead

In this chapter, students learn about the primary trigonometric ratios: tangent, sine, and cosine. These ratios can be used to find missing sides or missing angles. The chapter ends by combining all methods to solve triangles (find all the missing sides and angles).

Planning Notes

Explain that the chapter is about using the relationships among the sides and angles of a right triangle to find unknown measurements. Tell students that they will rely on their existing knowledge and skills of solving one-step equations and proportions. Direct students to the information about some of the different areas that trigonometry is used. Have students discuss what they know about the work in these areas and how trigonometry might be related to the work.

Unit Project

You might take the opportunity to discuss the Unit 1 project described in the Unit 1 opener on TR page 2. Throughout the chapter, there are individual questions for the Unit Project. These questions are not mandatory but are recommended because they provide some of the research needed for the final report for the Unit 1 project assignment.

You will find questions related to the project in the Check Your Understanding in sections 3.1 and 3.3.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 99 and how it might be used to summarize Chapter 3. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Discuss with them the advantage of keeping track of what they need to work on, and ask them to include a method for doing so.

As students progress through the chapter, provide time for them to keep track of what they need to work on. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 3–2 Chapter 3 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Consider having students staple a copy of **BLM U1–2 Unit 1 Project Checklist** to the back of the Foldable. They can check off items as they complete them. Students may also wish to start a formula section in their Foldable for trigonometric relationships.

- You may wish to post the student learning outcomes for the entire chapter in the classroom. You could color-code the outcomes by section in the chapter. Ensure that students understand the outcomes as written. This will help them to self-assess their progress and to identify areas of weakness.
- Make a bulletin board display or a large chart showing a large right triangle labelled with all the related definitions students have written.
- To reinforce the Key Terms, post seven sheets of paper around the room, each labelled with one Key Term. Have student pairs respond to the following prompts for each term:
  - definition in their own words
  - example in daily life
  - facts

  Have student pairs move around the room and use diagrams and words to contribute to each Key Term. Once each pair has contributed, have students review all the entries to see what others have written. As a class, debrief each sheet to conclude the activity. Leave the sheets on display throughout the chapter.
- Some students may find it useful to keep a taped or oral summary of what they are learning. Others may work best with a keyboarded version in a software application of their choice.
- Invite a carpenter or architect to talk to the class about design and how math skills related to trigonometry are used in their profession.
- If you have student do the Unit 1 project questions, consider offering them the opportunity to work on these alone or with a partner.
- Some students may benefit from completing all unit project questions.
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These questions provide a beginning for the Unit 1 project.

ELL

- For each section of the textbook, you may want to consider the following approach as a way to assist students:
  - Read the opening paragraphs as a class, and discuss any definitions given in the paragraph. Identify any words with which students are unfamiliar. Suggest that students restate the meaning of the material in their own words. You may even consider having students restate the meaning in their own language first, and then restate it again in English.
  - Discuss and identify examples of any definitions. Provide the class with any necessary materials. Work through the investigation as a class exercise. It may assist students to work in pairs to respond to the Reflect and Respond question. You may even have the class share their findings, and then construct a class response.
  - Work through each Example and Solution as a class, demonstrating the process of solving each problem. Ask the students to work in pairs to solve the Your Turn questions.

Enrichment

- Consider the following triangles, with the given side lengths:
  - Triangle A: 3, 4, 5
  - Triangle B: 1, 1, $\sqrt{2}$
  - Triangle C: 1, 2, $\sqrt{3}$

  As students learn right-triangle facts and processes, encourage them to find why these three triangles have special mathematical significance.

Gifted

- After reading the chapter opener, have students speculate about how finding distances in space terms might be particularly challenging.

Career Connection

You might want to discuss what it means to be an astronomer: what do they do? what kinds of questions do they attempt to answer? what are some of the emerging topics in this field? Many students may not recognize how integral (and complex) the mathematics of astronomy is. Invite them to research the training and qualifications, and employment opportunities in this field. Have them locate where astronomers work in Canada, in their province, and possibly even in their community (they might also look at some of the interesting locations astronomers work around the world). What role could they see trigonometry playing in this career? Have them visit the web link below as a starting point for their research.

Web Link

Students could visit www.mhrmath10.ca and follow the links to learn more about astronomers and astronomy.

The Tangent Ratio

3.1

***Mathematics 10*, pages 100–113**

Suggested Timing

120–140 minutes

Materials

- grid paper
- protractor
- ruler

Blackline Masters

BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Chapter 3 Unit 1 Project
BLM 3–5 Section 3.1 Extra Practice
BLM 3–6 Protractor

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Unit Project Note that #10 and 17 are Unit 1 project questions.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), 2, 3a), b), 4a), b), 5, 6, 9, 10, 17, 18, 20
Typical	#1, 3a)–c), 4a), b), 5–7, 9–13, 18–21
Extension/Enrichment	#10–21

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes for the chapter, encourage students to refer to those that relate to this section.

Before beginning this section, you may want to discuss with students

- that trigonometry is used to find heights and distances that might otherwise be inaccessible
- that right-angle trigonometry involves right triangles
- what a ratio is
- how to solve for a missing value in a proportion

You may want to use the following questions to engage students in a class discussion about sailing.

- Who is involved in sailing?
- How do sailors get to shore if there is an offshore wind?
- How is mathematics used in sailing?

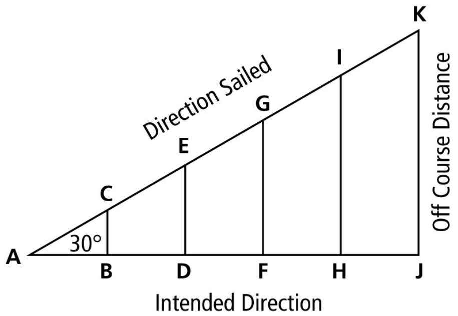
Before beginning the section, you may also want to suggest that students refer to the Key Terms list and make flashcards for any terms that are found in this section.

Investigate the Tangent Ratio

In this investigation, students discover that the basis of trigonometric ratios comes from similar triangles. Similar triangles are scaled versions of each other and, therefore, the ratio of the sides remains the same. Since similar triangles always have equal angle measurements, the ratio of the corresponding sides is the same for any given angle. As such, each angle has a unique tangent ratio.

You could begin this investigation by having students who sail explain how tacking works and describe when they have used it. You might also want to reactivate student knowledge of similar triangles since these are the basis for connecting the data from #2 to the tangent ratio in #4.

Have students work in pairs or small groups. Ask them to complete #1 to 6. You may need to coach some students through the steps from #1c) in order to get a diagram that looks like the one below.



As students work on the investigation, consider asking such guiding questions as the following:

- What is the relationship between △ABC and △ADE?
- How are △ABC and △AHI related?
- What does the $\frac{\text{off course distance}}{\text{intended direction}}$ ratio tell you about each of these triangles?
- How is the tangent ratio of 30° related to this ratio?
- Identify the hypotenuse, opposite side, and adjacent side of your triangles.
- Using this information, how is the $\frac{\text{off course distance}}{\text{intended direction}}$ ratio related to the tangent ratio?
- How can you use this information to develop a formula for determining the tangent ratio?
- Explain how to use this formula to determine the tangent ratios for ∠A and ∠B.

You may wish to refer students to the Did You Know? since this may be the first time students have seen an angle referred to as *theta*, and labelled with the symbol θ. Review the pronunciation with students (*thay* – *ta*) and explain that this method of labelling within the space of the angle avoids having to label the angle using three letters.

This is a good opportunity to discuss the language and symbols of math. You could illustrate how Inuktitut and other languages have their own alphabets. You could discuss how these could easily be used as variables, and then discuss why Greek symbols are more commonly used.

Meeting Student Needs

- Some students may find it easier to use **Master 8 Centimetre Grid Paper** and place the sheet horizontally to draw the triangles.
- Some students may need coaching to get the calculator in degree mode.
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

Enrichment

- Ask students to research other common Greek letters. Students may wish to use them to label the angles of their triangles.

Gifted

Ask students to explain why the tangent ratio can have values greater than one, whereas sine and cosine are never greater than one. Ask them if the tangent ratio has an upper limit and to explain their thinking.

Common Errors

- Students sometimes set up the proportion incorrectly. For example, for tangent, they will make the proportion $\frac{\text{adjacent}}{\text{opposite}}$, rather than $\frac{\text{opposite}}{\text{adjacent}}$.

Rx Go over labelling the triangle with opposite and adjacent sides. The tangent is the ratio of opposite side to adjacent side.

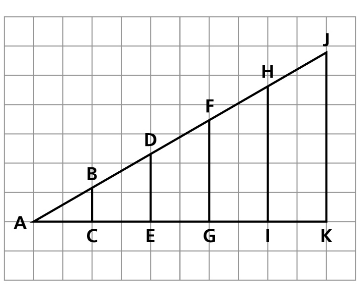
- Students solve the proportion incorrectly.

Rx Review how to solve a proportion by isolating the variable.

Answers

Investigate the Tangent Ratio

1. c)



2.

Triangle	Intended Direction	Off Course Distance	$\frac{\text{Off Course Distance}}{\text{Intended Direction}}$
△ABC	2 cm	1.15 cm	0.5750
△ADE	4 cm	2.3 cm	0.5750
△AFG	6 cm	3.4 cm	0.5667
△AHI	8 cm	4.6 cm	0.5750
△AJK	10 cm	5.75 cm	0.5750

Answers

Investigate the Tangent Ratio

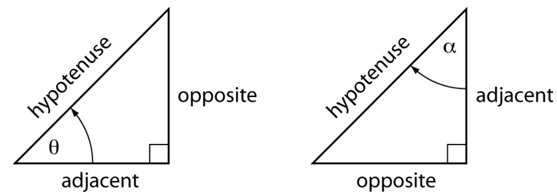
3. a) The triangles are similar because they have all three angles measuring the same.

b) The ratio $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$ is constant for any side lengths with the same angle measures.

4. a) tangent 30° = 0.5773...

b) The value of tangent 30° is equal to the ratio $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$.

5.



6. a) tangent of any angle = $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$

b) tangent A = $\frac{a}{b}$; tangent B = $\frac{a}{b}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the investigation. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to discuss the answers to #6 with the class to assist any students who may not understand the connection between the final column in the chart in #2 and the tangent ratio, and how to use the terminology explained in #5 to develop an appropriate formula.

Link the Ideas

The Link the Ideas discusses the tangent ratio. You may wish to have students draw a sample right triangle in their Foldable and label the sides. Have them explain in their own words how to use the diagram to help them remember how to determine the tangent ratio.

Some students may find it beneficial to note that tan A is the short form for the tangent ratio of angle A. Discuss other mathematical short forms and how these help simplify communication.

Example 1

In this first example, students practise writing trigonometric ratios and then solving them. Ask them how the answers to a) and b) differ from how they might usually see fractions (i.e., the fractions are not simplified).

In the Your Turn, students may wish to put their finger on the target angle before identifying the adjacent and opposite angle. Discuss whether the hypotenuse is ever an adjacent or opposite angle. (The answer is no.)

Example 2

In this example, students are asked to use technology to determine the tangent ratio of an angle, and to find the measure of an angle given the tangent ratio. It is extremely important that students understand what to do when they have a ratio of the sides of a triangle. For example, what does the triangle look like? Drawing and labelling the triangle helps them clarify the relationship between the ratio and how this might appear on the triangle. You may wish to have students draw the diagram for b) before discussing the use of technology to determine a solution.

Review with students how to put the calculator in degree mode before working on trigonometric ratios. Some students may wish to make a note about this in their Foldable.

Have students work in pairs on the Your Turn. They can talk through the calculator key sequence needed for the tangent ratios, solve them, and then compare their answers. Then, each pair can talk through the different key sequence needed for the angles, solve them, and compare their answers.

Example 3

This example is a practical application of the tangent ratio. Using the reference angle in the diagram, have students label the sides as opposite, adjacent, and hypotenuse in relation to the reference angle. They can then apply the tangent ratio and substitute the appropriate values.

Ask students to identify how the Your Turn involves tangent ratios. In terms of tangent ratios, what are they being asked to determine? Have them label the triangle's sides in relation to the reference angle before determining the answer.

Example 4

Before looking at the solution to Example 4, have students sketch the scenario and label the sides in relation to the reference angle. Ask the following:
- What are you being asked to determine?
- What method can you use to do that?
- How will you state your answer?

Have your students draw a diagram showing the scenario in the Your Turn. You may wish to discuss that a clinometer is an instrument that measures angles above or below horizontal. Once students have the diagram, ask questions such as the following:
- What are you being asked to determine?
- What information do you know for sure?
- How can you verify that the other information is correct?

This question provides a good opportunity for students to try verifying different parts of their data. For example, if the measurements are correct, what should the angle be? If the angle and one measurement are correct, what should the other measurement be? Encourage students to discuss which measurement may or may not be accurate and to develop mathematical arguments supporting their point of view.

Key Ideas

Have students discuss the Key Ideas with a partner. What information is new to them? What do they find easy to understand? What is still difficult? Have them add notes about the Key Ideas to their Foldable.

Meeting Student Needs

- Have one or more students develop large visuals showing the adjacent and opposite sides related to a particular angle. Post these visuals around the classroom.
- Some student will not be familiar with the functions on their calculators. Be sure to take time to discuss the use of individual calculators, and specifically how to determine the tangent of an angle and how to find the measure of an angle.
- Have students draw a scenario that shows how to determine the measurement of an acute angle when both legs of the triangle are known, and how to determine the side length if one acute angle and the length of one leg of a right triangle are known. Enlarge and display these visuals.

Gifted

- A unit die has sides one unit long. Ask students to use the tangent ratio to find the minimum distance an ant would have to travel to go from one corner of a unit die to the diametrically opposite corner (you may want to discuss what is meant by *diametrically opposite*).

Common Errors

- Some students may get incorrect answers because their calculator is not in degree mode.

R_x You could reset the calculator or check the mode and reset it. This could be an excellent opportunity to discuss estimation skills and the reasonableness of answers, especially if the calculator is in radians.

Answers

Example 1: Your Turn

a) $\tan L = \frac{5}{12}$ **b)** $\tan N = \frac{12}{5}$

Example 2: Your Turn

0.5095, 1, 1.5399; 27°, 29°, 56°

Example 3: Your Turn

The ladder will reach approximately 3.9252 m up the wall.

Example 4: Your Turn

The guy wire forms a 65° angle with the ground.

Assessment	Supporting Learning
Assessment for Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Provide additional angle measures and ask students to write the tangent values to four decimal places. • Provide additional tan values and ask students to determine the degree measure to four decimal places.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from labelling the sides of the triangle formed by the ladder, wall, and ground according to the reference angle (opposite side and adjacent side). Ask the following: – What is the missing value? – How does this scenario relate to tangent ratios? – How can you use tangent ratios to determine the missing value?
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw the scenario and label the measurements given. Challenge them to consider how to verify any one measurement using the other two. Allow different methods for answering this question.

Check Your Understanding

Practise

Questions #1 to 5 give students practice labelling the sides of a triangle, working with similar triangles, and using their calculator.

Question #6 gives students diagrams and contexts for using the tangent ratio. Have students identify the reference angle, adjacent side, and opposite side in each triangle.

Students need to develop their own diagram for Question #7. Considering that this question deals with an Olympic athlete, you could ask if any students have ever been to an Olympic event. What was their favorite event? Have they ever watched a floor exercise? Are any of them involved in gymnastics? Ask about taking the diagonal run for their routines.

Apply

Students might need help identifying the rise and run in the diagram for #8.

For #11, students should make a diagram with the information given in the problem. Again, remind them to label the sides of the triangle in relation to the reference angle.

For #13, you could link the idea of steepness of a line to the slope of a line. Either give students general guidelines as to what is considered steep, or have students decide amongst themselves.

Extend

For #14, students will have to work with the given diagram, solving the problem using two triangles.

In #15, students will have to be able to draw in the right triangles and appropriately label them in order to determine how the tangent ratio is used. This question is based on a relevant environmental theme, since green building practices are a growing concern in building homes and places of employment. Energy-conscious designers are always trying to come up with ways to help reduce the amount of energy being used. Using the energy from natural resources such as the sun is just one such step in reducing energy consumption.

In #16, students will need to read very carefully and add in the dimensions as they pertain to the problem. Remind students that when they label an angle, the vertex is always in the middle. For example, the vertex of $\angle ACD$ is at $\angle C$. The second part to this question will use the first calculation for the distance across the water. It should be noted that this is how surveyors find inaccessible distances.

Create Connections

Question #18 provides a good section summary. It is recommended that all students answer this question.

For #19, students will have to be able to understand what a ratio of 1 means in terms of the tangent ratio. Have them sketch a diagram and mark a reference angle and give a tangent ratio of 1 for the opposite and adjacent sides. Ask them what this angle measure is.

Question #20 demonstrates how trigonometry may be used to find heights. Devin used material around him to determine the height of the silo. Ask students how he was able to do this. Using things that are around them, could they come up with ways to measure heights of objects in the schoolyard?

For #21, have students work with a partner to measure angles. They would be doing similar things that a surveyor would do to measure distances that are inaccessible or difficult to measure.

Unit Project

The Unit 1 project questions give students opportunities to solve problems involving the use of the primary trigonometric ratios and the Pythagorean relationship to explore how wireless systems have impacted music distribution.

For #10, students use the information about a satellite radio cell tower and three substations to determine the distance of each substation from the intersection of two roads.

Question #17 compares the storage capacities of wax cylinders and today's digital storage systems. Most students will be able to relate to the number of songs that they are able to store in their MP3 players. It is interesting to note that early recording devices contained 2 to 4 min of songs. The standard for a 3-min song came about because of the storage capacity of the early wax cylinders. In this question, students will be able to see how technology has changed in how the music industry is able to bring their product to their fans. This might be a time to discuss what future technology might bring, and how things might change over the next generation of recording devices.

Meeting Student Needs

- Provide **BLM 3–5 Section 3.1 Extra Practice** to students who would benefit from more practice.
- For #21, you may wish to give kinesthetic/tactile learners a separate assignment. They could determine the height of a building in your community, or the rise : run ratio of a ramp (see #9).
- You could arrange for a surveyor to make a presentation in your classroom. The surveyor could demonstrate how to use the equipment as well as discuss the career.

Enrichment

- You can create further discussion for #19 by leading students to find the length of the hypotenuse ($\sqrt{2}$). This triangle allows us to find exact values of trigonometric ratios. You may wish to discuss what an exact value is.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do questions #1a), b), 2, 3a), b), 4a), b), 5, 6, 9. Students who have no problems with these questions can go on to the remaining questions.	• Have students who are having difficulty with #1 and 2, review Example 1. Coach students who are experiencing difficulty, and assign the balance of #1 to check for understanding. • For students who are having difficulty with #3, 4, and 5, coach them through the appropriate parts of Example 2. • Ensure students have a good working knowledge of their calculator, and ensure that they have their calculator in degree mode. You may wish to give students a quick check to perform, so that they always know that they are in the correct mode. Teach them that tan 45° = 1. If, when they enter this in their calculator, they do not get 1, then their calculator is in an incorrect mode. Assign some of the unused parts of questions to check for understanding. • Encourage students having difficulty with #6 and 9 to draw a diagram first and label it with the information given in the question. Have them verbally identify what they will solve for. Reviewing Examples 3 and 4 may assist them in starting their questions.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><th colspan="2">Assessment for Learning</th></tr>
<tr><td>Unit 1 Project
If students complete #10 and 17, which are related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• Have students use different colours to highlight the right triangle for each cell tower in #10. Have them identify the angle and opposite side, then explain how to determine the length of the missing side. Some students may need to talk through how to solve the first triangle before doing the others independently.
• Question #17 requires students to sketch a diagram first. Encourage them to compare their diagram with a partner before beginning to solve.
• Encourage students to label the sides of the given angles to help determine which ratio will be used to solve.
• Remind students to place their solution and all supporting documents into their project portfolio.</td></tr>
<tr><th colspan="2">Assessment as Learning</th></tr>
<tr><td>Create Connections
Have all students do questions #18, 19, and 20.</td><td>• Have students check each other’s notes for #18. This organizer should be added to the Foldable.
• Students having difficulty with #19 should be encouraged to start with labelling a right triangle of their choosing and then identify the opposite and adjacent sides. Prompt them with questions, such as the following:
– What is the tangent ratio, expressed as a ratio of sides?
– How could you use this ratio to solve the missing side lengths? Hint: if the ratio has to equal 1, what numbers are in your ratio?
– If you write a fraction and reduce it to 1, what types of numbers were in the fraction?
• Encourage students to draw two triangles separately for #20. Have them verbalize whether they will need to add or subtract values.</td></tr>
</table>

3.2 The Sine and Cosine Ratios

Mathematics 10, pages 114–124

Suggested Timing

100–120 min

Materials

- protractor
- ruler
- calculator
- 1 m of foam pipe insulation, cut lengthwise
- marble or small steel ball
- eight to ten thick books or bricks or a chair
- masking tape
- measuring tape
- table

Blackline Masters

BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Chapter 3 Unit 1 Project
BLM 3–7 Section 3.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), c), e), 2a), c), 3a), b), e), f), 4a), 5, 6a)–c), 7, 8, 13
Typical	#1a), c), 2a), c)–e), 3a), b), e), f), 4a), 5, 6a)–c), 7, 8, 11, 13, 17
Extension/Enrichment	#10, 12–17

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the chapter outcomes, have students consider those that relate to this section.

You might begin the class by asking students if they have ever been on a suspension bridge. What was it like? About how long was it? How would an engineer calculate the distance a bridge would have to span? Ask, “Would guessing be a good strategy to solve this problem?” Note that there are few bridges built in some parts of Canada because of the geography. For example, in Canada’s Arctic, ice and permafrost can make bridge construction difficult. If you are teaching in one of these areas, you may want to ask students if there is a river or a ravine near their community over which it would be useful to have a bridge.

Before moving on to the investigation, you may want to discuss the meaning of the word *solve*. Students need to understand that this means to find all missing sides and angles.

Investigate Trigonometric Ratios

Accuracy in the drawing of the triangles is important, so suggest that students draw the triangles with a ruler (alternatively, the triangles could be drawn using grid paper). You could ask the class how, if they know only one angle in a right triangle, they can be certain the triangles are similar.

When they are measuring the side lengths, partners should discuss the best unit of measurement to use. Which would be most accurate? Which would be least accurate?

When students fill in the charts and put the ratios in their lowest terms, they should share the results of the charts with the class. Have them discuss and compare patterns and generalizations.

Meeting Student Needs

- This Investigate works best if it is done in pairs or small groups. One student can be the recorder and could create the required tables on a computer. The other students create the triangles needed for the investigation. All students should be involved in the measurements and conclusions.
- This section assumes that students recall angle properties learned in previous math courses. You may want to review parallel line rules, such as alternate interior angles, corresponding angles, and interior angles. Use geometric diagrams that review these properties as well as vertically opposite angles and the angle sum of triangles.
- You could discuss traditional navigation methods. For example, Inuit have many different methods of navigation, such as looking at the stars, sun, and moon, as well as considering the wind direction, snow drifts, landmarks, and land shapes. What affect do you think these technologies are having on the traditional skills that are being used and passed down?
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

Common Errors

- Some students may not take accurate measurements.

R_x Encourage students to draw the triangle with a sharp pencil and straight edge. The side lengths should be measured twice and checked by another student.

- Some students may not draw the angles accurately.

R_x It might be a good idea to review the use of a protractor to draw angles. Students are often unsure whether to use the inner or outer scale. Also, students must be sure that the other angle measures exactly 90º. Encourage students to start with the 90º angle and then draw their chosen angle adjacent to it.

Answers

Investigate Trigonometric Ratios

7. Example: The opposite to hypotenuse ratio in each triangle is equal, as is the adjacent to hypotenuse ratio.

8. Example: The ratio of the opposite to the hypotenuse in one table is the same as the ratio of the adjacent to the hypotenuse in the other table.

9. Example: Since they are all similar triangles, the ratios for this angle will always be the same, regardless of the size of the triangle.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the investigation. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Remind students that their answers are relative estimates. • All students might benefit from writing their results on a common chart on the board. Doing so may provide a better opportunity for students to see the pattern. • It will be important for students to understand that the rules for their calculator mode (degree), angle calculations, and side lengths will be the same for cosine and sine as it was for tangent.

Link the Ideas

The tangent ratio was introduced in the last section. This section introduces two other trigonometric ratios: sine and cosine. Both of these ratios are based on the theory of similar triangles, which was developed in the tangent ratio. You might want to review labelling the sides of the triangle in relation to the reference angle and establish the ratios for sine and cosine.

Example 1

In this example, students write out the trigonometric ratios for sine and cosine, using both acute angles.

Have students work with a partner to do the Your Turn. Partner can explain to each other how they completed one of the questions.

Example 2

This example allows students the chance to familiarize themselves with how to use a calculator to calculate sine and cosine ratios, and how to find an angle given the sine or cosine ratio. If the angle measure is given and they need to find the ratio, they use the sin or cos button on their calculator. If the ratio is given and they need to find the angle, they use the inverse of sine ($\sin^{-1}$) or the inverse of cosine ($\cos^{-1}$).

In the Your Turn, have students work in pairs. Ask them to discuss with each other what is different about recording trigonometric ratios compared to recording degrees. If students have different calculators, encourage them to show each other how their particular calculator works.

Example 3

In this example, students are given a contextual framework and asked to solve for a missing angle. Sketching a diagram is an effective way to organize information. From the diagram, students should identify the reference angle and from this, label the sides of the triangle as hypotenuse, adjacent, and opposite. They must decide which ratio to use to solve the problem based on what was given.

Have students draw a diagram for the Your Turn, identify what they need to do, and then determine the answer.

Example 4

In this example, students are given a contextual framework and asked to solve for a missing side. Again, sketching a diagram is an effective way to organize information.

In the Your Turn, have students sketch a diagram to show the scenario, describe the diagram to a partner, and then discuss how to solve for the missing value.

Key Ideas

Use the following prompts to help students clarify their understanding of the Key Ideas:

- What diagram(s) would help you remember the difference between the sine and cosine?
- What do you need to do to determine each ratio?
- What calculator key sequences can you use to determine the value of the sine ratio? cosine ratio?

Meeting Student Needs

- You could have students create a mnemonic of their own to remember the three trigonometric ratios. Each student could add their acronym to their Foldable (bookmark, recipe card) for easy reference.
- Before studying Example 1, some students may benefit from a quick review of changing fractions to decimals.

Enrichment

- To give students the opportunity to measure sides and angles with and without technology, consider having them work in pairs to construct their own right triangles. They could use a protractor to precisely measure the right angle and one other internal angle. They can then use a ruler to measure either the adjacent or opposite side. Using what they have learned, they can use either the cosine or sine ratio (depending on which side they measured) to find the unknown side length. They can measure this side with a ruler to check their calculation. Pairs could then make two other right triangles, measuring all three sides for each. They could use $\sin^{-1}$ to find the internal angles on one of the triangles, and $\cos^{-1}$ to find the internal angles on the other. They can check their calculations by measuring with a protractor.

Gifted

- You could have students research the graphical representation of the sine and cosine ratios, for reference angles of 0º to 360º (see Web Link below).

  For the sine ratio, direct students' attention to the fact that it oscillates from –1 to +1. This means the absolute value of the sine ratio can never be greater than 1. You may also want to guide their attention to the fact that the graph shows that the sine ratio is 0 when the reference angle is 0º, 180º, and 360º. The sine ratio is exactly 1 when the reference angle is 90º.

  For the cosine curve, the ratio oscillates from –1 to +1. This means the absolute value of the cosine ratio can never be greater than the absolute value of 1. Students should also notice that the graph shows that the cosine ratio is 0 when the reference angle is 90º and 270º. The cosine ratio is exactly 1 when the reference angle is 0º and 360º.

Web Link

For links to graphical representations of the sine and cosine ratios, have students visit www.mhrmath10.ca, and follow the links.

Common Errors

- Some students may label the sides of the triangle incorrectly.

R_x You can review with students how to label a triangle using the reference angle. The hypotenuse and opposite side are often the easier sides for students to identify. The adjacent side would be the remaining side of the right triangle.

- Some students may write the sine or cosine ratios incorrectly.

R_x You can review the primary trigonometric ratios using memory aides. Encourage students to write a memory aid that works for them at the top of the page and for each question write out the trigonometric ratio before substituting known values.

WWW Web Link

Students can visit www.mhrmath10.ca and follow the links in order to find out more about the northern and southern lights, and how the parallax is used in astronomy.

Answers

Example 1: Your Turn

a) $\frac{5}{13}$ **b)** $\frac{5}{13}$ **c)** $\frac{12}{13}$ **d** $\frac{12}{13}$

Example 2: Your Turn

a) 0.8660, 0.5000, 0.7071 **b)** 26°, 78°

Example 3: Your Turn

45.1°

Example 4: Your Turn

423.8 m

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Suggest that students write out the sine and cosine ratios for each angle before starting to solve. • Remind students to have their calculators in degree mode.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students recall how to use the calculator to calculate sine and cosine ratios, and how to find an angle given the sine or cosine ratio. • Suggest that students write a quick rule as to when to use the *sin* and *cos* buttons, and when to use the inverse buttons on their calculator. These are concepts that many students confuse. Posting the ratios and a quick reminder as to when each is used would be helpful. • Provide students who are having difficulty with remediation and additional questions.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to verbalize which is the reference angle and what the relationship of the sides is to this angle. • Help students recall how identifying the sides determines the trigonometric ratio to use. Ask the following questions: – Which sides are used to calculate cosine? – Which sides are used to calculate sine? • Encourage students to draw a diagram and label the sides in relation to the reference angle.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Because students will be working with both sine and cosine questions, encourage them to draw a diagram and label the sides with the reference angle from the problem. This should be standard practice for students who struggle without visuals.

Check Your Understanding

Practise

Questions #1 and 3 will give students much needed practice using their calculators. Give students time to explain what is happening when they use their calculators and why they are pressing the buttons they are pressing. Encourage students to understand the difference between the ratio and the angle when using their calculator.

In #4 to 6 students should sketch and label the diagrams to help them decide which ratio to use and how to write the ratio.

Apply

Observe how students solve #7. Some will immediately go to the Pythagorean relationship to find the missing side. Others might use the tangent ratio to find the missing angle and then either the sine or cosine ratio to find the missing side. Discuss with students which method they use and encourage students to share their thinking.

For #8 to 10, students need to read through each problem to be able to gather all the information that is needed to solve it. Encourage them to sketch out the problem, list the components that they are given, and state what it is they are asked to find.

For #11, students need to draw a diagram for each scenario. You might also want to discuss why it is important to avoid using a calculated measurement to find another measurement whenever possible.

For #12, students may struggle with how to place the information on one right triangle. Since the question tells them the sag in the center is 4600 m long, students will need to bisect some of the measurements.

Watch what method students use to solve #13. You may want to provide guidance if any of your students are having difficulty.

Question #14 involves multiple triangles. You could encourage students to draw two separate diagrams for this question: one for the beginner slope and one for the expert slope.

Extend

For #15, students will need to put together all that they have learned about using the sine and cosine ratios. You might want to suggest to them that they list the components that they know, and what they need to find out. They will need to plan out how they will logically progress through this problem.

For #16, you may want to ask students how they could use the diameter to help them solve this problem. Also suggest that diagramming this problem would be very useful in solving the question.

Create Connections

Question #17 involves the Olympic event of ski jumping. For the ski ramps at Whistler, the length of the in-run is 116 m with an angle of 35º. This activity will allow students to see how the length of the run and the slope of the take-off ramp will affect the distance that a ball will travel. Students should work in small groups, where someone will measure the distance the ball travels, someone will let the ball roll down the ramp, and someone will control the slope of the take-off ramp. At the end of the activity, discuss as a class how the distance may be improved and what factors are involved. Once students have completed this question, you might want to have a discussion about ski jumping. What factors have they discovered in the question that may affect the length of a ski jump? You might also ask them if they can think of other factors that are not reflected in their experiment with the ramp and marble.

Meeting Student Needs

- Provide **BLM 3–7 Section 3.2 Extra Practice** to students who would benefit from more practice.
- For #11, some students may need help with the diagram, particularly those that have not played golf. You may want to discuss some of the terms, such as teeing area, fairway, and hole/flag.
- For #12, some students may not be familiar with gondolas and zip lines. You may want to have a discussion, asking questions such as the following:
  - Who has been on a gondola?
  - Why does the line sag?
  - Who has ever gone on a zip line?
  - Does the zip line sag like the gondola line?
  - How could engineers take into account the sag when constructing such lines?
- For #13, if students are struggling, you may want to suggest that they complete part b) before part a). If they find the angle first, the side length can be found using either the tangent or sine ratio.
- Students who are unfamiliar with skiing may have difficulty with #14. You could have a discussion, asking questions such as the following:
  - How many of you ski?

– Do any of you ski the difficult, or black diamond, runs?
– What makes a run difficult?
– What is the steepest run you have ever been on?

- For #16, you may want to remind students what it means for a triangle to be equilateral. Some students may also need to be reminded what *diameter* means. This may be a good question to have students work on in pairs so that they can strategize together about how to solve this problem.
- For #17, you may want to review the sport of ski jumping; some students may be unfamiliar with this event. Ski jumping is an event where skiers go down an in-run to a take-off ramp, attempting to jump as far as possible.

Web Link

You may suggest students visit www.mhrmath10.ca to learn how trigonometry is reflected in sports. They can also find information on the gondola at Whistler Blackcomb (in support of #12), and about the sculptures of Floyd Wanner (in support of #13).

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply #1a), c), e), 2a), c), 3a), b), e), f), 4a), b), 5a), b), 6a)–c), 7, 8. Students who have no problem with these questions can go on to the remaining questions.	• Students who have difficulty with either #1 or 2 should review Example 1, and be coached through the solution to the assigned questions. To check for understanding, use one of the additional parts in each question. Review the identification of the sides. • For questions #3 and 4, direct students to Example 2. Providing them an opportunity to try additional questions, and then the unassigned ones in #3 and 4, will help check for understanding. • Questions #5 and #6 are linked to Examples 3 and 4 respectively. Reviewing them and the mode and entry of values into the calculator would be beneficial. • Encourage all students to make it standard practice to draw a diagram reflecting the question, labelling the sides and angles before looking at what the question is asking them to find. • It would be beneficial for all students to revisit the term *angle of elevation*.
Assessment *as* Learning	
Create Connections Have all students do questions #17.	• It might be beneficial to have students of mixed ability work in groups of 3 or 4 to complete this mini lab. • Students will need sufficient space to complete the trials. Borrowing some material from the science lab may be required.

3.3 Solving Right Triangles

Mathematics 10, **pages 125–135**

Suggested Timing

100–120 min

Materials

- metre stick or measuring tape
- masking tape
- calculator

Blackline Masters

BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Chapter 3 Unit 1 Project
BLM 3–8 Section 3.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), c), 2, 3, 4 a), b), 5a), e), 6, 7, 11, 16
Typical	#1a)–c), 2, 3, 6, 7, 11, 12, 16
Extension/Enrichment	#9, 13, 14–16

Unit Project Note that #11 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the chapter outcomes, refer students to those that relate to this section.

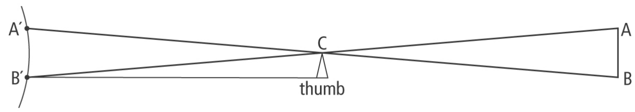
Ask students what they know about the *polar aurora*. You may need to expand on their responses by explaining that auroras are natural light displays in the sky, which are usually seen at night. Polar auroras are sometimes called the northern and southern lights. This phenomenon is more common the closer you are to the magnetic poles, so many student from lower latitudes may not have witnessed it. You may want to explain to Northern students that this experience, while common for them, is rare for most people in southern Canada.

For interest, you may want to tell students that many Aboriginal and Inuit legends attribute the northern lights to spirits at play or dancing. Some stories say that when you whistle, the lights will dance. However, the aurora was also feared, and in some Inuit traditions, children were warned never to whistle at the lights because the auroras could come closer and snatch them away.

Investigate Estimation of Distance

You could begin this investigation by discussing with students possible ways of estimating distances. You might also want to take some time to clarify the term *parallax*, since it is the basis or main idea introducing the investigation. (A parallax is an apparent change in the position of an object that is viewed along two different lines of sight.) To help students grasp this concept, draw their attention to the Did You Know? feature on page 126. Have them stretch out an arm, holding up their thumb and closing one eye. Have them line their thumb up with an object in the classroom, and then alternate their open and closed eyes. They should notice how the position of their thumb in relation to the object appears to shift. This shift is the parallax.

For most people, the angle between the lines from the eyes (A′B′) to the outstretched thumb is about 6°. That angle is the parallax of your thumb, viewed from your eyes. The triangle A′B′C is similar to the larger triangle ABC. With an angle of approximately 6°, the similarity ratio is 10:1. Therefore, if the distance B′C to the thumb is 10 times the distance A′B′ between the eyes, the distance AC to the far landmark is also 10 times the distance AB.



$$\frac{B'C}{A'B'} = \frac{AC}{AB}$$
$$= \frac{10}{1}$$

Encourage students to draw a diagram and label it with their measurements. Students should repeat their measurements to see if a pattern exists.

Meeting Student Needs

- To illustrate the concept of a parallax, you could take your class outside and ask them to focus on a distant object (a house, a tree, a soccer post, etc). Students should hold their hand out with one finger raised and close one eye. They need to line up their raised finger and try to block out the sight of their chosen object. Now without moving their arm, they should close their open eye and open their closed eye. They will find that the object, when viewed through the other eye, will no longer be blocked. Students should then move their finger to block that eye's line of sight. This change in finger position is the parallax. The distance to the chosen object can be determined by finding how far apart the finger positions are.
- Consider having students do this investigation with a partner or in groups of three, where one person could do the measuring of the distances. The same person should do the measuring to eliminate measurement error as much as possible.
- Students could complete the investigation outside in a large area and then use either a pedometer or a trundle wheel to determine the distances.
- Some student benefit from hands-on activities. If you teach in a community where there is an observatory, you could consider arranging an evening field trip to visit it. Invite an astronomer to give a presentation and to discuss what astronomers do, and how mathematics is used in their field.
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

Enrichment

- Ask students to investigate how trigonometry could be used to find distances in space where direct measurement is impossible.

Gifted

- Suppose the first triangle of a series of right isosceles triangles has legs of 1 cm and an unknown hypotenuse. If the hypotenuse forms a leg of the next triangle in the series, have students find the dimensions of the 6th triangle in the series. (Note that you may want to have students recall the meaning of isosceles).

Answers

Investigate Estimation of Distance

4. a) Example: The distance between my partner and I was approximately six times the distance between the two locations where my partner was standing.

b) Estimate the parallax distance at your object and multiply it by six.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the investigation. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• The class could develop a large chart on the board, and students could compare two of their triangles with the rest of the classes' results. • Encourage students to identify whether there is a best ratio to use. • Many students would benefit from completing the investigation on grid paper.

Link the Ideas

To be able to solve trigonometric problems, students must understand the vocabulary used in the problem. Angle of elevation and angle of depression are commonly used in trigonometry problems:

- An angle of elevation is an angle made above the horizon or horizontal line.
- An angle of depression is an angle made below the horizon or horizontal line.

Because of alternate interior angles, the angle of depression is equal to the angle of elevation. You might want to review alternate angles with students. Have them recall that alternate angles are angles that are either both inside or both outside a set of parallel lines, but on opposite sides of the transversal. An angle of depression and an angle of elevation are alternate angles because they are inside a set of parallel lines (the horizontal) and on opposite sides of the transversal (the line cutting the horizontal).

Example 1

Have students talk through the strategy used to solve Example 1 and discuss how the visual assists them in solving the problem.

When students start the Your Turn, point out that a transit measures angles of elevation.

Have students draw a diagram to show the scenario in the question. You may wish to use coaching questions such as the following:

- Which side of the triangle represents the height of the silo?
- What information does the problem give you?
- Where does the reference angle go?
- How does drawing and labelling the triangle help you decide which trigonometric ratio to use?
- What trigonometric ratio is that?

Example 2

You could discuss with students what it might be like to be standing on the top of a rock and looking down. Emphasize that this is the angle of depression. Students should sketch a diagram for the problem, drawing the top horizontal line and labelling this angle 73º. Remind students that the angle of depression and the angle of elevation are congruent angles.

You may wish to have students work in pairs for the Your Turn. They can each draw and label a diagram of the scenario, then talk through how to determine the height of the balloon.

Example 3

In this example, students are asked to solve a triangle. You will need to discuss with the class what *solving* a triangle means. To *solve* a right triangle means to solve for all the missing side lengths and angles in the triangle. Encourage students, whenever possible, to use only information given in the question to find the missing values.

Students should begin by listing the quantities they know and placing them in a diagram. There are several ways to begin answering this question. You could ask the class for input on how they would start. Students will see from the varied responses that there are several ways to begin this type of question. Some students may want to use triangle sum to determine the measure of the third angle, while other students might want to use a trigonometric ratio later to determine the missing angle.

For the Your Turn question, encourage students to use a different method than the one they used in Example 3.

Example 4

Trigonometry is used to solve several types of problems in mathematics, science, and industry. Some of these problems require two right triangles to model the situation. It is often best to separate these problems into two problems.

You might want to ask students which ratio they would need to use to solve the problem. Once the two triangles are solved, ask students what they do with the answers.

You may wish to have students work in pairs on the Your Turn. They can draw a diagram representing the scenario, decide what two triangles need to be solved, and each develop that solution. Then, the students can decide what to do with their two answers in order to determine how far the bus travelled.

Key Ideas

You may wish to use coaching questions such as the following to help students talk through the Key Ideas.

- How are the angle of depression and angle of elevation similar?
- How do they differ?
- How can you remember which is which?
- What is entailed in solving a triangle?
- What three trigonometric ratios might you use in solving a triangle?
- What other strategies might you use?
- What information about the angles of a triangle might you use to help with or verify your solution?

Meeting Student Needs

- For the Link the Ideas, consider having students create posters representing examples of "Angle of Elevation" and "Angle of Depression" as they relate to something familiar to them. For example, if students are skiers, they could sketch a diagram showing the angle to the top of the ski slope.
- In Example 1, you may have to remind some students that the height of the transit has to be added to the height calculation of the totem pole.
- In Example 4, you might want to review compass directions.

Enrichment

- To give students the opportunity to solve triangles with and without technology, consider having them work in pairs to construct their own right triangles. They can use a protractor and ruler to measure and provide either two sides lengths, or one side and one internal angle. They can then use what they have learned in sections 3.2 and 3.3 to solve the triangle. They can check their calculations by measuring with a ruler and protractor. (You might extend this exercise by having students mark the unknown sides and angles as variables. They can then solve for these, writing the corresponding values on the back of the triangle. Pairs can then exchange triangles with other pairs, solve the triangles, and check their answers with the ones provided on the back.)

Common Errors

- Some students will make a mistake on one value and use this incorrect value throughout the question.

R_x Encourage students, whenever possible, to use only the original values given in the question. If students make a mistake, it will not affect the remaining measurements if they always go back to the original question.

Answers

Example 1: Your Turn
85 m

Example 2: Your Turn
150 yd

Example 3: Your Turn
52 m, 54°, 36°

Example 4 Your Turn
151 m

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw and label a diagram for all angles. • Students commonly have difficulty with deciding where to draw shadows and typically draw them on the hypotenuse. Have students go outside on a sunny day to visually see where a shadow would appear (on the ground). • You may need to coach students through the meaning of *elevation*, and how it applies to angle of elevation or the angle measured from the base. • Encourage students to write the meanings of elevation and depression into their Foldable, along with a definition and example in their own words.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw and label a diagram for all angles. • Students commonly have difficulty with deciding where to draw and how to measure, the angle of depression. Have students go to a window or higher place and look down at an object. Help them relate their line of sight to the triangle needed. • You may need to coach students through the meaning of *depression* and how it applies to angle of depression or the angle measured from the imaginary horizon line.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label a diagram for each question. Label the corresponding sides and determine which angle is being referenced. • Have students verbalize which method is easier for them to understand when solving. • Have students check that their calculator is in degree mode.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may find multiple right triangles confusing. Encourage them to redraw and label separate triangles when solving. Encourage them to label the sides and highlight the common sides.

Check Your Understanding

Practise

In #1, students should label the sides of each triangle and then decide which trigonometric ratio to use.

Questions #2 and 3 are multiple-triangle questions: one triangle is dependent upon the other. Students will decide from the diagram which elements are missing and what ratio they need to solve them with.

Question #5 is based on the fact that many towns and cities across Canada have roadside attractions that represent an aspect of the community. You could ask students if anyone has seen an attraction like this. If not, ask students about other attractions they have come across when travelling through communities.

Apply

Ask students where the given angle for #6 should be placed and how to draw the diagram.

For #7, students should sketch and label a diagram. There is extra information provided in this problem; students will have to decide where it fits in. What will they need to do to justify their answer?

Question #9 may provide material for an interesting discussion. You could ask students the following questions:

- How are cranes raised as the height of the building increases?
- Why don't the cranes tip over?

Students will need to be aware of how the arm moves and the different angles that the operator would see as he looks down to negotiate the boom.

In #10, you might want to ask students what kind of work would be needed to arrange people into the design shown in the Arctic Wisdom picture. Arctic Wisdom was a special two-day briefing on the scientific, cultural, and political issues related to climate change in the Arctic.

Students should sketch a diagram for #12. They may need to understand where the helicopter is in relation to the boats. Remind them that this is a two-triangle problem.

Extend

Question #14 involves another step. Once students have determined both distances, they must determine the truck's speed. Another twist to the problem is that the initial measurement is given in metres, but the speed limit is given in kilometres per hour. Students will have to convert these values to the same units. Also, since the speed of the truck is given in kilometres per hour, and the time span is given in seconds, students will have to compute the equivalent speed.

Question #15 is the first three-dimensional trigonometric problem students have seen. A possible first step to solving this problem is to draw the right triangle that has the rod as its hypotenuse, and then decide what pieces are missing and what needs to be done to find those missing pieces.

Create Connections

In #16, students will need to sketch the problem from both Jennie's and Mike's perspectives. They need to do both sets of calculations to establish whether Richard is correct.

Unit Project

The Unit 1 project questions give students opportunities to solve problems involving the use of the primary trigonometric ratios and the Pythagorean relationship to explore how wireless systems have impacted music distribution.

Question #11 is a multi-triangle question in which students have to find a length in one triangle to find the length in another triangle. You might want to discuss with students the fact that cell phones are now equipped with a GPS that will allow the user to be located. At least three towers will pick up the signal, transferring your call to the tower that is least busy or that gives the strongest signal. It is possible to triangulate a cell phone as the radio waves from the cell phone are sent out. As a result of the triangulation, cell phone users are able to be located when they initiate any kind of call with the cell phone.

Meeting Student Needs

- Provide **BLM 3–8 Section 3.3 Extra Practice** to students who would benefit from more practice.
- Some questions in the Check Your Understanding rely on the idea of complementary angles adding to 90º. You might want to review this concept with students who may not recall it from earlier math courses.
- Some students may benefit from building physical models of some of the questions. These models can be very simple, built from items such as a cardboard box and a rope or string.

- For #8, you might want to explain what an alidade is a device, originally used in astronomy, that allows someone to sight a distant object and use the line of sight to measure the angle.
- Some students may have difficulty with the conversions in #14. You may want to provide guidance or have students work in pairs.
- Some students may have difficulty visualizing and drawing the situation in #15. They may benefit from using manipulatives to model the problem and then draw it.

Enrichment

- One of the many uses of trigonometry is to aid in navigation. Encourage students to create questions that involve airplanes traveling with winds at right angles blowing them off course.

Gifted

- Ask students to explore how triangles with angles greater than 90º could be solved using combinations of right triangles.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students complete questions #1a), c), 2, 3, 4a), b), 5a), e), 6, 7. Students who have no problems with these questions can go on to the remaining questions.	• By labelling the sides of the triangles for #1, students should be able to make choices as to which trigonometric ratio to use. Problems in this section will combine all the trigonometric ratios that the students have worked with. • Questions #2 and 3 use multiple right triangles. Encourage students to redraw the triangles separately, and highlight the side common to them. Labelling the sides using their reference angle should assist in deciding which trigonometric ratio to use. • For question #4, have students review Example 2. They should remember that the elevation angle is measured from the ground, and angle of depression is measured from the line of sight. Have students label the sides of the triangle. • Question #5 provides an opportunity to solve triangles in real world scenarios, using tourism as an example. Encourage students to draw and label a diagram before beginning. • For any questions from #1 to 5 that required extra coaching, use the unassigned questions to check for understanding. • Questions #6 and 7 require students to model a scenario with a diagram and solve it. Remind students that they need to support their answer with work and calculations.
Unit 1 Project If students complete #11, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• For question #11, remind students that, as part of their submission for the Unit 1 project, they need to draw and label their diagram and show their thinking, including all calculations. • Remind students to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students do #16.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the question, and then have them provide individual responses orally or in written form. • Encourage students to offer a good rationale for angles of elevation and depression. Remind students that they are to explain their thinking in both parts.

3 Chapter 3 Review

***Mathematics 10*, pages 136–137**

Suggested Timing

60–90 min

Materials

- ruler
- protractor
- calculator

Blackline Masters

BLM 3–5 Section 3.1 Extra Practice
BLM 3–7 Section 3.2 Extra Practice
BLM 3–8 Section 3.3 Extra Practice

Planning Notes

Have students work independently on #1 and 2, and then compare their answers with those of a classmate. After students have made corrections, have them work independently on the remaining questions. After completing the questions for each of sections 3.2 and 3.3, you might have students check their solutions with a classmate.

If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their chapter Foldable, the worked examples, and previously completed questions in the related sections of the student resource.

Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the practice test.

Meeting Student Needs

- Encourage students to refer to the Foldable or other materials created during the chapter.
- You may wish to hand out a copy of the student learning outcomes with a happy face and sad face beside each objective. Students could circle the appropriate icon as it refers to their understanding of each objective. You could use the student responses to direct them to various questions within the review.
- Encourage students to sketch and label diagrams in their notebooks. It may be helpful to label the hypotenuse, as well as the sides that are opposite and adjacent to the reference angle.
- Encourage students to construct small models using available materials, such as paper, pencils, and erasers. This may help tactile learners take the next step of drawing a diagram.
- Students should show the setup for each question. You may want them to explain how they determined which trigonometric ratio to use.
- Be prepared if students ask about rounding the answer for an angle. Although the examples in the chapter rounded the answer to the nearest degree, these questions require students to round to the nearest tenth of a degree.
- Remind students that *solve* means to find all missing sides and angles.
- Students who require more practice on a particular topic may refer to **BLM 3–5 Section 3.1 Extra Practice**, **BLM 3–7 Section 3.2 Extra Practice**, and **BLM 3–8 Section 3.3 Extra Practice**.

Gifted

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 3 Review The Chapter 3 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

Chapter 3 Practice Test

3

***Mathematics 10*, pages 138–139**

Suggested Timing

40–50 min

Materials

- ruler
- calculator

Blackline Masters

BLM 3–9 Chapter 3 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need a lot of help with, a little help with, or no help with. Have students first complete the questions they know they can do, followed by those they know something about. Finally, suggest that students do their best on all the questions, even those with which they are struggling. Tell them that this test is for practice, and struggling with a question serves to show them what concepts they will want to revisit.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–4, 6–9.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1, 5	3.2	Example 1	✓ use similar triangles to find missing sides
#2	3.3	Example 3	✓ use the Pythagorean theorem to solve a right triangle
#3, 8	3.3	Example 4	✓ use the sine or cosine ratio to find a missing side
#4, 9b)	3.2	Example 3	✓ use the tangent ratio to find a missing side
#6a)	3.2	Example 2a)	✓ evaluate the tangent ratio
#6b), c)	3.3	Example 2a)	✓ evaluate the sine or cosine ratio
#7a), c)	3.3	Example 2b)	✓ calculate the measure of an angle for the sine or cosine ratio
#7b)	3.2	Example 2b)	✓ calculate the measure of an angle for the tangent ratio
#9	3.4	Example 1	✓ solve a triangle involving angle of elevation
#9a)	3.2	Example 4	✓ use the tangent ratio to find a missing angle

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 3 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their chapter Foldable.	• Make grid paper available to students. • Have students use their responses on the practice test and work they completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 3 Test After students complete the practice test, you may wish to use **BLM 3–9 Chapter 3 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable.

UNIT 1

Unit 1 Project

Mathematics 10, page 140

Suggested Timing

60–90 min

Blackline Masters

Master 1 Project Rubric
BLM U1–3 Unit 1 Project Final Report

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

General Outcome

Develop spatial sense and proportional reasoning.

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:

- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Planning Notes

Start the class by asking students to brainstorm what changes might occur in the music industry during their lifetime. You may wish to introduce this by outlining the changes that have occurred since you were a teenager.

List and post ideas from the the brainstorming session. Then, explain that students will complete their unit project by visualizing what will happen next in the music industry. Review the expectations for the presentation outlined under Unit Connections on page 140 in the student resource. Clarify with the class that the final part of the project involves the following:

- providing details about a future advancement in music distribution, including a description of the equipment, dimensions, and volume (in SI and imperial units)
- explaining how this equipment would allow people to access music

Emphasize that students are being encouraged to use their imagination to come up with ideas, and then to use the mathematics skills they have acquired to describe the advancement and how it might work.

Suggest that students review the contents of their project portfolio and ensure that they have completed all the required components for their final report or presentation.

Meeting Student Needs

Gifted

- *The Brain that Changes Itself* by Norman Doidge, M.D. (Penguin Books, 2007) outlines many of the techniques researchers use to study the brain. Challenge students to consider which ones could be adapted for music distribution and how they might be used.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 1 Project This unit project gives students an opportunity to apply and demonstrate the concepts, skills, and proccesses learned in Unit 1. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on the Unit 1 project.	• You may wish to have students use **BLM U1–3 Unit 1 Project Final Report**, which provides a checklist for students to identify where in their project they demonstrate the concepts, skills, and processes explored in Unit 1. • Reviewing **Master 1 Project Rubric** with students will help clarify the expectations and the scoring. It is recommended to review the scoring rubric at the beginning of the unit, as well as intermittently throughout the unit to refresh students about the project assessment.

The Specific Level Notes below provide suggestions for using **Master 1 Project Rubric** to assess student work on the Unit 1 project.

Score/Level	Specific Level Notes
5 (Standard of Excellence)	• provides a complete and correct response with clear and concise communication; may include a minor error that does not affect the understanding of the overall project; may include weak communication in no more than one calculation
4 (Above Acceptable)	Provides *one* of the following: • a complete response to all parts of the project, with missing justification in at most two calculations; includes good communication that addresses how the various changes have affected the music industry • a complete response with one error that is carried through correctly, (i.e., uses a diameter in the calculation of area of a circle or solves for the lengths of sides of multiple right triangles correctly, but uses the measures in an incorrect trigonometric ratio); includes good communication that addresses how the various changes have affected the music industry • a response which correctly addresses all parts of the project but is difficult to follow and lacks organization; does not provide support for the recommendations or conclusions; includes good communication that addresses how the various changes have affected the music industry
3 (Meets Acceptable)	Demonstrates *one* of the following: • makes initial correct start to all sections of the project • correctly completes basic measurement estimates and calculations in SI and imperial systems; completes surface area and volume calculations with some errors; demonstrates a basic understanding of trigonometry by correctly solving for the sides of right triangles and generally setting up the trigonometric ratios correctly; includes good communication with some connections • provides answers to all questions without supporting work or justification
2 (Below Acceptable)	• makes initial starts to various sections of the project; provides some correct links • makes some correct estimates and reasonable measurements in one or both measurement systems • demonstrates the ability to find diameters and perimeter with relative accuracy; further calculations have some errors • attempts surface area and volume calculations with some errors • attempts trigonometric ratio calculations, with some or limited success • includes some communication
1 (Beginning)	• makes initial starts to various sections of the project but is unable to carry through or link concepts together • has difficulty estimating in one or both measurement systems • includes responses that are basic and limited to only one measurement system • demonstrates the ability to find diameter and perimeter with some accuracy; includes numerous errors in further calculations • attempts surface area and volume calculations, but makes numerous errors • may attempt trigonometric calculations, with little or no success • includes little or no communication

UNIT 1

Review and Test

***Mathematics 10*, pages 140–145**

Suggested Timing

60–90 min

Materials

- ruler

Planning Notes

Have students work independently to complete the review and then compare their solutions with a classmate. Alternatively, assign the Unit 1 Review to reinforce the concepts, skills, and processes learned so far. If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their notes in each chapter Foldable, and then to the specific section in the student resource and/or their notebook. Once they have found a suitable strategy, have students add it to the appropriate section of their chapter Foldable.

Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the unit test.

Meeting Student Needs

- Encourage students to draw and label diagrams, when appropriate.
- Make grid paper available to students.
- Encourage students to use their chapter Foldables and to add new notes if they wish.

Enrichment

- Students interested in *The Glass Wheatfield* in #9 may wish to visit the site of the Regina Plains Museum, the home of this unique and internationally recognized work of art. See the Web Link below.

Web Link

To visit the Regina Plains Museum site, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 1 Review The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students review their notes from each Foldable and the tests from each chapter to identify items that they had difficulty with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section that they are having difficulty with.
Assessment *of* Learning	
Unit 1 Test After students complete the unit review, you may wish to use the unit test on pages 144 and 145 as a summative assessment.	• Consider allowing students to use their chapter Foldables.

UNIT 2

Algebra and Number

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:
- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN2 Demonstrate an understanding of irrational numbers by:
- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Mathematics 10, pages 156–159

Suggested Timing

30–40 min

Blackline Masters

BLM U2–1 Unit 2 Project
BLM U2–2 Unit 2 Project Checklist

What's Ahead

In Unit 2, students investigate and extend their knowledge of powers and exponents. Students solve problems involving square roots and cube roots and apply the exponent laws to expressions involving powers with integral and rational exponents. They learn how to represent, identify, and simplify irrational numbers and order irrational numbers. Students also learn how to multiply and factor polynomials. Students extend these skills to factor the difference of squares and perfect square trinomials.

Planning Notes

Introduce Unit 2 by pointing out the algebra and number organizer on page 156 of the student resource. This concept map shows how the topics in this unit—the real number system, including rational and irrational numbers, exponents, and polynomials—are related. The concept map is repeated at the beginning of each chapter and is shaded to show which topics are covered in that particular chapter.

The Looking Ahead box at the bottom of page 157 identifies the types of problems students will solve throughout the unit. You may wish to reactivate students' knowledge of these topics.

Unit 2 Project

The Unit 2 project focuses on the real-world application of mathematics in art. The project is continuous in nature and is explicitly divided by chapters.

Introduce the Unit 2 project by reading and discussing the introductory notes on page 158 of the student resource as a class. Consider distributing **BLM U2–1 Unit 2 Project** to inform students about how the project develops throughout the unit. This master provides an overview of the project as well as the requirements for completing the Unit 2 project.

You may wish to point out the questions related to the Unit 2 project that are indicated throughout Chapters 4 and 5 with a project logo. Note that these questions are not mandatory but are recommended because they provide some of the background and research needed to complete the Unit 2 project. The questions are also available on masters, one for each chapter. You may decide to use these masters to create a student booklet and have students record their finalized answers in the booklet either after they have completed their in-class work, during assigned project work time, or in conjunction with chapter assignments. Alternatively, you may wish to provide students with **BLM U2–2 Unit 2 Project Checklist**, which lists *all* of the related questions for each chapter. Students can use the checklist to monitor their progress and prepare their presentation and report. Have students store all the work for the Unit 2 project in a portfolio.

For additional information on the Unit 2 Project, see the Unit 2 Connections on page 256 in the student resource or TR pages 202–203.

Career Connection

Use the collage of photographs to direct a discussion about careers that are related to math. For example, students may mention how math is involved when architects design buildings, scientists monitor water quality, and accountants analyse financial performance. Give an example such as a builder who used the Pythagorean relationship to ensure that a deck she is building will have square corners. You might mention some other careers that involve math skills. For example, graphic designers create effective ways to get messages across in print, electronic, and film media using methods such as print and layout techniques, illustration, photography, and animation. Multimedia artists and animators create special effects and animation for film and video, advertising, and computer systems design. They draw by hand and use computers to create the sequence of pictures that form animated images or special effects. Ask students what they know about how each of these careers involve math.

Web Link

For information about careers related to math, training and qualifications, employment, and job outlook, go to www.mhrmath10.ca and follow the links.

Exponents and Radicals

4

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:
- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN2 Demonstrate an understanding of irrational numbers by:
- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
4.1	✓ determine the square root of a perfect square and explain the process
	✓ determine the cube root of a perfect cube and explain the process
	✓ solve problems involving square roots or cube roots
4.2	✓ apply the exponent laws to expressions using rational numbers or variables as bases and integers as exponents
	✓ convert a power with a negative exponent to an equivalent power with a positive exponent
	✓ solve problems that involve powers with integral exponents
4.3	✓ apply the exponent laws to expressions using rational numbers or variables as bases and rational exponents
	✓ solve problems that involve powers with rational exponents
4.4	✓ represent, identify, and simplify irrational numbers
	✓ convert between powers with rational exponents and radicals
	✓ convert between mixed radicals and entire radicals
	✓ solve problems involving radicals

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 4–1 Chapter 4 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 150 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal to explain what they personally know about exponents and powers. You might provide the following prompts: • Where have you encountered exponents and powers? • Why might exponents be important, and to whom? • In what instances in your life did you need to know about exponents and powers?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 4–2 Chapter 4 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 4 Foldable As students work on each section in Chapter 4, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter.
Assessment *for* Learning	
BLM 4–3 Chapter 4 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 4 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 30–40 min (TR page 113)	Students should be familiar with • prime factors • operations on powers with integral bases and whole number exponents • calculating area of squares and volume of cubes		BLM 4–1 Chapter 4 Self-Assessment BLM 4–2 Chapter 4 Prerequisite Skills BLM 4–4 Chapter 4 Foldable BLM 4–5 Chapter 4 Unit 2 Project BLM U2–2 Unit 2 Project Checklist		TR page 112 Chapter 4 Foldable, TR page 112	TR page 112	
4.1 Square Roots and Cube Roots • 80–100 min (TR page 115)	Students should be familiar with • square roots • perfect squares • prime factors • calculating area of squares and volume of cubes	• square dot paper • isometric dot paper • ruler	Master 3 Square Dot Paper Master 4 Isometric Dot Paper BLM 4–3 Chapter 4 Warm-Up BLM 4–5 Chapter 4 Unit 2 Project BLM 4–6 Section 4.1 Extra Practice	**Essential:** #1, 4, 5, 7, 9, 10, 13, 15, 20 **Typical:** #1, 4, 6–10, 11 *or* 12, four of 13–17, 20, 21 **Extension/Enrichment:** #1, 4, 6, 7, 13, 15, 18–21	TR pages 117, 122 Chapter 4 Foldable, TR page 112	TR pages 120, 122	
4.2 Integral Exponents • 80–100 min (TR page 123)	Students should be familiar with • integers and operations with integers • order of operations • powers with integral bases and whole number exponents • exponent laws for powers with integral bases and whole number exponents	• ruler	BLM 4–3 Chapter 4 Warm-Up BLM 4–4 Chapter 4 Foldable BLM 4–7 Section 4.2 Extra Practice	**Essential:** #2, 4, 5, 9, 10, 11, 14, 21, 25, 26 **Typical:** #1, 2, 4, 5, 7, 9–11, 13–15, 17, 19–21, 25–27 **Extension/Enrichment:** #1, 3, 5, 8–10, two of 11–13, 16, 18–27	TR pages 125, 133 Chapter 4 Foldable, TR page 112	TR pages 130, 133	
4.3 Rational Exponents • 80–100 min (TR page 134)	Students should be familiar with • rational numbers and operations with rational numbers • powers with integral bases and whole number exponents • exponent laws for powers with integral bases and whole number exponents		BLM 4–3 Chapter 4 Warm-Up BLM 4–8 Section 4.3 Extra Practice	**Essential:** #1, 3a)–c), 4a), c), e), 5a)–c), 6, 7, 9–11, 18, 19 **Typical:** #1, 3a)–c), 4a), c), e), 5a)–c), 6 *or* 7, two of 8, 10, 12 *or* 13, three of 9, 11, 14 *or* 15, 18, 19 **Extension/Enrichment:** #3–5, 7, two of 11–15, 16–19	TR pages 135, 140 Chapter 4 Foldable, TR page 112	TR pages 138, 140	
4.4 Irrational Numbers • 100–120 min (TR page 141)	Students should be familiar with • powers with rational and variable bases and integral and rational exponents • exponent laws • approximating and calculating square roots and cube roots	• ruler • compasses • square dot paper	Master 3 Square Dot Paper BLM 4–3 Chapter 4 Warm-Up BLM 4–5 Chapter 4 Unit 2 Project BLM 4–9 Section 4.4 Extra Practice	**Essential:** #1a)–c), 2b)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 11, 13, 21, 24 **Typical:** #1a)–c), 2a)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 13, three of 12–17, 21, 22 *or* 23, 24 **Extension/Enrichment:** #1d), 2a), d), 3c), d), 4e), f), 5d), f), 6c), d), 7c)–f), 9, 13, 18, 19 *or* 20, 19, 21, 22 *or* 23, 24	TR pages 143, 150 Chapter 4 Foldable, TR page 112	TR pages 143, 146, 147, 150	
Chapter 4 Review • 60–90 min (TR page 151)			BLM 4–6 Section 4.1 Extra Practice BLM 4–7 Section 4.2 Extra Practice BLM 4–8 Section 4.3 Extra Practice BLM 4–9 Section 4.4 Extra Practice	Have students do at least one question related to each concept, skill, or process that has been giving them trouble.	Chapter 4 Foldable, TR page 112	TR page 151	
Chapter 4 Practice Test • 40–50 min (TR page 152)			BLM 4–10 Chapter 4 Test BLM 4–11 Chapter 4 BLM Answers	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1, 2, 4–7, 9–13	TR page 153		TR page 153 BLM 4–10 Chapter 4 Test

Exponents and Radicals

Opener

***Mathematics 10*, pages 150–151**

Suggested Timing

30–40 min

Blackline Masters

BLM 4–2 Chapter 4 Prerequisite Skills
BLM 4–4 Chapter 4 Foldable
BLM 4–5 Chapter 4 Unit 2 Project
BLM U2–2 Unit 2 Project Checklist

Key Terms

perfect square	square root
perfect cube	cube root
prime factorization	irrational number
radical	radicand
index	mixed radical
entire radical	

What's Ahead

In this chapter, students extend their understanding of powers and exponents. They begin by learning more about square roots and cube roots. Students then apply the exponent laws to expressions involving rational numbers or variables as bases and integers and rational numbers as exponents. They are then introduced to irrational numbers and learn to convert between powers with rational exponents and radicals and between mixed radicals and entire radicals.

Planning Notes

Introduce the chapter by having students discuss what they know about powers and exponents from previous math and science classes. Then, as a class, read the chapter opener and direct students to the collage of visuals on pages 150 and 151 in the student resource.

Point out Plimpton 322, which has three columns listing Pythagorean triples. Explain that a Pythagorean triple consists of three whole numbers that form the sides of a right triangle. For example, 3, 4, and 5 form a Pythagorean triple because $3^2 + 4^2 = 5^2$. Some scholars say that the tablet is the oldest surviving work in number theory, while others say that the tablet was a notebook used to learn and teach the Pythagorean triples. The Babylonians were using the Pythagorean theorem as early as 1750 B.C.E.

Brainstorm what the other visuals might represent. Ask about any connections between the visuals and the use of powers (or repeated multiplication).

Explain to students that they will build on their knowledge of powers and exponents and learn how to model situations involving growth and decay, such as in nature and finance.

Note that the pine cone, sunflower, and pentagram represent irrational numbers.

Direct students to the information about artists. Ask them about careers in art that they are familiar with. Have students discuss what they know about the work that these artists do, and how math skills might be related to their work. Point out that crafts such as stained glass, basket weaving, iron work, and ceramics often involve mathematics.

Unit Project

You might take the opportunity to discuss the Unit 2 project described in the Unit 2 opener on TR page 107. Throughout the chapter, there are individual questions for the unit project. These questions are not mandatory but are recommended because they provide some of the research needed for the final report for the Unit 2 project assignment.

The Unit 2 project is integrated throughout the chapter. You will find questions related to the project in the section 4.4 Investigate and the Check Your Understanding for sections 4.1 and 4.4.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- What disadvantages do Foldables have?
- Which, if any, designs were hard to use?

- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 151 and how it might be used to summarize Chapter 4. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

As students progress through the chapter, provide time for them to keep track of what they need to work on. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 4–2 Chapter 4 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Some students may find it useful to use **BLM 4–4 Chapter 4 Foldable**, which summarizes the exponent laws. Have students staple or tape a copy to the centre panel of their Foldable. Alternatively, students can use the centre panel of the Foldable to list the exponent laws. If they do so, they can use the back of the inserts for sections 4.2 and 4.3 to provide their own examples of integral and radical exponents for each exponent law. Consider having students staple or tape a copy of **BLM U2–2 Unit 2 Project Checklist** to the centre panel on the back of the Foldable. They can check off items as they complete them.
- Some students may find it useful to keep a taped or oral summary of what they are learning. Others may work best with a keyboarded version using software of their choice.
- To reinforce the Key Terms after they are introduced, post sheets of paper around the room, each labelled with one Key Term. Have student pairs respond to the following prompts for each term: definition, example, and facts. Have student pairs move around the room and use diagrams and words to contribute to each Key Term. Once each pair has contributed, have students review all the entries. As a class, debrief each sheet to conclude the activity. Leave the sheets on display throughout the chapter.
- Throughout the chapter, encourage students to use strategies such as making models and drawing diagrams to help them move from the concrete to the abstract level. Encourage students to "say their thinking." Look and listen for unorthodox, yet mathematically correct procedures as much as you observe for correcting mathematically unsound procedures.
- Consider allowing students to work with a partner on all Unit 2 project questions.
- Some students may benefit from completing all unit project questions.
- **BLM 4–5 Chapter 4 Unit 2 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 2 project report.

ELL

- Work through each worked example and solution as a class, demonstrating the process of solving each problem. Have students work in pairs to complete the Your Turn questions.

Enrichment

- Encourage students to consider forms of art from their cultural heritage that involve math. For example, students might research Aboriginal artists. They might use bead work as a starting point. Ask them to provide a sample. As a class, discuss how the samples might use mathematics.
- Encourage students who are interested in artists, training and qualifications, employment, and job outlook to research and present a report on a category of artist in which they are interested. Have them address how math concepts and skills are important in their area of interest.

Gifted

- Have students speculate as to why exponents exist and in what situations exponents are useful.

Career Connection

Use the photograph and the text to highlight career opportunities for artists, including craft artists, fine artists, art directors, multimedia artists, and animators. Invite students to research different careers within the arts. They may find the related Web Link that follows helpful.

Web Link

For information about careers within the arts, go to www.mhrmath10.ca and follow the links.

Square Roots and Cube Roots

4.1

***Mathematics 10,* pages 152–161**

Suggested Timing

80–100 min

Materials

- square dot paper
- isometric dot paper
- ruler

Blackline Masters

Master 3 Square Dot Paper
Master 4 Isometric Dot Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–5 Chapter 4 Unit 2 Project
BLM 4–6 Section 4.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

AN1 Demonstrate an understanding of factors of whole numbers by determining the:

- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 4, 5, 7, 9, 10, 13, 15, 20
Typical	#1, 4, 6–10, 11 *or* 12, four of 13–17, 20, 21
Extension/Enrichment	#1, 4, 6, 7, 13, 15, 18–21

Unit Project Note that #13 and 15 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, read and discuss the opening text about how workers such as painters and designers apply math when working with area and volume. Ask students why it is important for painters and designers to take accurate measurements. Tell students that tradespeople need to be effective problem solvers. Ask students to discuss what strategies a painter and a designer might use to solve the problems posed.

Investigate Square Roots and Cube Roots

In this Investigate, students use patterning to connect the side length of a square with the square root of a number (area) and the edge length of a cube with the cube root of a number (volume).

Have students work in pairs. Make **Master 3 Square Dot Paper** and **Master 4 Isometric Dot Paper** available to students. They may find it easier to draw cubes using the isometric dot paper.

Circulate as students work and note the way they are recording area and volume. Ask what units are used for measuring area and volume. Use the following prompts:

- How are the procedures for finding the area of a square and the volume of a cube different? similar?
- How could you estimate the side length of a square with an area of 20 square units using your results?
- How could you use your calculator to determine the area of a square? the volume of a cube?

Have students discuss #3 in pairs, then in large groups. As a class, have students discuss their response to #3c).

In a follow-up class discussion, have students discuss how different types of numbers have certain features that can be grouped together. One way to look at numbers is to create geometric representations. Ask students for examples of numbers that form squares and cubes.

You might reinforce the idea that mathematicians have been relating to numbers using geometric representations for a long time by mentioning that Pythagoras (about 580–500 B.C.E) believed that all of nature is based on numbers and a mathematical structure. For instance, you might show an array of 9 beads and explain that the number 9 can be represented concretely as a perfect square. You might help students make the connection between numbers and their geometric representation by asking the following questions:

- For 5^7, we say "5 to the 7th." For 6^4, we say "6 to the 4th." For 7^2, why do we say "7 squared" instead of "7 to the 2nd?"
- For 4^3, why do we say "4 cubed?"

Explain that during the chapter, they will learn about numbers that form rectangles (golden ratio) and diagonals of rectangles (irrational numbers).

Meeting Student Needs

- Some students may not relate to the opening scenario. For example, in most Northern communities, house paint is not readily available. It must be ordered and shipped by air freight or on the annual resupply. You might adapt the context by asking students how a painter knows how much paint to order for the local school.
- Review concepts of length, area, and volume of rectangular prisms briefly before beginning the Investigate. The Investigate relies on students having a clear understanding of the differences between these concepts. They also need to be clear about using the correct corresponding units.
- Consider having students create posters illustrating perfect squares composed of smaller blocks, or tape sections of the classroom floor, if it is tiled. Display the posters in the classroom.
- For #2, encourage students to use manipulatives such as building blocks (e.g., connect-a-cubes, LEGO®) to build models of perfect cubes.

Enrichment

- Challenge students to devise a method for predicting perfect cubes in large numbers.

Gifted

- Have students look for a pattern in the number of perfect cubes that appear as the base values increase.
- Elephants' ears have a high surface area and act to cool the elephant's blood. Challenge students to give reasons why the volume-to-surface-area ratio of an animal might affect mammals in hot climates. Why might animals in the north be expected to have small ears?

Common Errors

- Some students may confuse the concepts of volume and surface area of a cube.

R_x Help students recall the meaning of volume and the formula for finding the volume of a cube. Recall that volume is always measured in cubic units and surface area is measured in square units. Use a model of a cube to help students to visualize the difference. Have students verbalize their understanding.

Answers

Investigate Square Roots and Cube Roots

1. a), b)

Side Length	Area in Exponential Form	Area
1	1^2	1
2	2^2	4
3	3^2	9
4	4^2	16
5	5^2	25
6	6^2	36

c) The area is the square of the side length so $A = s^2$.

2. a), b)

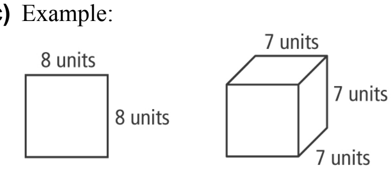
Edge Length	Volume in Exponential Form	Volume
1	1^3	1
2	2^3	8
3	3^3	27
4	4^3	64
5	5^3	125
6	6^3	216

c) The volume is the cube of the edge length so $V = s^3$.

3. a) Take the square root of the area to get the side length or $s = \sqrt{A}$.

b) Take the cube root of the volume to get the edge length or $s = \sqrt[3]{V}$.

c) Example:



$A = 64$ units^2 $V = 343$ units^3

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to show a pictorial representation of the square and the cube of a number. For example, using 5, show a square with 25 small squares inside it, and a cube with 125 small squares inside it. • Ensure that students understand that the square root is derived from a 2-D shape and the cube root is derived from a 3-D object (initially). • For students who struggle with #3c), encourage them to rewrite the numbers in prime factorization form. Help them recall that this involves writing a number as the product of its prime factors.

Link the Ideas

It is important that students have a clear understanding of the terms in this section in order to apply the terminology correctly later with radicals. As a class, have students discuss the terms *perfect square*, *square root*, *perfect cube*, and *cube root*. You might have students recall what they know about perfect squares and square roots. Reinforce using the example that 25 is a perfect square because it forms a square.

After students have discussed the examples of perfect squares, ask the following questions:
- What is another example of a perfect square?
- How do you know it is a perfect square?

Help students clarify any misunderstandings by asking questions such as the following:
- What is the difference between squaring a number and taking the square root of a number?
- Using the number 9, what is the difference between squaring 9 and taking the square root of 9?

Using an example such as $\sqrt{9}$, explain that this means the square root of 9. The 2 (in the index) is understood. This is important so students get a clear understanding of and can differentiate between square roots and higher-order radicals. Also, use an example with a leading negative sign, such as (-4^2) and (-4^3), and point out the negative sign. Explain how the negative sign affects the final answer.

Direct students to the example showing how perfect cubes and cube roots are related. You might reinforce that the cube root of 27 equals the cube root of (3)(3)(3), which equals the cube root of 3 to the power of 3, which, therefore, equals 3.

After students have discussed the examples of perfect cubes, ask the following questions:
- What is another example of a perfect cube?
- How do you know it is a perfect cube?
- What is the difference between cubing a number and taking the cube root of a number?
- Using the number 8, what is the difference between cubing 8 and taking the cube root of 8?

After the discussion, use the following prompts:
- What did you learn by discussing these terms?
- What, if any, misconceptions did you correct?

Example 1

In this Example, students determine whether given numbers are perfect squares, perfect cubes, both, or neither.

Before students consider the strategy shown for each problem, encourage them to try other strategies such as guess and check, prime factorization, and using a calculator. Have student pairs compare their answers with another pair of students who used different strategies. Have them correct any discrepancies.

As a class, walk through the provided solutions. For part b), discuss the term *prime factorization* and consider working through a second example of prime factorization using a number such as 1000 or 216 that is not both a perfect square and a perfect cube. This will illustrate how to use prime factors to identify a perfect cube.

For part c), you might model using prime factorization to further illustrate that since the factors do not form groups of two, 356 is not a perfect square. Since the factors do not form groups of three, 356 is not a perfect cube.

Since students have tried different strategies, you might ask which strategy they prefer and why.

Make the following points:
- Prime factorization is an effective method to establish that a number is a perfect square if the prime factors can be equally distributed into two groups. A number is a perfect cube if the prime factors can be equally distributed into three groups.
- A calculator is useful to check if an answer is correct.

Direct students to the Did You Know? on page 155 that explains how square roots and cube roots were studied and applied by ancient civilizations. You might ask students what they know about how ancient civilizations applied this knowledge.

Have students work in small groups to complete the Your Turn questions. Challenge each member of the group to use a different method to answer the questions and then discuss their solutions. Alternatively, students could work individually and use prime factorization for all three questions. Have them check their answers using a calculator.

Example 2

In this Example, students calculate the cube root of a large number that is a perfect cube.

Use the information in the Did You Know? about uranium to help set the context. Explain that uranium is a silvery-grey element that occurs in nature in the form of minerals. It is often used as a fuel in nuclear power plants.

Before having students consider the solution, ask them to explain how they know that the question requires working with cube roots and not square roots. You may wish to have students work in pairs and use their own strategies to solve the problem. Use the following prompts to assist students:
- How did you determine the dimensions of the cube?
- How did you determine the cube root?
- How else could you determine the cube root?

You might encourage students to use prime factorization to solve and then check with a calculator.

Walk through the given solutions. Have students compare their solutions and discuss any discrepancies with their partner.

Have students complete the Your Turn questions using the methods of their choice and then explain their methods to a classmate.

Key Ideas

The Key Ideas summarize determining square roots of whole numbers that are perfect squares, and cube roots of whole numbers that are perfect cubes. Check understanding by asking students for an additional example of a perfect square and a perfect cube. Check that students understand the process of prime factorization. Ask the following questions:
- How do you rewrite the number 36 as a product of prime factors?
- How can you use prime factorization for 36 to determine if 36 is a perfect square? a perfect cube?

Have students use their Foldable to record their own definition and example for each Key Term. Additionally, have them create their own summary of the Key Ideas and include it in their Foldable.

Meeting Student Needs

- Review the difference between *factors* and *prime factors*. Factors are all the numbers that a number is divisible by, whereas prime factors are the primes that divide evenly into a number. Have students recall that 1 is not a prime number. The reason is because 1 is divisible by 1 and 1, which is not two distinct (different) factors.
- Some students may benefit from reviewing divisibility rules for the prime numbers 2, 3, and 5. Remind students that the number 7 has no rule.
- Some students may benefit from practising prime factorization using the interactive tool described in the related Web Link at the end of this section.
- Ensure that students know how to use their calculator for operations involving square roots and cube roots. They need to identify and be able to use the keys to square, cube, take the square root, and take the cube root.
- Some students may benefit from working in pairs to develop a summary of the methods for determining that a number is a perfect square or a perfect cube. Encourage them to practise using all the methods.
- At the end of each lesson, consider using one of the following tools to informally assess student understanding of new concepts and determine where students need clarification:
  - Develop an exit slip with a few key questions to be answered by students during the last 5 to 10 min of class. Have them turn in the exit slips as they leave. Use the assessment to help determine where to begin in the next class or which students require assistance.

- Use Fist-to-Five at the end of the lesson. Ask students how well they understand a key concept. Be specific. Ask each student to show the number of fingers that correspond to their level of understanding. The responses will range from a fist, which indicates no understanding, to five fingers, which indicates complete understanding. Generally, fewer than three fingers indicates a need for revisiting a concept.

- For the Did You Know? on page 155, clarify that wet clay was formed into flat shapes to create tablets. While the tablets were still wet, the Babylonians used a stylus to make wedge-shaped letters on the surface. The clay was then sun dried or kiln fired.

Enrichment

- For the Example 1: Your Turn, assign the following additional questions:
  - Which of the following are both perfect squares and perfect cubes?
    216 400 1024 46 656
  - Think of a larger perfect square that is also a perfect cube.
- Challenge students to think of a number that contains only the number 5 in its prime factorization and that is both a perfect square and a perfect cube. Ask them to explain how they solved the problem and whether there is more than one solution.

Gifted

- For Example 2, have students use the information in the Did You Know? on page 156 in the student resource as a springboard to research the annual uranium production in Canada or the world to develop and solve a similar problem.
- Challenge students to research the smallest number expressible as the sum of two positive cubes in two different ways ($1729 = 1^3 + 12^3$ or $9^3 + 10^3$). They may find the Web Link at the end of this section interesting.

Common Errors

- Some students may struggle with determining the prime factorization of a number.

Rx Review the procedure and the divisibility rules for 2, 3, 5, and 10.

WWW Web Link

For a factor tree tool that illustrates both prime factorization and common factors, go to www.mhrmath10.ca and follow the links.

To read a story about the discovery of the smallest number expressible as the sum of two positive cubes in two different ways, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) Perfect cube; $125 = 5^3$

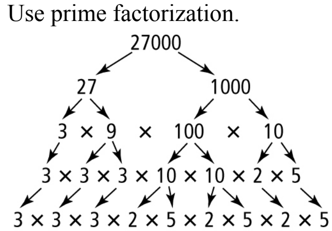
b) Perfect square; $196 = 14^2$

c) Both a perfect square and a perfect cube; $4096 = 64^2$ and 16^3.
Look for a variety of methods for a)–c), such as using diagrams, prime factorization, or a calculator.
Example for c): $4096 = (2)(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)(2)$
$= 2^{12}$
Square rooting gives two groups of $(2)(2)(2)(2)(2)(2) = 2^6$ or 64.
Cube rooting gives three groups of $(2)(2)(2)(2)$ or 2^4 or 16.

Example 2: Your Turn

a) 14 m

b) 30 in. by 30 in. by 30 in. Look for two methods. Example:
- Use prime factorization.



For a cube root, look for triplets, so one of 3, one of 2, and one of 5. Therefore, the cube root is $(3)(2)(5) = 30$.
- Use a calculator. Example:
  C 27 000 2nd $\sqrt[x]{y}$ 3 = 30.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use more than one method. Encourage them to record and store multiple methods in their Foldable. • Some students may benefit from using grid paper and isometric dot paper to help visualize side lengths and their squares, and edge lengths and their cubes, respectively. • Encourage students to use prime factorization for large numbers. • Some students may benefit from reviewing factor trees. Point out that for large numbers, not all students will start with the same factors, but that the factors at the end will be the same. Explain that when there are two equal groups of factors, you have found the square root. When there are three equal groups of factors, you have found the cube root. Have students compare their factor trees with those of a partner.
Example 2 Have students do the Your Turn related to Example 2.	• Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking. • For each of parts a) and b), have students verbalize what they are finding and how they know. • If students are using a calculator, you might ask them to explain what the index on the radical indicates.

Check Your Understanding

Practise

For #5 and 6, encourage students to use prime factorization in their solutions for the larger numbers.

For #9 and 10, tell students to check that the units in their solutions are correct. For #9, you may need to coach students to realize that the problem involves calculating the perimeter of the rug, not the length of one side.

For #10, some students may struggle with using prime factorization if they do not check division by 7. Check that students who use a calculator use it correctly to find the cube root.

Apply

It may not be necessary to assign all questions to all students. Allow students some choice in the questions they need to do. Remind students to read the problems carefully and to note the given units of measurement and the units of measurement needed in the solution.

For #11a), students use mental estimation to estimate a lower and upper boundary for the square root of a large number. You might prompt students who have difficulty with this to make a reasonable guess of the square root of 1000.

For #14, although it is appropriate for students to use a calculator to find the cube root of 46 656, encourage them to use prime factorization.

For #16, some students may benefit from recalling the formula for the surface area of a cube ($SA = 6s^2$). You may wish to take the opportunity to show students how to construct the formula using a net of a cube.

Extend

For #17b), encourage students to isolate the variable, t^2, prior to taking the square root on both sides of the equation.

For #18, students may need the hint to use the Pythagorean relationship.

For #19, the volume of the basketball is expressed in terms of pi. Discuss with students why it would be impossible to measure the volume of a basketball with exact precision. Students may need the hint to use the formula of a sphere to help solve the problem.

Create Connections

For #20, consider having students work in pairs before developing an individual response.

For #21, have students discuss their solutions in a class discussion.

Unit Project

The Unit 2 project questions, #13 and 15, provide an opportunity for students to solve problems involving square roots and cube roots.

For #13, check that students note the different units used. Encourage students to sketch a diagram to help visualize what the problem is asking. You may need to clarify that the classroom wall is not a square but a rectangle. Point out that the shape of the mural does not matter, as it is the shapes made by the square tiles that are important. Reinforce that it is the design of the mural (not the overall dimensions of the mural) that needs to be a geometric representation of square roots.

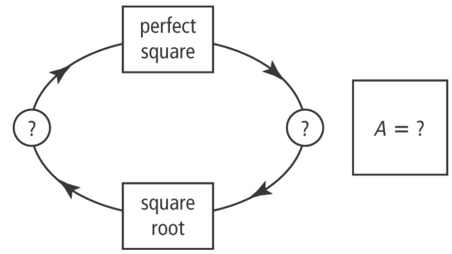
Students will approach #13 in a variety of ways. You might have them work in pairs before developing an individual response. Afterward, have students compare and discuss their designs in a class discussion. For instance, a design might involve squares with diagonals (e.g., a square that is 1 unit by 1 unit would have a diagonal that is $\sqrt{2}$ units in length). This activity and follow-up discussion will help set up students' learning about the golden ratio in section 4.4.

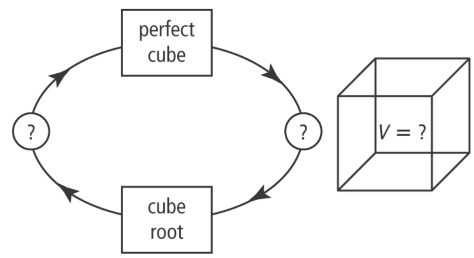
For #15, you might explain that sculptor Tony Bloom works mostly in steel, copper, bronze, and aluminum. He designed the WaterWork, a water-driven fountain for the head office of the BC Hydro Authority in Vancouver, BC. His work has been exhibited in North America, Europe, and Japan, and he has been the recipient of national, provincial, and civic awards.

For #13 and 15, encourage students to use a combination of sketches, words, and numbers to explain how the mural and the sculpture are geometric representations.

Meeting Student Needs

- Allow students to work in pairs.
- For #13, you may need to remind students to convert to the same units.
- For #19, review the formulas for the volume of a cube and of a sphere.
- Some students may benefit from using an example and a diagram to explain the relationship between perfect squares and square roots and between perfect cubes and cube roots. They may find the diagrams shown helpful. Have students use their example and the diagram to explain the relationships to a classmate.





- Provide **BLM 4–6 Section 4.1 Extra Practice** to students who would benefit from more practice.

ELL

- For #12, use the photograph and the related Did You Know? to describe star quilts.
- For #13, explain that a *mural mosaic* is a design constructed from smaller pieces and is displayed on a wall.
- Teach the following terms in context: *recycling depot, bales, copper leaf, meteorologists, tornadoes,* and *hurricanes.*

Enrichment

- Give students one of the following challenges:
  - Research how people working in some of the following occupations apply knowledge of square roots and cube roots: carpenters, electricians, machinists, architects, civil engineers, computer scientists, surveyors, chemists, physicists, and biologists. Allow students to present their findings in a format of their choice.
  - Research the Dominion Land Survey system and how it works. Mention that land surveyors used the Dominion Land Survey (DLS) as the method to divide much of Western Canada into 1 mi^2 sections, mainly for agricultural purposes. This survey system and its related terms, including meridians, baselines, townships, sections, and quarter sections, are an important part of rural culture in the Prairies today. Have students prepare and present a report of their findings.

– Some students may be interested in researching the 1869 Red River Rebellion or 1885 Battle of Batoche. The Red River is in present-day Manitoba. The Battle of Batoche took place in Saskatchewan. Both conflicts were precipitated by surveyors encroaching on Métis lands to break up the seigneurial land system and replace it with the imperial township grid system. One of the issues involved grievances about federal land surveyors using the DLS system to divide the river-front lots owned by the Métis, and their refusal to acknowledge traditional Métis land holdings.

Web Link

For information about the Dominion Land Survey system, go to www.mhrmath10.ca and follow the links.

For information about the Battle of Batoche, go to www.mhrmath10.ca and follow the links.

For information about the Red River Rebellion, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 4, 5, 7, 9, 10, 13, and 15. Students who have no problems with these questions can go on to the remaining questions.	• For #1, students may benefit from reviewing the fractional forms, when to square the denominator, when to square the numerator, and when to square both the numerator and the denominator. Provide additional coaching and use #2 to check for understanding. • For #4, ensure students understand the difference between an index of 2 and 3. Model examples to help students conclude that the square root is one of two identical values and the cube root is one of three identical values. Provide additional coaching and use #3 to check for understanding. • Provide additional coaching with Example 1 to students who need support with #5 and 7. Work with them to correct their errors and then have them do #6 and 8 on their own. • Provide additional coaching with Example 2 to students who need support with #9 and 10. Work with them to correct their errors, and then have them try solving for 64 m^2 for #9 and 216 m^3 for #10. • Encourage students to draw diagrams for #9, 10, and 13. • Some students may find it helpful to list the squares of the numbers 1 to 20 and the cubes of the numbers 1 to 10 in their Foldable. However, ensure they are able to determine square roots and cube roots.
Unit 2 Project If students complete #13 and 15, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 4–5 Chapter 4 Unit 2 Project**, and have them finalize their answers. • Remind them to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #20.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the question, and then have them provide an individual response orally or in written form.

Integral Exponents

4.2

Mathematics 10, pages 162–173

Suggested Timing

80–100 min

Materials

- ruler

Blackline Masters

BLM 4–3 Chapter 4 Warm-Up
BLM 4–4 Chapter 4 Foldable
BLM 4–7 Section 4.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#2, 4, 5, 9, 10, 11, 14, 21, 25, 26
Typical	#1, 2, 4, 5, 7, 9–11, 13–15, 17, 19–21, 25–27
Extension/Enrichment	#1, 3, 5, 8–10, two of 11–13, 16, 18–27

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, discuss the opening paragraph and the photograph of the Rhind papyrus. You might use the following arithmetic puzzle that is recorded in the RMP to help introduce exponents: Seven houses contain seven cats. Each cat kills seven mice. Each mouse had eaten seven ears of grain. Each ear of grain would have produced seven hekats of wheat. (The hekat was an ancient Egyptian unit of volume.) Ask students how they could use exponents to express the total number of houses, mice, ears of grain, and hekats of wheat. What is the total? You might ask them to share similar puzzles they are familiar with.

Use the lead-in about the age of the RMP to read and discuss the information about carbon-14 dating. Ask students to discuss what a decreasing amount of carbon-14 means for the sign of the exponent in the formula.

You might explain that carbon-14 is formed by cosmic rays changing the nuclear structure of nitrogen-14 atoms. The ratio of carbon-14 atoms to carbon-12 atoms is relatively constant in nature. There is approximately one carbon-14 atom for every trillion carbon-12 atoms. This ratio remains constant in living organisms. When an organism dies, no new carbon-14 is incorporated into the organism and the carbon-14 that is present decays at a very slow rate. Half of a sample of carbon-14 decays in 5700 years. The ratio of carbon-14 to carbon-12 in a sample is used to date the sample. Refer students to the Did You Know? about the accuracy of carbon-14 dating for samples up to 60 000 years of age. This dating technique was devised by Willard Libby in 1949. He received the Nobel Prize for chemistry in 1960 for his discovery. Note that the formula for determining the age of a sample is beyond high school math. A formula that can be used to calculate the amount of carbon-14 remaining in a sample after t years could be approximated using the formula $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{5700}}$, where N_o is the amount of carbon-14 at time 0 and N is the amount of carbon-14 after t years. By replacing $\frac{1}{2}$ with 2^{-1}, the formula can also be written as $N = N_0(2)^{\frac{-t}{5700}}$.

Investigate Negative Exponents

In this Investigate, students determine the meaning of a power with a negative exponent through a patterning exercise.

Have students work individually or with a partner to construct the number line and work through #1 to 4.

While students work, circulate and ask questions to help them focus on the key ideas about exponential form. Ensure that students halve the distance each time between 0 (left endpoint) and the previous midpoint. Some students may incorrectly halve the distance on both sides of the midpoint with the nearest endpoint.

For #2, you may need to coach students to record the value for x in base 2 after each division. After students have marked two or more midpoints, ask the following questions:

- How does the power change for each successive halving process?
- How does the power relate to the distance from 0 (the left endpoint)?

For #3, some students may record the distances less than 1 using decimal form. If so, ask them for the fractional form for each distance. Check that students have included distances up to the value for $\frac{1}{8} = 2^{-3}$.

For #4, students may organize their results differently. Consider asking a student pair to record the results on a table on the board. As a class, discuss the results and then have students compare with their own results. Have them discuss any differences in values.

Give students sufficient time to complete #5 and 6 either individually or with a partner. For #5b), some students may benefit from rewriting the denominator as a power with base 2 in order to make the connection between negative and positive exponents. For example, $\frac{1}{8} = \frac{1}{2^3}$. This may help them to develop a general form for writing any power with a negative exponent as an equivalent power with a positive exponent for #5c).

For #6a), direct students to the Did You Know? that explains that the half-life of a radioactive element is constant. Ask why this is important to know for answering part b). You might coach students to divide the total time of decay by the half-life to determine the number of half-lives. Then, calculate what fraction of the sample will remain after the calculated number of half-lives. For example, the fraction of the sample remaining after two half-lives is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 4^{-1}$, and after three half-lives is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 8^{-1}$.

As a class, have students discuss their findings. Invite students to discuss their conclusion about when negative exponents might be used.

Meeting Student Needs

- Consider inviting a science teacher to talk about radioactive decay and carbon-14 dating. Alternatively, you might show a video clip illustrating carbon-14 dating. If so, you may find the related Web Link at the end of this section useful.
- Students may use grid paper instead of measuring line segments with a ruler.
- Some students may benefit from working through an example of half-lives using the base 10, since this base is familiar to them. If so, draw a series of adjacent boxes and show the place value of each digit in both whole-number and power-of-10 form. When you move to the right of the decimal point, show the values as a fraction, a decimal, and a negative exponent. Using place values that students are familiar with may help them to understand the concept of negative exponents.
- Work with students to develop a table such as the one shown. Have students decide on the headings for each column and row.

Halfway Points	Value	Exponential Form
1	16	2^4
2		
3		
4		
5		
6		
7		

- For #6a), check that students understand the meaning of half-life. Ask students to imagine eating half a bag of snacks and then half of what is left and then half of that, etc. Will they ever finish all the snacks? Discuss this as a class.

ELL

- Explain that *integral exponents* refer to exponents that are integers.
- Explain the term *papyrus*. Papyrus is a thick material produced from the papyrus plant used to make paper. It was once abundant in the Nile delta of Egypt. Some students may be more familiar with birchbark. Birchbark, which is the bark of the Paper Birch tree, has been used in Aboriginal traditions to make items ranging from canoes to scrolls, art, and maps.

Enrichment

- Challenge students to write a report explaining how carbon-14 works or a discovery made with carbon-14 dating. For instance, they might research discoveries of artifacts such as arrowheads.

Common Errors

- For #3, students may struggle with determining and labelling the fractional distances between 0 and 1.

R_x Coach students to observe the pattern as the fractions are halved each time. The denominator doubles.

- Students may struggle with labelling the three powers between 0 and 1.

R_x Encourage them to expand the area between 0 and 1 on their number line by drawing a zoom-out above or below the original number line in order to label the values.

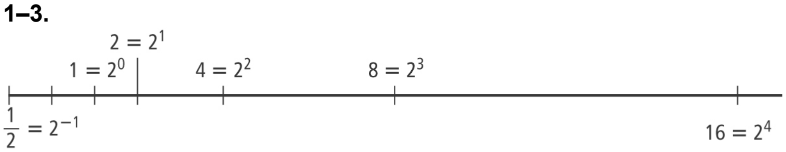
Web Link

For a video clip illustrating how carbon-14 dating works, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Negative Exponents

1–3.



2. a) 4 times **b)** decreased by one each time

3. a) $\frac{1}{2}$ **b)** 2^{-1} **c)** $\frac{1}{4} = 2^{-2}; \frac{1}{8} = 2^{-3}$

4. Example:

Line Segment Lengths	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Exponential Form	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

5. a) Each halving reduces the exponent by 1.

b) $\frac{1}{8} = \frac{1}{2^3}$

$= \left(\frac{1}{2}\right)^3$

$= 2^{-3}$

Example: The pattern is "1 over a power is the same as the power to a negative exponent."

c) $n^{-x} = \left(\frac{1}{n}\right)^x$

6. a) $2^{-2} = \frac{1}{4}$ remaining; $2^{-3} = \frac{1}{8}$ remaining

b) Examples: Situations involving light intensity, gravitation, or erosion.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students fill in the corresponding entry on a table after each number line division. Ask how $\left(\frac{1}{2^2}\right)$, $\left(\frac{1}{3^2}\right)$, and $\left(\frac{1}{4^4}\right)$ change to 2^{-2}, 3^{-2}, and 4^{-4}. Then, have students attempt to generalize a formula. • Have students complete #6a) and check for understanding before moving on.

Link the Ideas

Have students recall what they know about the exponent laws from previous math courses. Direct their attention to the table that summarizes the exponent laws on page 164 in the student resource. For a product of powers, you might model how to expand an example, such as $(4^5)(4^2)$ $= (4)(4)(4)(4)(4)(4)(4)$. Explain that exponent laws can be used to simplify calculations of five 4s multiplied by two 4s, which means multiplying seven 4s together (by adding the exponents). For a quotient of powers, you might ask students to verbalize what an expression, such as $\left(\frac{5^6}{5^2}\right)$ means (six 5s divided by two 5s) and how to simplify using the related exponent law (by subtracting the exponents).

Then, have students form groups of four and verbalize the meaning of each exponent law. If necessary, coach students by using prompts such as the following:

- How can you multiply powers with the same base? (Add the exponents.)
- How can you divide powers with the same base? (Subtract the exponents.)
- How can you raise a power to an exponent? (Multiply the exponents.)
- How can you raise a product to an exponent? (Rewrite each number in the product with the same exponent.)
- How can you raise a quotient to an exponent? (Rewrite each number in the quotient with the same exponent.)
- What is the value of the power when the exponent of the power is 0? (1)

For the exponent law about zero, help students recall how they know that $a^0 = 1$ by using division to show this. Choose any power of 4, such as 3. Divide it by itself.

$$\frac{4^3}{4^3} = 4^{3-3}$$
$$= 4^0$$
$$\frac{4^3}{4^3} = 1$$

So, $4^0 = 1$.

Challenge students to develop their own example to prove that $a^0 = 1$.

You might have students use **BLM 4–4 Chapter 4 Foldable** to record the verbal explanation for each exponent law.

After time for discussion, ask the following questions:

- What did you learn by sharing your ideas with another pair?
- How did this sharing modify your ideas?
- What did you learn?
- What, if any, misconceptions did you correct?

Based on what students say, you might consider re-teaching any exponent laws that students find challenging.

Explain that some powers in this section will have *fractional bases*. Explain to students what these are and how to recognize them and differentiate between them (e.g., $\left(\frac{3}{4}\right)^2$ and $3^{\frac{2}{4}}$). Ask which power has a fractional base and which does not.

Help students recall for an integral exponent that *integers* are positive and negative whole numbers and include zero. This section features integers as the exponents in a power. The new concept is having negative whole numbers and zero as exponents.

As a class, walk through the principle that the power with a negative exponent is equal to the reciprocal of the base raised to the positive of the exponent. Show the example $10^{-3} = \left(\frac{10^{-3}}{1}\right)\left(\frac{10^3}{10^3}\right)$

$$= \frac{1}{10^3}$$

Then, you might walk through the following example and show that a negative exponent in the denominator of a fraction is equal to the reciprocal of the base raised to the positive of the exponent.

If $10^{-3} = \frac{1}{10^3}$, then $(10^{-3})(10^3) = \frac{10^3}{10^3}$
$$= 1$$

Then, $\frac{1}{10^{-3}} = \left(\frac{1}{10^{-3}}\right)\left(\frac{10^3}{10^3}\right)$
$$= 10^3$$

Have students restate the meaning of the new principle in their own words. For example, a base with a negative exponent can be written as a fraction with the base and the positive of the exponent in the denominator. Also, a base with a negative exponent in the denominator can be written as a base with a positive exponent.

Example 1

In this Example, students multiply or divide powers with a common base. Work through both methods for multiplying and dividing powers as a class.

For part a) Method 1, some students may benefit from seeing the division of 5^8 and 5^3 in expanded form before applying the exponent law: $\frac{(5)(5)(5)(5)(5)(5)(5)(5)}{(5)(5)(5)}$.

For part b), Method 2, ask how they know they can convert the negative exponents to positive exponents.

For part c), you might ask students what other method could be used. Using positive exponents,

$$\begin{aligned}\frac{x^5}{x^{-3}} &= (x^5)(x^3)\\ &= x^8\end{aligned}$$

Have them try it.

For part d), you might show students an alternative way of arriving at the same solution for Method 2 using division of fractions:

$$\begin{aligned}\frac{(2x)^3}{(2x)^{-2}} &= \frac{(2x)^3}{1} \div \frac{1}{(2x)^2}\\ &= \left[\frac{(2x)^3}{1}\right]\left[\frac{(2x)^2}{1}\right]\\ &= (2x)^5\end{aligned}$$

Use the following prompts to assist students:

- Why is it important to maintain the brackets?
- Why do you take the reciprocal of the second fraction?

You might then have students work in small groups to discuss the solutions and why and how the methods shown work. Ask questions such as the following:

- What methods can be used to multiply or divide powers with the same base?
- Which method do you prefer? Why?
- For which types of questions would you use positive exponents as your preferred strategy?
- For which types of questions would you use adding or subtracting exponents as your preferred strategy?

Have students work with a partner and challenge them to use different methods to solve each Your Turn question. For part c), ask what the parentheses around the negative bases mean. Reinforce that the negative sign is part of the base and that the exponent also applies to the negative sign. Have partners discuss their answers with each other and work together to resolve any differences in the solutions.

Example 2

In this Example, students simplify and then evaluate powers of powers. Work through each problem as a class.

Before students attempt part a), you might demonstrate the law for raising a power to an exponent by using repeated multiplication.

For example,
$$\begin{aligned}(2^4)^3 &= 2^{4+4+4}\\ &= 2^{12}\end{aligned}$$

Another way of expressing this is $2^{(4)(3)}$.

Have students discuss and try solving part b) using the exponent law about zero exponents. The solution is

$$\begin{aligned}[(a^{-2})(a^0)]^{-1} &= [(a^{-2})(1)]^{-1}\\ &= (a^{-2})^{-1}(1)^{-1}\\ &= (a^{(-2)(-1)})(1)\\ &= a^2\end{aligned}$$

For part c), ask students what other method could be used to evaluate the expression. How can they use what they know about negative exponents? Invert and power positively:

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= \left(\frac{2^6}{2^4}\right)^3\\ &= (2^2)^3\\ &= 64\end{aligned}$$

Ask students which method they prefer and why.

For part d), have students explain why the base is the reciprocal of the original (to divide by a fraction, take the reciprocal of the base). Draw students' attention to the Mental Math box that asks if this is true in all cases. If so, does this allow them to create a mental math shortcut for expressing fractions such as $\left(\frac{2}{3}\right)^{-4}$ as $\left(\frac{3}{2}\right)^4$?

As students analyse the methods shown in Example 2, challenge them to notice the similarities and the differences from the strategies used in Example 1. Have students discuss why certain methods are shown and the advantages and possible disadvantages of these methods. Ask students the following questions:

- What methods do you find easier to use? Why?
- What strategies do you use to help decide on a method?

- When might one method be better than another one? Explain.

Direct students to the Did You Know? on page 166 that explains John Wallis' work. Clarify that *infinity* means without end. In mathematics, it is often used in contexts as if it were a number (e.g., an infinite number of terms). Explain that the precise origin of the symbol is unknown but it is sometimes called a "lazy eight". You might have students discuss where they have seen the infinity symbol.

Did You Know?

The Métis flag features an infinity symbol. Many Métis people believe the symbol represents the joining of two peoples as a new nation into infinity or that the two nations will survive into infinity.

Have students work with a partner to complete the Your Turn questions using the methods of their choice. Have partners discuss their answers with another student pair and work together to resolve any differences in the final solutions.

Example 3

In this Example, students solve a population density problem involving powers with integral exponents using two different methods.

As a class, read the problem about grasshopper population density. Use the Did You Know? to help provide the context. Explain that grasshopper surveys and forecast maps help determine the need for control measures. You might point out the table in the related Your Turn to reinforce that control measures would be needed for moderate to severe infestations.

Before having students consider the solution, you may have them use their own strategies to solve the problem. Ask the following questions:

- How did you determine the approximate population per square kilometre?
- What answer did you get?

As a class, work through the solution. Note that the first method uses arithmetic while the second method uses exponent rules. You might ask students which method they prefer and why.

Have students do the Your Turn using the method of their choice and then explain their method to a classmate.

Key Ideas

The Key Ideas reinforce that the pattern $2^0 = 1$, $2^{-1} = \frac{1}{2}$, $2^{-2} = \frac{1}{2^2}$, $2^{-3} = \frac{1}{2^3}$, and $2^{-4} = \frac{1}{2^4}$ allows us to define the expression $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$, when a does not equal 0 and n is a positive integer. Have students develop their own example and record it in section 4.2 of their Foldable.

Direct students to the table summarizing the exponent laws. You might have students verify that the equivalent expressions are equal using technology. Have students use the back of the insert for section 4.2 to provide their own example of an integral exponent for each exponent law. You might encourage students to exchange their examples with those of a partner and check the solutions.

Meeting Student Needs

- For the Link the Ideas, consider modelling an example of each exponent law to help reinforce students' understanding. Alternatively, have students work with a partner to verify each example.
  - Product of Powers: $(4^5)(4^2) = 4^7$
  - Quotient of Powers: $\frac{5^6}{5^2} = 5^4$
  - Power of a Power: $(4^3)^5 = 4^{15}$
  - Power of a Product: $(4y)^2 = (4^2)(y^2)$
  - Power of a Quotient: $\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$
  - Zero Exponent: $(-5)^0 = 1$
- Consider having students work in pairs to create posters that illustrate the exponent laws. Each student pair could develop a poster illustrating one exponent law. Have students use the posters displayed in the classroom as a reference tool.
- Some students may benefit from recalling the order of operations. You might have students record the acronym BEDMAS into their Foldable and using it as a reference in solving problems involving order of operations. Coach them to recall the correct order of operations: brackets, exponents, divide and multiply in order from left to right, and add and subtract in order from left to right. Alternatively, some students may prefer the phrase **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally (parentheses, exponents, multiplication and division, addition and subtraction).

- For Example 1, some students may benefit from practising multiplying powers by expanding first before applying the exponent law.
- Some students may need to recall how to divide fractions (multiply by the reciprocal).
- Some students may find it helpful to think of rewriting expressions in positive exponent form as moving factors from a denominator to a numerator or vice versa. For example, $\frac{2x^{-3}}{5}$ becomes $\frac{2}{5x^3}$ and $\frac{2}{5x^{-3}}$ becomes $\frac{2x^3}{5}$. The example shown for the power of a quotient in the Key Ideas clearly demonstrates this shortcut.
- The Your Turn for Example 2 asks students to simplify and evaluate where possible. You might explain that *simplify* means to make more simple. For example, you might use the order of operations to simplify an expression. Explain that *evaluate* in this case means to determine the value of an expression. For example, $10 - 3 = 7$.

ELL

- Explain that an *integral exponent* is an exponent that is an integer, such as $2a^3$ and 9^{-4}.

Enrichment

- For Example 2, challenge students to solve a problem in which they need to convert the powers to the same base first. For example, $\left(\frac{2^4}{8^2}\right)^{-3}$.
- Challenge students to research the origins of the infinity symbol and how it is used. For example, the Métis flag features an infinity symbol. Students may find the related Web Link at the end of the section useful.

Gifted

- Challenge students to explain the reasoning that produces negative exponents.

Common Errors

- Some students may be confused about the exponent rules for multiplying and dividing powers.

R_x Coach students through solving additional problems using expansion and then simplification. Help them discover that when powers are multiplied, the exponent in the final power is the sum of the initial exponents. Similarly, when powers are divided, the exponent in the final power is the difference between the initial exponents.

- Some students may multiply and divide positive and negative integers incorrectly.

R_x Coach students through additional problems to help them recall the rules for multiplying and dividing positive and negative integers.

- Some students may add exponents when a power is raised to another exponent.

R_x Coach students through additional problems using expansion and simplification to help them recall the rule for multiplying the exponents when a power is raised to another exponent.

- Some students may add the bases and the exponents. For example, $(2^3)(2^4) = 4^7$.

R_x Remind students to keep the bases the same and add only the exponents.

- When working with negative exponents, some students may change the base to a negative number.

R_x Have students talk through their thinking as they work with a negative exponent.

WWW Web Link

For information about the Métis flag, which includes an infinity symbol, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $\frac{1}{8}(32) = 4$ or $2^{(-3+5)} = 2^2$ **b)** $7^{(-5-3)} = 7^{-8}$ or $\frac{1}{7^8}$

c) $(-3.5)^{[4-(-3)]} = (-3.5)^7$ **d)** $(3y)^{[2-(-6)]} = (3y)^8$

Example 2: Your Turn

a) $(0.6^0)^{-5} = 1$ **b)** $(t^{-1})^{-3} = t^3$ **c)** $(x^2)^{-2} = x^{-4}$ **d)** $\left(\frac{1}{y^3}\right)^{-3} = \frac{1}{y^9}$

Example 3: Your Turn

There are 16.04 grasshoppers per square metre. This is a severe infestation.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that when they are multiplying or dividing powers with the same base, the exponents change but the base does not. • Encourage students to apply the rules of their choice. • Provide similar questions before having students try the Your Turn. For example, $(3x)^4(3x)^3$ and $\frac{(4x)^5}{(4x)^{-3}}$. • For part d), reinforce the importance of maintaining the brackets by asking what the difference is between $(2x)^2$ and $2x^2$.
Example 2 Have students do the Your Turn related to Example 2.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize the process for simplifying each question. • You may wish to have students work with a partner. • Remind students to evaluate only after simplifying.
Example 3 Have students do the Your Turn related to Example 3.	• Some students may prefer to solve using arithmetic. You might point out the possibility of making an error when entering a number with multiple zeros. Encourage multiple approaches. Encourage students to use a second method and check that the solution is correct. • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.

Check Your Understanding

Practise

For #1, students need to understand the meaning of a negative or a positive exponent used to represent a situation. You might prompt students to recognize that a positive exponent means increasing in value and a negative exponent means decreasing in value.

For #3, have students correct the error and then provide an explanation of the error and correction to a classmate.

For #4 and 5, remind students that for #5c), $t \neq 0$ and for part e), $n \neq 0$. The introduction of coefficients in #5c) and d) requires special consideration. For #5d), check that students apply the exponent 3 to the coefficient 2.

In #6, the bases are rational numbers. Encourage students to simplify using the exponent laws before applying technology to determine the answer.

For #7, you may need to remind students that in time, *four years ago* can be represented by –4. The word *ago* refers to the past, which requires using a negative exponent.

For #8, you may need to help students understand that the base in an exponential expression that simulates growth must be larger than 1 (or 100%). In this case where the growth rate is 1.05 or 5%, the base is 1.05 or 105%.

Apply

It may not be necessary to assign all questions to all students. Allow students some choice in the questions they need to do. Encourage students to solve the problems using a method of their choice.

You might have students compare their method for some problems with that of a classmate who solved the problems in a different way.

For #9, encourage students to express the diameter as a power with base 10 in order to use exponent laws for solving it. Consider extending the question by asking students to estimate how many bacteria could fit on the head of the pin.

For #10b), encourage students who are stuck to solve the exponential equation for $t = 0$.

For #11, you might mention that the Great Galaxy in Andromeda, also called M131, is the nearest spiral galaxy to our own galaxy, the Milky Way. M131 is one of the farthest objects visible to the naked eye.

For #13, refer students to the Did You Know? about whooping cranes to help set the context.

For #16, refer students to the Did You Know? about algae. You might have students discuss problems with algae that they are familiar with.

For #18, refer students to the Did You Know? about oil spills to help explain the context. Emphasize the importance of fast and effective action to contain an oil spill. For instance, a spot of oil no larger than a quarter may be enough to kill a seabird. Ask students what strategies they could use to determine the time needed for the bacteria to reach the required concentration. Encourage them to solve the problem using mental math and estimation.

Extend

For #20a), you might suggest that students raise each side of the equation to the exponent –1. Students will find that the right side of the equation will convert to the reciprocal. For part b), ask students how the power can be rewritten with a base of 3. For part c), encourage students to rewrite the fraction on the right side as powers with base 3 or 4.

For #21, you might provide a clue about rewriting 8 in base 2.

For #22a), some students may need help to realize that $n = 0$ in this case.

Create Connections

For #25, consider having students work in pairs to develop their own pattern to demonstrate the relationship between negative and positive exponents. Have students verbally describe the relationship to their partner before recording a response.

For #26, consider having a class discussion and then have students use the ideas as a springboard to develop their own response.

For #27, students determine how to compare the sizes of different powers. The questions are set up to help students conclude that it is possible to compare different powers when either the bases are the same or the exponents are the same.

For part c), encourage students to try evaluating the powers using technology. They will note the limitation of the calculator in that it cannot display such large values. Allowing students to see the limitations with technology is important in the math classroom. In order to successfully evaluate the powers, students must be prepared to rewrite the exponents in factored form and identify the greatest common factor of the exponents. Subsequently, students can apply the exponent law of powers to rewrite each power with a base that can then be compared. In this case, the greatest common factor of the exponents is 111. The first power could then be rewritten in the following equivalent form:

$$\begin{aligned} 2^{666} &= (2^6)^{111} \\ &= 64^{111} \end{aligned}$$

Have students rewrite each power in an equivalent form with an exponent of 111 and a base that can be evaluated and then compared with the other powers.

As a class, have students discuss their results and conclusions.

Meeting Student Needs

- Allow students to work in pairs.
- For #16, coach students to explain the percent of the pond covered today and last week.
- For #27, some students may benefit from coaching before developing their response. Use prompts and examples to help students realize it is possible to compare powers when
  - the bases are the same. (2^5 is larger than 2^4 since the exponent 5 is larger than 4, and the bases are both 2.)
  - the exponents are the same. (4^5 is greater than 3^5 since the base 4 is larger than the base of 3, and the exponents are both 5.)
- Some students may benefit from doing the tutorial about exponent laws described in the Web Link at the end of this section.
- Students need to build a solid understanding of the exponent laws using integral exponents in order to be successful with rational exponents in the next section. Some students may benefit from extra practice questions to reinforce understanding.
- Provide **BLM 4–7 Section 4.2 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *radioactive*, (bacterial) *culture*, *French-language publishing*, *bacterium*, *red blood cell*, *endangered species*, *atoms*, *rechargeable batteries, nickel-metal hydride battery, voltage, pledges, oil spill*, *crude oil*, *degrading*, *concentration*, *intensity*, *coloured gels,* and *radium.*
- For #11, clarify that a galaxy is a massive system of stars, gas, and dust. A spiral galaxy is one of the three main classes of galaxies. Use the photo to help explain that M131 consists of a flat disk that looks like spirals with long arms winding toward a central concentration of stars called a bulge.

Enrichment

- Students with an interest in the Paleolithic caves and rock art discovered in the Pyrenees, in France, may enjoy researching dating methods such as carbon dating to determine the age of the rock paintings. Have students present a report of their findings with a particular focus on the problems encountered in using carbon dating. Students may find the related Web Link at the end of this section helpful.
- Discuss some examples of exponential growth and decay (e.g., calculating the growth of a caribou population of 1400 using the expression $1400(1.05)^n$; calculating the growth of bacteria using the expression 0.85^{-n}; calculating the decline in value of a mutual fund using the expression $500(1.5)^{-n}$; calculating the amount of a radioactive substance remaining using the expression 0.5^n). Have students observe the base and the exponent of the power in each example and identify when a positive or a negative exponent is used to model a situation. Then, prompt them to draw conclusions. Ask students how they can use the base and the exponent of a power to determine if the power represents a growth or a decay situation.

  You might have students develop a table similar to the one shown to organize their findings.

Growth Model	Decay Model
• a base larger than 1 and an exponent that is positive	• a base larger than 1 and an exponent that is negative
• a base between 0 and 1 and an exponent that is negative	• a base between 0 and 1 and an exponent that is positive

  (These conditions are due to the fact that raising a base to –1 results in the reciprocal of the base.)

- Have students who are interested in bioremediation research how scientists use nutrient enrichment to clean up oil-contaminated shorelines. They may find the related Web Link at the end of this section helpful.

Gifted

- Challenge students to develop and solve their own carbon-14 dating problem: Research a painting from a cultural heritage of your choice. The related Web Link at the end of this section may be helpful. For example, you might choose a painting titled Hero and Heroine by French painter Jacques Iverny (active from 1411–1435). Create a question that requires using the carbon-14 formula to date the painting *or* determine if it is fake based on the amount of carbon-14 remaining on a sample taken from the painting today. Hint: Choosing an older painting is preferable to illustrate carbon-14 dating.
- Challenge students to show the exponential growth of a circular oil spill and resulting environmental damage for every hour or day of delay in containing the spill. Encourage students to graph the results and present their findings to the class.
- Challenge students to solve a problem that illustrates exponential growth: Ms. Lee has a class rule that students who show up late for class must contribute 1¢ toward a class pizza party and every day that they show up late afterward, they must double the amount contributed. Write the formula. How much would a student owe after showing up late 4 days? 8 days? 20 days? 30 days?

Web Link

For a tutorial about the exponent laws, go to www.mhrmath10.ca and follow the links.

For information about cave paintings and the dating methods used to determine the age of the artifacts, go to www.mhrmath10.ca and follow the links.

For a database of European painting and sculpture and a web gallery of artists' works, go to www.mhrmath10.ca and follow the links.

For a video about how bioremediation was used to clean up an oil-contaminated shoreline in Nova Scotia, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise, Apply, and Extend Have students do #2, 4, 5, 9, 10, 11, 14, and 21. Students who have no problems with these questions can go on to the remaining questions.	• Provide additional coaching to students who need support with #2, 4, and 5. These questions provide students with practice writing expressions with positive exponents, and in the case of #4 and 5, practice simplifying expressions. Work with students to correct their errors and then have them try #6 on their own. • Some students may benefit from reviewing the exponent laws using integral values before attempting questions with variables. Refer students to the notes in their Foldable or a class poster of the exponent laws to help them. • For #9, 10, 11, and 14, students apply their understanding of exponents to real-world situations. It may be beneficial to discuss situations when a negative or a positive exponent should be used. Encourage students to add any additional notes to their Foldable. • For #21, students apply the exponent laws in different ways to solve for a missing exponent or base. For each case, encourage students to identify the exponent law that is being used and then attempt to solve the problem.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • For #25, students could record their response in their Foldable and use it as a study tool. • For #26, consider allowing students to adapt a real-life situation from the student resource.

4.3 Rational Exponents

***Mathematics 10,* pages 174–183**

Suggested Timing

80–100 min

Blackline Masters

BLM 4–3 Chapter 4 Warm-Up
BLM 4–8 Section 4.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
Visualization (V)

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3a)–c), 4a), c), e), 5a)–c), 6, 7, 9–11, 18, 19
Typical	#1, 3a)–c), 4a), c), e), 5a)–c), 6 or 7, two of 8, 10, 12, *or* 13, three of 9, 11, 14, *or* 15, 18, 19
Extension/Enrichment	#3–5, 7, two of 11–15, 16–19

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

Discuss the opening text and visual about the piano keyboard. Explain that a piano keyboard has 88 musical pitches organized in ascending order of pitch. The lowest note is at the far left of the instrument, the highest note on the far right. Each black key and white key on a keyboard produces a pitch or note. The distance between one key and the next adjacent key is the smallest difference in pitch between two notes that humans can hear.

Highlight that the piano keyboard is an example of the connection between mathematics and music and that this relationship supports ideas in the Unit 2 project.

Investigate Rational Exponents

In this Investigate, students revisit the use of the exponent laws to develop the concept and the meaning of a rational exponent. The key idea is for students to connect the definitions of square root (and cube root) from section 4.1 with the notation using powers with rational exponents. Students explore two different ways of evaluating powers with integral exponents. They predict values of powers with rational exponents and check their predictions using technology.

Have students work individually or with a partner to work through #1 to 4. For #1, you might have students try their own example that they can evaluate without a calculator, using both notations (e.g., $25^{\frac{1}{2}}$). Encourage students to write one of the powers in #2 using the two forms shown in #1. As you circulate, have students verbalize the connection between square roots and powers with the fractional exponent $\frac{1}{2}$. Consider using the following prompts:

- What does raising 9 to the exponent $\frac{1}{2}$ mean? (Students may say it is the same as taking the square root of 9.)
- What is the connection between a power with the exponent $\frac{1}{2}$ and its square root? (Students may say that multiplying a value by itself results in the square of the value and that the inverse is taking a square root. Or, if a number times itself has a value of 9, then its value must be the square root of 9.)

For #3, have students express $8^{\frac{1}{3}}$ in statements similar to those in #1. As you circulate, you might have students verbalize the connection between cube roots and powers with the fractional exponent $\frac{1}{3}$.

Ask what raising 8 to the exponent $\frac{1}{3}$ means.

(Students may say that it is the same as taking the cube root of 8.)

Give students sufficient time to complete #4 either individually or with a partner. As a class, have students discuss their responses.

Meeting Student Needs

- Allow students to work in pairs or small groups. Provide coaching as needed to help students practise with both notations.

ELL

- Clarify that *pitch* refers to how low or how high a note sounds.

Common Errors

- Some students may erroneously calculate powers with rational exponents as the product of the base and the exponent (e.g., $4^{\frac{1}{2}}$ as $(4)\left(\frac{1}{2}\right) = 2$), which would appear to be correct in this case).

R$_x$ Remind students that in a power such as 5^2, the exponent 2 indicates that the base 5 is multiplied by itself 2 times, as in $5^2 = (5)(5)$
$= 25$

Answers

Investigate Rational Exponents

1. $9^{\frac{1}{2}} = 3$ because $(3)(3) = 9$

2. $4^{\frac{1}{2}} = 2$; $16^{\frac{1}{2}} = 4$; $36^{\frac{1}{2}} = 6$; $49^{\frac{1}{2}} = 7$

3. $\sqrt[3]{8} = 2$. Example: $(2)(2)(2) = 8$. An exponent to one half results in a square root, so an exponent to one third should result in a cube root.

4. a) Example: A fractional exponent is a root.

b) $2^{\frac{1}{12}} = 1.059463...$

c) $1.059463^{12} = 1.999998$. Example: It is not because 1.059463 is rounded that the answer is close to 2. If the full calculator screen is used (with the extra registers still full), then the answer is 2 on the screen. The twelfth root and the twelfth power are inverse operations.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Encourage students to verbalize the connection between the definition for square root and a power to the exponent $\frac{1}{2}$. Similarly, have students verbalize the connection between the definition for cube root and a power to the exponent $\frac{1}{3}$.

Link the Ideas

Have students recall what they know about rational numbers. Explain that they will work with rational exponents. Discuss that a rational exponent is an exponent that is a rational number. It can be expressed as $x^{\frac{a}{b}}$, where a and b are integers, and $b \neq 0$. Use some examples to reinforce that rational exponents can be expressed in decimal or fractional form.

Students will be familiar with the exponent laws from working with integral exponents in section 4.2. Use their earlier work to make the connections to applying the exponent laws using rational numbers as bases and exponents.

As a class, have students recall that a power with a negative exponent is equal to the reciprocal of the base raised to the positive of the exponent. Remind students that the exponent 0.2 in the example is rational because it can be expressed as a ratio of two integers. Ask the class what this fraction is in lowest terms.

Model expanding the given example:

$$3^{-0.2} = \left(\frac{3^{-0.2}}{1}\right)\left(\frac{3^{0.2}}{3^{0.2}}\right)$$
$$= \frac{1}{3^{0.2}}$$

Then, walk through the following example and show that a negative exponent in the denominator of a fraction is equal to the reciprocal of the base raised to the positive of the exponent.

If $3^{-0.2} = \frac{1}{3^{0.2}}$, then

$$(3^{-0.2})(3^{0.2}) = \left(\frac{1}{3^{0.2}}\right)(3^{0.2})$$
$$= \frac{3^{0.2}}{3^{0.2}}$$
$$= 1$$

Then,

$$\frac{1}{3^{-0.2}} = \left(\frac{1}{3^{-0.2}}\right)\left(\frac{3^{0.2}}{3^{0.2}}\right)$$
$$= \frac{3^{0.2}}{1}$$
$$= 3^{0.2}$$

Have students summarize how the sign of an exponent can be changed from a negative to a positive value.

Example 1

In this Example, students multiply or divide powers with a common base and rational exponents.

Since students have multiplied or divided powers with integral exponents, consider having them work in pairs or small groups to answer the questions using their own strategies.

For part c), ask students what they need to do to the exponents before solving. Coach students through converting exponents to fractional or decimal form, if necessary. For parts b) and c), you might have students write one product or quotient using exponents in fractional form and the other one in decimal form.

For part d), ask students how they can use powers of 2 to convert to the same base.

As a class, walk through the given solutions. Have students compare their solutions to the given solutions. You might have students work in small groups to discuss the given solutions and explain why and how the methods shown work. Ask questions such as the following:

- What methods do you use to multiply or divide powers with the same base?
- Which method do you prefer? Why?

Have students work with a partner and challenge them to use different methods to answer the Your Turn questions. Have partners discuss their solutions and work together to resolve any differences in the solutions.

Example 2

In this Example, students simplify and then evaluate powers with rational exponents.

Work through the solutions as a class. Alternatively, since students have simplified powers with integral exponents, consider having them work in pairs or small groups to answer the questions using their own strategies before working through the given solutions as a class.

For part a), emphasize the importance of raising the coefficient of the power, 4, as well as the variable base to the outside exponent. You may need to remind students of the connection between 0.5 and $\frac{1}{2}$.

For parts b) and c), show both methods. Ask students which method they prefer and why.

For part b), Method 1, have students use mental math to add $3 + \frac{3}{2}$.

For part c), ask the following questions:

- How can you use prime factorization to convert to the same base?
- Why do you convert the exponent from –0.75 to $-\frac{3}{4}$?

Discuss that in general, it may be helpful, but is not necessary, that all rational exponents be in the same form after being simplified. Discuss situations when it would be better to leave rational exponents in fractional form. For example, $\frac{2}{3}$ does not convert into an exact decimal, so it should be left in fractional form, for better accuracy.

Have students work with a partner and challenge them to use different methods to solve the Your Turn questions. Have partners discuss their solutions and work together to resolve any differences in the final solutions.

As a class, have students discuss the method they used to simplify part b). Ask the following questions:

- Did you simplify within the brackets first or did you raise each power to the exponent 9 first?
- Which method do you prefer? Why?

Example 3

In this Example, students solve a population growth problem involving powers with rational exponents.

As a class, read the problem and walk through the solution. For part a), ask students how they know from the values in the formula that it represents a growth model. (The base is greater than 1.)

Discuss the fractional exponent and reinforce that the doubling of the bacteria occurs every 42 h. Ask students what exponents represent 84 h and 126 h.

For parts c) and d), ask students if it is reasonable to round up. Why? (Fractions of bacteria do not make sense.)

Before assigning the Your Turn, you might explain that a fund is an investment that pools money from many individuals and invests it according to the fund's objectives. Explain that depending on factors such as the risk of the investment, mutual funds can increase or decrease in value. If needed, clarify that *quarterly* means every three months. You might ask students about the connection between 12.6% and 1.126 in the formula. Point out that the base of the power in the formula is equal to 112.6%. Then, break this percent into the sum: 100% + 1.26%. Have students do the Your Turn and then explain their solution to a classmate.

Key Ideas

The Key Ideas reinforce the principle that a power with a negative exponent can be written as a power with a positive exponent. Ask students to develop their own example and record it in section 4.3 of their Foldable.

Direct students to the table summarizing the exponent laws as they apply to powers with rational exponents. Have students verify the expressions by converting each exponent to either decimal or fractional form and then simplifying. Have students use the back of the insert for section 4.3 to provide their own example of a rational exponent for each exponent law. Have them include an example of a power with a rational exponent that is written in both decimal and fractional form. Encourage students to exchange their examples with those of a partner and check the solutions. Check that students apply the exponent laws to coefficients of powers that are raised to another exponent.

Meeting Student Needs

- For the Link the Ideas, consider modelling an example of each exponent law to help reinforce students' understanding about rational exponents. Alternatively, have students work with a partner to verify each example.
  - Product of Powers: $(3^{\frac{1}{2}})(3^{\frac{1}{2}}) = 3$
  - Quotient of Powers: $\dfrac{5^{0.5}}{5^{0.25}} = 5^{0.25}$
  - Power of a Power: $(3^{\frac{1}{5}})^{\frac{1}{2}} = 3^{\frac{1}{10}}$
  - Power of a Product: $(4y)^{0.6} = (4^{0.6})(y^{0.6})$
  - Power of a Quotient: $\left(\dfrac{2}{5}\right)^{\frac{2}{3}} = \dfrac{2^{\frac{2}{3}}}{5^{\frac{2}{3}}}$
  - Zero Exponent: $(-5)^0 = 1$
- For the Link the Ideas, some students may benefit from restating the principle about powers with negative exponents in their own words. Refer them to the notes in their Foldable to help them. For example, a negative exponent in the numerator can be written as a positive exponent in the denominator. Also, a negative exponent in the denominator can be written as a positive exponent.
- Encourage students to refer to the classroom posters illustrating the exponent laws. Reinforce that the same laws apply to rational exponents.
- For Example 1, have students restate the exponent laws for multiplying or dividing powers with the same bases:
  - To multiply powers with the same base, add the rational exponents.
  - To divide powers with the same base, subtract the rational exponents.
- Encourage students to perform the calculations involving fractions without technology. Allow students access to calculators when simplifying questions involving rational exponents.
- Reinforce that when simplifying rational exponents that are in fractional and decimal form, the exponents need to be converted so all are fractions or decimals. Be prepared to help students review how to change from one form to the other.
- For Example 3, ask students how the base or the exponent of the power indicates a growth model or a decay model.

- For Example 3: Your Turn, check students' understanding of the concept of interest and how it is earned. For example, students living in Northern communities may not have access to banks. You may wish to explain simple interest and compound interest and how the number of compounding periods affects the final amount of the investment.

Common Errors

- Some students may have difficulty evaluating powers with rational exponents when using their calculator.

R$_x$ Coach students to use proper key sequencing on their particular model. Check that they place brackets around the fractional exponents.

- Students may struggle with fraction operations when applying the exponent laws to powers with rational exponents.

R$_x$ Help students review fraction operations and how to convert between fractions and decimals.

WWW Web Link

For a tutorial on rational exponents, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) x^5 **b)** $p^{\frac{-3}{4}}$ **c)** $4^0 = 1$ **d)** $1.5^{\frac{7}{6}}$ or $\left(\frac{3}{2}\right)^{\frac{7}{6}}$

Example 2: Your Turn

a) $27^{\frac{2}{3}}x^{\frac{12}{3}} = 9x^4$ **b)** $(t^{12})(t^3) = t^{15}$ **c)** -27 **d)** $\left(\frac{64}{x^3}\right)^{\frac{2}{3}} = \left(\frac{4}{x}\right)^2$

$$= \frac{16}{x^2}$$

Example 3: Your Turn

a) Example: 12.6% = 0.126; 1 = full investment, so 1.126 = full investment plus interest

b) The value is $5465.42 after the third quarter.

c) The value is $7138.14 after three years.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For each question, ask students to verbalize the exponent law they would use. • For part c), ask students whether they would change both exponents to fractional form or decimal form. Consider using some examples to help students generalize that powers with rational exponents that convert to non-terminating decimals should be kept in fractional form.
Example 2 Have students do the Your Turn related to Example 2.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from coaching to decide which exponent laws to apply and how to simplify a question such as $(27x^6)^{\frac{2}{3}}$. Ask them to indentify the exponent of 27.
Example 3 Have students do the Your Turn related to Example 3.	• Have students identify and record the meaning of each variable in the formula. • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.

Check Your Understanding

Practise

For #1 and 2, encourage students to practise working with exponents in both decimal and fractional form. Remind students to raise any coefficients to the exponent.

For #1f), you might model converting to a positive base. Using an example, ask students to explain how to convert –8 to a positive:

$$\begin{aligned}(-8)^{\frac{2}{3}} &= [(-1)(8)]^{\frac{2}{3}}\\ &= (-1^{\frac{2}{3}})(8^{\frac{2}{3}})\\ &= (-1)^{\frac{2}{3}}(2)^{(3)\left(\frac{2}{3}\right)}\\ &= (1)(2^2)\\ &= 4\end{aligned}$$

For #5, remind students to simplify using the exponent laws before using a calculator to evaluate.

For #6, ask students how the base of 1.1 in the formula relates to the growth rate. Refer students to the Did You Know? about species of fish that are stocked in BC.

Apply

Allow students some choice in the questions they need to do. Encourage students to solve the problems using a method of their choice. You might have students compare their method for some problems with that of a classmate who solved the problems in a different way.

For #7, students perform error analysis. For each problem, encourage them to confirm that the initial expression and the final expression are not equivalent by evaluating the expressions using a whole number in place of the variable.

For #9c), students must remember that the exponent is negative.

For #10c), discuss why exponential growth results in overcrowding and the subsequent decline of the population as the fish compete for limited resources. Ask students to provide some limiting factors (e.g., space, food) to exponential growth.

For #11, some students may not understand why the base is 0.88 (100% – 12%). A base less than 1 implies that depreciation is occurring.

For #14, you might explain that the formula is derived from Isaac Newton's Law of Cooling. The law states that the rate at which a warm object cools is approximately proportional to the difference between the temperature of the object and the temperature of its environment.

For #15, refer students to the Did You Know? about Johannes Kepler.

Extend

For #16, encourage students to try folding a sheet of paper the number of times that corresponds to the answer. Some students may be successful if you provide the hint that 100% represents the starting area and 1% represents the ending area. Have students discuss with a classmate how to use these values in the formula. Encourage them to test their answer.

Create Connections

For #18, consider having students brainstorm some real-life applications of rational exponents. Students can use the ideas as a springboard to develop their own response.

For #19, encourage students to develop an example to help with their explanation, and provide a correction. Have them discuss their correction with a partner.

Meeting Student Needs

- Allow students to work in pairs.
- Have students refer to the exponent laws in their Foldable notes and the classroom posters when working through the questions.
- For word problems, encourage students to identify the key terms (e.g., half-life, doubles) to help determine what values are needed.
- For #15, some students may be successful if you restate the meaning of the variables in the formula and prompt them about the order to use for solving the problem.
- Provide **BLM 4–8 Section 4.3 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *stocked annually*, *trout*, *char*, *kokanee*, *guppies*, *photographic enlarger*, *planetary system*, *orbital radius*, *planetary motion*, *consecutive folds*, and *bloodstream*.

Enrichment

- Tell students about the Rule of 72, which is used to estimate the amount of time it takes to double the value of an investment earning compound interest, depending on the interest rate. For a term deposit invested at 3%, it would take 24 years to double your money.

  $$\frac{72}{3} = 24$$

  For a term deposit invested at 5%, it would take 14.4 years.

  $$\frac{72}{5} = 14.4$$

  Have students try out the rule using different interest rates.

- Challenge students who are interested in famous mathematicians to research Sir Isaac Newton and Gottfried Leibnitz and the controversy about who invented calculus. Have them present a report on their findings. They may find the related Web Link at the end of this section helpful.
- For the Did you know? on page 183, have students research Johannes Kepler's work on planetary motion. They might present a report using a format of their choice. They may find the related Web Link at the end of this section helpful.

Gifted

- One of the major concerns about climate change involves the melting of sea ice, in both area and volume. Have students research this issue and speculate how scientists use mathematical models of melting sea ice and shrinking polar ice sheets in order to predict future trends.

- Challenge students interested in the formula related to musical pitch to explore the related Web Link on this page.

Web Link

For information about Sir Isaac Newton and Gottfried Leibnitz and the calculus controversy, go to www.mhrmath10.ca and follow the links.

For information about Johannes Kepler's work on planetary motion, go to www.mhrmath10.ca and follow the links.

For a lesson involving the formula that expresses the relationship between the pitches of adjacent musical notes, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3a) to c), 4a), c), and e), 5a) to c), 6, 7, and 9 to 11. Students who have no problems with these questions can go on to the remaining questions.	• Encourage students to verbalize their thinking. • Provide additional coaching with Example 1 for #1 and with Example 2 for #3 to 5. Coach students through correcting their errors before having them try some of the questions that were not assigned. • For #1, encourage students to refer to the notes about the exponent laws in their Foldable. • For #3, some students may benefit from reviewing the process of opposite operations. Model solving equations such as $x + 1 = 4$ and $\frac{x}{2} = 3$, and ask them to identify the opposite operations that will allow them to solve for x in each case. Encourage them to apply the same thinking to #3c). • Provide additional coaching with Example 3 for #6. Have students talk through their thinking and help them correct their errors. Check for understanding before moving on to #9 to 11. • For #7, prompt students to verbalize the strategy and the exponent law they would apply for each solution. Have them work through simplifying the initial expression and then identify the error. • For #9 and 11, have students record the meaning of each variable as part of the solution. For #11, clarify the meaning of *decreasing in value* and discuss how the wording affects what values are used in an equation.
Assessment *as* Learning	
Create Connections Have all students complete #18 and 19.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions and then develop an individual response orally or in written form. • For #18, encourage students to generate their own problem. You might allow students to adapt a problem from the student resource. • You might encourage students who have difficulty in choosing an example for #19 to use one of the questions in #3. The solutions to these types of questions typically show errors in thinking. In particular, #3f) might be appropriate as students often erroneously multiply the base numbers 2 and 3.

Irrational Numbers

4.4

Mathematics 10, pages 184–195

Suggested Timing

100–120 min

Materials

- ruler
- compasses
- square dot paper

Blackline Masters

Master 3 Square Dot Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–5 Chapter 4 Unit 2 Project
BLM 4–9 Section 4.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

AN2 Demonstrate an understanding of irrational numbers by:

- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a)–c), 2b)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 11, 13, 21, 24
Typical	#1a)–c), 2a)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 13, three of 12–17, 21, 22 *or* 23, 24
Extension/Enrichment	#1d), 2a), d), 3c), d), 4e), f), 5d), f), 6c), d), 7c)–f), 9, 13, 18, 19 *or* 20, 21, 22 *or* 23, 24

Unit Project Note that #13, 18, and 24 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, discuss the painting in the opener. Use the prompts to promote discussion:

- What is a ratio?
- What do you know about the golden ratio? (You might have students who are familiar with the golden ratio give some examples of golden ratios.)
- Why might artists and architects use the golden ratio in their designs?
- What questions do you have about the golden ratio? irrational numbers?

Direct students to the Did You Know? about Ayla Bouvette, a self-taught Métis artist. Encourage students to discuss other artists who use the golden ratio.

Investigate the Golden Rectangle

Unit Project

During this Investigate, which is part of the unit project, students

- draw a geometric representation of the golden ratio
- write an exact expression for the golden ratio
- describe the golden rectangles in a painting
- find rectangular shapes that may be in the golden ratio and determine how close the shapes are to golden rectangles

Tell students that they will construct a rectangle that expresses the golden ratio.

Have students work individually or in pairs. Have them use a blank sheet of paper, a ruler, and compasses to construct the golden ratio as outlined. Refer students to the diagrams in the student resource to help them. The non-drawing point of the compass should be placed at vertex D of the square. Emphasize the importance of accuracy in drawing and measuring.

As you circulate, use some of the following prompts:

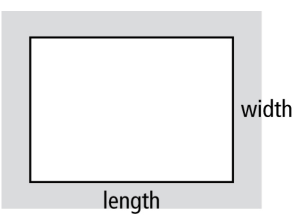
- How might you apply the Pythagorean relationship to calculate the length of DE?
- How close was your measured value of DE to the calculated value?
- In terms of computing the golden ratio, why are the dimensions of the original square irrelevant?
- Looking at your model of the golden rectangle, why do you think people find the ratio pleasing to the eye (compared to other rectangles)?

You may wish to have students use geometry software to construct the golden rectangle.

Give students enough time to answer #5 and 6. For #5a), you might ask students what values they would use to write an exact expression for the golden ratio. For #5, students should conclude that the ratio of the sides of a golden rectangle is approximately 1.618. For #6, have students compare their results with those of a classmate. You might then have students discuss as a class which items in the classroom, if any, are in the golden ratio.

In a follow-up discussion, prompt students to observe that one way to look at a number such as 1.618 is as a geometric representation, not unlike what they have done previously with square roots and cube roots.

You might display a diagram of the golden rectangle similar to the following to help students visualize the golden ratio and the ratio of the length to the width.



$$\text{golden ratio} = \frac{\text{length}}{\text{width}} \approx 1.618\ldots$$

Direct students to the Did You Know? on page 185 that introduces the term *phi* and the symbol used to represent the golden ratio. Use the diagram of the Great Pyramid of Giza to point out the proportions that form the golden ratio.

Meeting Student Needs

- Some students may be more successful if they use grid paper to construct the golden rectangle.
- For #5a), tell students to keep the length of DE in exact radical form in order to determine the exact expression of the golden ratio.
- For #6a), you might have students search for rectangular shapes at home as well as in the classroom (e.g., picture frames, playing cards, photos) and bring them in as part of the Unit 2 project.
- You might invite an artist or a craftsperson to talk to the class about how they use math concepts and skills related to the golden ratio in their work. For instance, a basket maker might use the Fibonacci sequence in creating designs.
- Invite a visual arts teacher to display and discuss some samples of the golden rectangle in student-created artwork. Alternatively, have a visual arts teacher coach students through creating a piece of art using the golden rectangle they created in the Investigate.

Common Errors

- Students may not accurately construct the arc to locate point F.

R$_x$ Reinforce the importance of being careful in constructing the golden rectangle. Students should measure the square accurately and use the compass appropriately. Consider demonstrating how to use the compass.

- Students may not understand how they can use the Pythagorean relationship to determine DE.

R$_x$ Coach students to identify the right triangle containing hypotenuse DE.

Web Link

For an animation of the golden rectangle, go to www.mhrmath10.ca and follow the links.

Answers

Investigate the Golden Rectangle

1. Example: 2 cm

4. a) Example: 2.2361

c) Example: 3.2361

5. a) $\dfrac{1+\sqrt{5}}{2}$

b) 1.62

c) Example: There are two sets of golden rectangles. One set is horizontal: ground, sky, and centre. One set is vertical: centre with the birds only (not including the entire golden circle around them).

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 2 Project Have students complete the Investigate.	• Consider having students work in pairs. • Listen as students discuss how to solve the problems. As you circulate, clarify any misunderstandings. • Suggest to students that they make their drawings large enough so that they can easily measure and draw using a compass. • You may wish to provide students with **BLM–5 Chapter 4 Unit 2 Project**, and have them finalize their answers. • Remind students to store all project-related materials in a portfolio for that purpose.
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students discuss #5 in pairs, in large groups, and then as a class before developing their own response. • You might ask students whether they can predict the ratio for any rectangle, and if so, explain how.

Link the Ideas

In this section, students build on their knowledge of the number system to learn about real numbers and the relationships among the subsets of the real numbers. They are introduced to irrational numbers and radicals.

Use the graphic organizer in the student resource to help define the real numbers and the subsets of real numbers. Explain why the subsets of natural and whole numbers and integers are nested inside the rational numbers. Use examples to reinforce that numbers can belong to more than one subset.

Define and differentiate between rational and irrational numbers. Have students recall that a rational number is a number that can be expressed as a fraction in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$. Ask for other examples of rational numbers (e.g., –2, $\frac{65}{4}$, $-\frac{3}{4}$, and 1.85).

Use examples and non-examples to develop the concepts. Emphasize that irrational numbers are decimals that neither terminate nor repeat. Point out the Did You Know? on page 187 in the student resource about pi as an example of a famous irrational number.

Ask students the following questions:
- Which subset do irrational numbers belong to?
- Which subsets do integers belong to? whole numbers? natural numbers?

After the discussion, use the following prompts:
- What did you learn by discussing real numbers?
- What, if any, misconceptions did you correct?

Discuss the definition of a *radical* and the terms *index* and *radicand*. Explain that a radical is a term for root. The root operation is represented by the radical sign. Use examples such as the following to show that radicals can be rational or irrational numbers.

Irrational examples include $\sqrt{2}$, $\sqrt{\frac{1}{5}}$, and $\sqrt[4]{7^3}$.

Rational examples include $\sqrt{4}$ and $\sqrt{\frac{25}{49}}$. Reinforce that where a radical that is an irrational number appears in a calculation (e.g., Pythagorean relationship, circle geometry), it has an actual value even though the radical cannot be expressed as a fraction (rational number).

As a class, walk through the examples demonstrating how powers with fractional exponents can be written as radicals. Have students discuss the meaning of the diagram showing radical form and exponential form.

Direct students to the example, $2^{\frac{3}{4}}$, written as $\sqrt[4]{2^3}$ or $\left(\sqrt[4]{2}\right)^3$. Reinforce the difference in reading these two expressions. The first one is read "the 4th root of 2 to the power of 3." The second is read "the 4th root of 2 all to the power of 3." Use a few examples (e.g., the 4th root of 2 to the power of 4, $\sqrt[5]{6^3}$) and have students practise reading and writing radicals using proper form.

Walk through the example of rewriting a power with a fractional exponent in decimal form and then evaluating. You might walk through a few additional examples of numerical radicands and have students practise converting and evaluating them.

Reinforce that in the example $\sqrt{5^3}$, the index, and therefore denominator of the rational exponent, is 2 ($5^{\frac{3}{2}}$).

Example 1

In this Example, students rewrite powers with fractional exponents in radical form.

As a class, work through the solutions. Encourage students to explain their thinking for each.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 2

In this Example, students rewrite radicals as powers with fractional exponents.

As a class, work through the solutions. Remind students that if no index is indicated for a radical, the index is understood to be 2. Ask students: Which part of a radical corresponds to the denominator of the exponent? the numerator of the exponent?

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 3

In this Example, students convert mixed radicals into entire radical form.

As a class, walk through the definitions of mixed radicals and entire radicals. For the examples of mixed radicals, have students practise reading them: $3\sqrt{2}$ as "3 times the square root of 2"; $\frac{1}{3}\sqrt{5}$ as "$\frac{1}{3}$ times the square root of 5", and $4\sqrt[3]{6}$ as "4 times the cube root of 6".

As a class, work through the solutions. Alternatively, consider having students work in pairs or small groups to answer the questions using their own strategies before working through the given solutions as a class. Point out that every mixed radical can be converted to entire radical form but not every entire radical will simplify to mixed radical form.

For each question, ask students to discuss any other methods they used. Ask which methods they prefer and why.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their methods and answers with another student pair and work together to resolve any differences in the solutions.

Example 4

In this Example, students convert entire radicals into mixed radical form.

As a class, work through the solutions.

For part a), ask students what two numbers multiply to 27 so that you can take the square root of one of the factors. Think of all the numbers that multiply to 27. The answer is (1)(27) and (9)(3). Nine is a perfect square. Think $\sqrt{(9)(3)} = 3\sqrt{3}$. The perfect square 9 can be simplified so the final answer is $3\sqrt{3}$. This may help students understand multiplying radicals later on and reinforces that $3\sqrt{3}$ means we are multiplying.

Some students may benefit from a reminder about how to determine the prime factorization of a number. Make the connection between the index of the radical and the number of factors that must be present in order to simplify the radical. Emphasize that the index of the radical must be recorded in the mixed radical form.

Direct students to the Did You Know? About pentagrams. Point out the features of the pentagram and ask students how the pentagram represents a golden ratio.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 5

In this Example, students compare and order a set of radicals.

As a class, work through both methods shown in the student resource. Ask students what other method might be used. Ask them what they notice about the index of each radical. (The index is the same.) Since the index of each radicand is the same, you might have students express each radical as a mixed radical with the same radicand, and then compare.

$$\begin{aligned} 2\sqrt{18} &= 2\sqrt{(2)(9)} \\ &= 2\sqrt{(2)(3^2)} \\ &= (2)(3)\sqrt{2} \\ &= 6\sqrt{2} \end{aligned} \qquad \begin{aligned} \sqrt{8} &= \sqrt{(4)(2)} \\ &= \sqrt{(2^2)(2)} \\ &= 2\sqrt{2} \end{aligned}$$

$$3\sqrt{2} \qquad \begin{aligned} \sqrt{32} &= \sqrt{(16)(2)} \\ &= \sqrt{(4^2)(2)} \\ &= 4\sqrt{2} \end{aligned}$$

Ask students why it is not always possible to use this method.

Have students work in pairs to complete the Your Turn question. Have each partner use a different method to solve, and then discuss their method and answer with each other.

Example 6

In this Example, students solve a problem involving irrational numbers.

As a class, read the problem and the Did You Know? features about gold production. Then, have students work in pairs to solve the problem.

As a class, have students discuss the method they used and the solution before walking through the given solution. Ask the following questions:

- What method did you use?
- Which method do you prefer? Why?

Have students do the Your Turn and then explain their method and solution to a classmate.

Key Ideas

The Key Ideas summarize the set of real numbers. As a class, discuss the following points as you review the organizer about real numbers:

- Real numbers include the rational numbers and irrational numbers.
- Rational numbers include the integers, whole numbers, natural numbers, fractions, and decimals that terminate or repeat. For example, $\frac{15}{4} = 3.75$.
  - Integers include the whole numbers and natural numbers (i.e., 0, –1, –2 … and 1, 2, 3 …).
  - Whole numbers include the natural numbers (i.e., 0, 1, 2 …).
  - Natural numbers are 1, 2, 3 … .
- Irrational numbers cannot be expressed in the form $\frac{a}{b}$, where a and b are integers, or as decimals that terminate or repeat. For example, $\sqrt{\frac{4}{5}}$ and $\sqrt[4]{35}$. Irrational numbers can be represented using decimal approximations. For example, $\pi \approx 3.142$ and $\varphi \approx 1.618$.

Remind students that radicals can be rational or irrational numbers. Ask for examples of each.

Review expressing radicals as powers with fractional exponents. You might have students use an example to show how a power with a fractional exponent can be written in decimal form, such as $\sqrt[5]{2^4} = 2^{\frac{4}{5}}$
$= 2^{0.8}$

Use the following prompts to discuss converting between radical and exponential forms:

- Do you prefer working with rational exponents that are in fractional or decimal form? Why?
- Is there ever a situation where you should use one form over the other? Explain.
- Explain how to convert from radical to exponential form.

Have students explain how to convert between entire radicals and mixed radicals. Have them demonstrate using examples such as the following:

- Using prime factorization:

$$\begin{aligned} \sqrt{72} &= \sqrt{(6^2)(2)} \\ &= 6\sqrt{2} \end{aligned}$$

- Multiplying:

$$\begin{aligned} 2\sqrt[5]{3} &= \sqrt[5]{(2^5)(3)} \\ &= \sqrt[5]{(32)(3)} \\ &= \sqrt[5]{96} \end{aligned}$$

Have students explain how they would order radicals that are irrational numbers.

Have students write their own summary of the Key Ideas and record it in section 4.4 of their Foldable. Also, have students use their Foldable to define the Key Terms in this section. Have them develop examples for each term. You might have students exchange their examples with those of a classmate and check that they are appropriate.

Meeting Student Needs

- Display a poster that illustrates a radical and labels the radical sign, radicand, and index. Encourage students to draw their own diagram and store it in their Foldable.
- Reinforce the idea that in order to remove a number from a square root, it needs to appear twice in the radicand. In order to remove a number from a cube root, it needs to appear three times in the radicand. Have students extend the pattern for other root indexes.
- For Example 4, discuss that $\sqrt{4} = 2$ and
  $$\begin{aligned}\sqrt{4} &= \sqrt{(2)(2)}\\ &= 2\end{aligned}$$
  Students can use these big ideas of multiplying or dividing radicals to help convert radicals.

Show students the general form $\left(\sqrt{a}\right)\left(\sqrt{b}\right) = \sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$. Have them apply the general form using examples.

Enrichment

- Encourage students to use the Web Link in the student resource on page 189 about pentagrams to help them draw one.

Common Errors

- Some students may struggle with converting between mixed and entire radical forms when the index of the radical is larger than 2.

R$_x$ Give students extra practice to determine the prime factorization of numbers. Also, stress the connection between the index of the radical and the number of identical factors that must be present in the radicand in order for a number to become a coefficient of the radical.

Web Link

For a video about expressing roots with fractional exponents, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $\sqrt[4]{10}$ **b)** $\sqrt[3]{1024}$ **c)** $x^{\frac{3}{2}} = \sqrt{x^3}$

Example 2: Your Turn

a) $125^{\frac{1}{2}}$ or $5^{\frac{3}{2}}$ **b)** $y^{\frac{5}{3}}$ **c)** $27^{\frac{2}{n}}$ or $3^{\frac{6}{n}}$

Example 3: Your Turn

a) $\sqrt[3]{2916}$ **b)** $\sqrt{317.52}$ **c)** $\sqrt{\frac{5}{2}}$ or $\sqrt{2.5}$

Example 4: Your Turn

a) $2\sqrt{10}$ **b)** $6\sqrt{3}$ **c)** $2\sqrt[3]{4}$

Example 5: Your Turn

Look for two methods. Example:

- Compare entire radicals: $2\sqrt{54} = \sqrt{216}$; $\sqrt{192}$; $5\sqrt{10} = \sqrt{250}$.
  From greatest to least: $5\sqrt{10}, 2\sqrt{54}$, and $\sqrt{192}$.
- Use a calculator to determine decimal equivalents and compare:
  $2\sqrt{54} = 14.696938...$; $\sqrt{192} = 13.856406...$; $5\sqrt{10} = 15.811388...$.
  From greatest to least: $5\sqrt{10}, 2\sqrt{54}$, and $\sqrt{192}$.

Example 6: Your Turn

$\sqrt[3]{(360)(5)} = 12.164403...$. The edge length is of the cube is approximately 12.2 cm.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Provide similar questions before having students try the Your Turn. • You may wish to have students work with a partner. • Some students may benefit from reviewing exponents and indexes and how they relate to a power. You might have students work in small groups to create a poster that can be displayed in the classroom and serve as a reference tool. • Remind students that when converting powers to radicals, it is the exponents that change and not the base. • Remind students to use brackets around a radicand that has more than one element.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner.
Example 3 Have students do the Your Turn related to Example 3.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Reinforce that the index becomes the exponent of the coefficient. For example: $4\sqrt{8} = \sqrt{(4^2)(8)}$
Example 4 Have students do the Your Turn related to Example 4.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from an alternative approach to finding factors of the radicand that multiply together where one factor is a perfect square or cube. For example: $\sqrt{27} = \left(\sqrt{9}\right)\left(\sqrt{3}\right)$ $= 3\sqrt{3}$
Example 5 Have students do the Your Turn related to Example 5.	• Provide a similar question before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to estimate their answers first. • Allow students to use technology as appropriate and as needed.
Example 6 Have students do the Your Turn related to Example 6.	• It may be beneficial to review the rules for isolating a variable in a linear equation. Emphasize the use of opposite operations. Ask students to verbalize the opposite of squaring and cubing. • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.

Check Your Understanding

Practise

For #1, students convert powers to equivalent radical form. Except for #1a), students need to simplify using the exponent laws before converting to radical form. Since the emphasis is on the process of expressing powers as radicals, students are not required to evaluate the expressions. However, some students may wish to evaluate the powers with a numerical base.

For #2, encourage students to simplify the final expression.

For #3, check that students use brackets around the exponent in order to ensure that the calculator is performing the proper order of operations.

For #4 and 5, you might have students observe how the index of the radical becomes the exponent for the coefficient in order to bring it into the radicand.

For #6 and 7, students need to use prime factorization in order to convert the larger radicands into mixed radicals.

For #8 and 9, the indexes of the radicals are not all the same. Therefore, students may need to use a calculator to evaluate these numbers.

Students will need to solve #10 using a calculator.

For #11, direct students to the Did You Know? about Pacific halibut to help set the context for the problem.

Apply

Allow students some choice in the questions they need to do. Encourage students to solve the problems using a method of their choice. You might have students compare their method for some problems with that of a classmate who solved the problems in a different way.

For #14, students may mistakenly place the denominator under the radical sign. For part b), refer students to the Did You Know? for an explanation of the term *geosynchronous orbit*. Check that students make the connection that 1 orbit of Earth is equivalent to 1 day (24 h).

For #15, tell students to use the pi button on their calculator to maintain reasonable accuracy.

For #16, a partial solution is shown. Check that students isolate the radical. Then, to undo the cube root, students need to cube both sides of the equation.

For #17, students should make the connection that the expression on the right is equivalent to the square root of the division of the power bythe resistance.

Extend

For #19, you may need to coach students to convert each radical sign into the corresponding exponential form. In part a), the power in the radicand needs to be raised 2 times successively to the exponent $\frac{1}{2}$.

For #20, students can begin by evaluating the expression with different types of numbers: positive and negative integers and rational numbers. Then, encourage students to develop an explanation that would extend to any number in general.

Create Connections

Question #24 is a Unit 2 project question. Refer to the Unit 2 project notes for this question.

For #21a), make **Master 3 Square Dot Paper** available. Have students exchange and compare their solution with that of a classmate.

For #22, have students work individually to describe the relationship between a radical and its equivalent power with a rational exponent and then describe the relationship to a classmate or small group.

For #23, students research the history of algebraic or mathematical symbols. They may find the related Web Link at the end of this section helpful.

Unit Project

The Unit 2 project questions, #13, 18, and 24, provide opportunities for students to apply their understanding of irrational numbers.

For #13, you might extend the question by providing the shorter dimension of a painting and asking students to determine the perimeter or the area.

For #18, direct students to the Web Link and the image of a pine cone showing the Fibonacci sequence. Explain this is an illustration of the spiral patterns of seeds. Have students try to locate two sets of spirals. Going clockwise, there are 8 spirals radiating from the centre and going counterclockwise, there are 13 spirals. Ask how the spirals are related to the Fibonacci sequence.

Have students try to simplify the expression in #18 using technology.

For #24, ensure that students describe a different example of golden geometry than what has already been presented during the chapter. You might direct them to the related Web Link at the end of this section to help them research examples of the golden ratio. You might have students present their visuals in a whole-class activity.

Meeting Student Needs

- Allow students to work in pairs.
- When expressing a radical as a power, some students may find it helpful to remember that the number in the notch goes in the denominator.
- Some students may rely heavily on using technology. Ensure they know the order in which to solve problems using technology.
- For #10, show students some Rubik's Cubes. Consider allowing students to use the actual measurements of the cube to answer the question. Invite students to solve the cube.
- Provide **BLM 4–9 Section 4.4 Extra Practice** to students who would benefit from more practice.

ELL

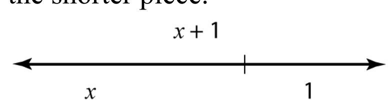
- For #14, clarify that an orbit is the path that an object, such as a satellite, takes as it travels around another object.
- Teach the following terms in context: *skid marks*, *coefficient of friction*, *weaver*, *tapestry*, *telecommunications satellite*, *swing of a pendulum*, *limited-time sale*, *sales discount*, *appliance*, *watts*, *resistance*, *ohms*, and *spiral patterns*.

Enrichment

- For #6, challenge students to express an entire radical such as $\sqrt[3]{\frac{16}{27}}$ as a mixed radical. Tell them to write the numerator and denominator in prime factorization form and simplify each independently.
- For #12, you may wish to invite a police officer to talk to the class about speed and braking distance and the importance of keeping a safe distance between moving vehicles. This may be a timely presentation for students who are learning to drive.
- Challenge students to create a geometric representation of an irrational number of their choice. Students may benefit from some guidance to represent an irrational number geometrically. You might suggest drawing a right triangle with short sides of 1 and 1 and a hypotenuse of $\sqrt{2}$, or a right triangle with short sides of 1 and 2 and a hypotenuse of $\sqrt{5}$. An interesting question would be to create a right triangle with a hypotenuse of $\sqrt{3}$. Alternatively, students might use a diagram that proves the Pythagorean theorem, such as placing three squares along the sides of a right triangle that models $a^2 + b^2 = c^2$, for a triangle with dimensions of 3, 4, and 5 units. Ask them how this model might be used to generate an irrational number for the hypotenuse (e.g., $9 + 4 = 13$).
- Have students research examples of the golden ratio in art and architecture. They might consider Canadian architect Douglas Cardinal and identify golden ratios in his designs.
- Have students explore the connection between the Fibonacci sequence and the golden ratio. Explain that in the Fibonacci sequence, each term is the sum of the two previous terms: $a_n = a_{n-1} + a_{n-2}$. For example, the 12th and 13th terms (144 and 233 respectively) are added to form the 14th term in the sequence (377). Dividing consecutive pairs of numbers in the sequence gives results that get closer and closer to the approximation of the golden ratio, which is $\frac{1+\sqrt{5}}{2} = 1.6180339887\ldots$. Consider $\frac{233}{144} = 1.6180556\ldots$ and $\frac{377}{233} = 1.6180258\ldots$. Challenge students to try using a different pair of consecutive terms and compare their results with those of other students.
- Have students research the Fibonacci sequence in art, architecture, or nature. Allow them to present their findings in a format of their choice.
  - For the Fibonacci sequence in nature, have students access the related Web Link on page 195 in the student resource.
  - For the Fibonacci sequence in art and architecture, have students access the related Web Link at the end of this section.
- For #23, some students may be interested in researching the origin of the term *algebra* and al-Khwarizmi, who is considered by some writers to be the father of algebra. They may find the related Web Link at the end of this section helpful.

Gifted

- Have students prove that the square root of 2 is irrational. Have them find a proof and work through it.
- Have students try the animation illustrating the connection between the Fibonacci sequence and the golden ratio described in the related Web Link at the end of this section. Have them summarize the relationship between the Fibonacci sequence and the golden ratio.
- For #24, some students may use the following idea to describe an example of the golden ratio. Divide a line segment of any length into two pieces so that the ratio of the entire segment to the longer piece equals the ratio of the longer piece to the shorter piece.



The ratios produce $\frac{x+1}{x} = \frac{x}{1}$.
So $x^2 = x + 1$ and $x^2 - x - 1 = 0$.
Then, by the quadratic formula,
$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$ and the positive value is $x = \frac{1+\sqrt{5}}{2}$, which is the golden ratio. Even though students are not familiar with the quadratic formula in grade 10, they could experiment with different lengths of line segments and determine where to divide the segment in order to come as close as possible to the golden ratio.

Common Errors

- Students may struggle with correct calculator sequencing for formulas.

Rx Ensure that students use brackets around fractional exponents. Also, make sure that students use a closing bracket at the end of a radicand.

- Some students may struggle with solving for a variable inside a radicand in a formula.

Rx Coach students through some worked examples that involve isolating a variable inside a radicand.

WWW Web Link

For information about the Fibonacci sequence in art and architecture, go to www.mhrmath10.ca and follow the links.

For an animation of the connection between the Fibonacci sequence and the golden ratio, go to www.mhrmath10.ca and follow the links.

For information about several famous irrational numbers, go to www.mhrmath10.ca and follow the links.

For information about the origin of the term *algebra*, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a) to c), 2b) to d), 3, 4a) to c), 5a) to c), 6a) to c), 7a) to c), 8, 10, 11, and 13. Students who have no problems with these questions can go on to the remaining questions.	• Provide additional coaching with Example 1 to students who need support with #1, with Example 2 for #2, with Example 3 for #4 and 5, with Example 4 for #6 and 7, and with Example 5 for #8. Coach students through correcting their errors before having them try some of the questions that were not assigned. • Provide additional coaching with Example 6 for #10. Have students talk through their thinking and help them correct the errors. Check for understanding before assigning #11 and 13. • For #8, review the meaning of *irrational* and the bar over the number in part a). • For the word problems, have students record the meaning of each variable before beginning to solve.
Unit 2 Project If students complete #13, 18, and 24, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• For #13, you may need to help students recall what they learned in the investigation about the golden ratio. • For #18, you might provide scaffolding to help students evaluate the brackets first before completing the solution on their own. • For #24, encourage students to use the guidelines to help organize their research plan. • You may wish to provide students with **BLM 4–5 Chapter 4 Unit 2 Project**, and have them finalize their answers. • Remind them to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #21.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions and then have them provide individual responses orally or in written form. • Allow students to make revisions before handing in their response.

Chapter 4 Review

4

Mathematics 10, pages 196–198

Suggested Timing

60–90 min

Blackline Masters

BLM 4–6 Section 4.1 Extra Practice
BLM 4–7 Section 4.2 Extra Practice
BLM 4–8 Section 4.3 Extra Practice
BLM 4–9 Section 4.4 Extra Practice

Planning Notes

Have students work individually or in pairs to work through the review questions. Provide opportunities for students who work individually to compare their solution strategies and answers with a classmate.

If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their Foldable, the worked examples, and previously completed questions in the student resource.

Have students make a list of the questions that they are unsure about. They can then use the list to help them prepare for the practice test. For any problematic question, consider having students construct a similar question using different values, solve it, and then exchange questions with a classmate. Afterward, compare the strategies used and the solutions. Have students correct any errors.

Meeting Student Needs

- Encourage students to use their Foldable and to add new notes if they wish.
- For #11, encourage students to use the methods of their choice.
- For #16, refer students to the Did You Know? that explains the term *tsunami*.
- For #24, students should know that drum head means only one of the circular surfaces.
- Consider having students work in small groups to create a game similar to Jeopardy. Have students review the Key Ideas in the chapter, choose five topics, and then develop five questions and answers for each topic. Questions should range from easy to difficult. You may wish to have the “host” or “hostess” check the questions and solutions before playing the game. You can either project the game from a computer or create squares on a whiteboard.
- Students who require more practice on a particular topic may refer to **BLM 4–6 Section 4.1 Extra Practice**, **BLM 4–7 Section 4.2 Extra Practice**, **BLM 4–8 Section 4.3 Extra Practice**, and **BLM 4–9 Section 4.4 Extra Practice**.

ELL

- Teach the following terms in context: *rebounds, computer chips,* and *drum head.*

Enrichment

- Challenge students to develop a formula to represent the response of a bacterial culture to an antibiotic test. Tell students to assume that the antibiotic halves the bacterial count every hour.

Gifted

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 4 Review The Chapter 4 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

4 Chapter 4 Practice Test

***Mathematics 10*, pages 199–201**

Suggested Timing

40–50 min

Blackline Masters

BLM 4–10 Chapter 4 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need a little help with, a lot of help with, or no help with. Have students first complete the questions they know they can do, followed by those they know something about. Finally, have students do their best on the questions that they are struggling with. If the assignment is done in class, consider posting correct solutions after students have had an opportunity to complete the test. Encourage students to compare their solutions.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1, 2, 4–7, and 9–13.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	4.1	Example 1	✓ determine the square root of a perfect square and explain the process ✓ determine the cube root of a perfect cube and explain the process
#2	4.2	Link the Ideas Example 2 Key Ideas	✓ convert a power with a negative exponent to an equivalent power with a positive exponent ✓ apply the exponent laws to expressions using rational numbers or variables as bases and integers as exponents
#3, 4	4.2	Examples 3, 4	✓ solve problems that involve powers with integral exponents
#5	4.4	Example 3	✓ convert between mixed radicals and entire radicals
#6	4.4	Example 2 Key Ideas	✓ convert between powers with rational exponents and radicals
#7	4.4	Example 6	✓ solve problems involving radicals
#8	4.1	Example 2	✓ determine the cube root of a perfect cube and explain the process ✓ solve problems involving square roots or cube roots
#9	4.4	Link the Ideas Key Ideas	✓ represent, identify, and simplify irrational numbers
#10	4.1	Example 2	✓ solve problems involving square roots or cube roots
#11	4.3	Link the Ideas Example 1 Key Ideas	✓ apply the exponent laws to expressions using rational numbers or variables as bases and rational exponents
#12	4.3	Example 3	✓ solve problems that involve powers with rational exponents
#13	4.4	Example 6	✓ solve problems involving radicals
#14	4.3	Investigate Example 3	✓ solve problems that involve powers with rational exponents
#15	4.4	Example 5	✓ represent, identify, and simplify irrational numbers

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 4 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Have students use their responses on the practice test and work they completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 4 Test After students complete the practice test, you may wish to **use BLM 4–10 Chapter 4 Test** as a summative assessment.	• Consider allowing students to use their Foldable.

Polynomials

5

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:
- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
5.1	✓ multiply polynomials
	✓ explain how multiplication of binomials is related to area and to the multiplication of two-digit numbers
5.2	✓ determine prime factors, greatest common factors, and least common multiples of whole numbers
	✓ write polynomials in factored form
	✓ apply their understanding of factors and multiples to solve problems
5.3	✓ develop strategies for factoring trinomials
	✓ explain the relationship between multiplication and factoring
5.4	✓ factor the difference of squares
	✓ factor perfect square trinomials

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 5–1 Chapter 5 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 202 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about polynomials and the skills of multiplying and factoring. You might provide the following prompts: • Have you ever seen or used polynomials before in your life? Where? • Have you ever taken something apart and put it back together again? How does that task relate to multiplying and factoring polynomials? • In what instances in your life have you performed a task over and over and found patterns in the process? • Can you describe anything from your life that resembles the operations of multiplying? factoring?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 5–2 Chapter 5 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 5 Foldable As students work on each section in Chapter 5, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter. • Encourage students to write examples of their own into their Foldable. They should have an example for each method that is covered in the chapter.
Assessment *for* Learning	
BLM 5–3 Chapter 5 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 5 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 45–60 min (TR page 159)	Students should be familiar with • Internet searches • word definitions • career searches	• one sheet of 11 × 17 paper • three sheets of 8.5 × 11 paper • scissors	BLM 5–1 Chapter 5 Self-Assessment BLM 5–2 Chapter 5 Prerequisite Skills BLM 5–4 Chapter 5 Unit 2 Project BLM U2–2 Unit 2 Project Checklist		TR page 158 Chapter 5 Foldable, TR page 158	TR page 158	
5.1 Multiplying Binomials • 100–120 min (TR page 161)	Students should be familiar with • algebra tiles • multiplication of two monomials • collection of like monomials using the operations of addition and subtraction	• algebra tiles	Master 5 Algebra Tiles (Positive Tiles) Master 6 Algebra Tiles (Negative Tiles) BLM 5–3 Chapter 5 Warm-Up BLM 5–4 Chapter 5 Unit 2 Project BLM 5–5 Section 5.1 Extra Practice	**Essential:** #1a), b), d), 2, 3a)–d), 4a)–c), 5, 6a), 8–10, 12, 18 **Typical:** #1b), e), f), 3c), e), f), 4b)–d), 5, 6b), c), 7–9, 11, 12, 14, 18 **Extension/Enrichment:** #1f), 3f), 4e), f), 6d), e), f), 8, 9, 17, 19	TR pages 164, 169 Chapter 5 Foldable, TR page 158	TR pages 166, 167, 169	
5.2 Common Factors • 100–150 min (TR page 170)	Students should be familiar with • factoring whole numbers • finding common factors and common multiples of whole numbers • dividing monomials by monomials • exponents • distributive property • problem solving strategies		BLM 5–3 Chapter 5 Warm-Up BLM 5–6 Section 5.2 Extra Practice	**Essential:** #1a), c), 2a), c), e), 3a), c), e), 4a), c), e), 5a), b), e), 6a), c), e), 7a), c), d), 8–10, 12, 14, 19 **Typical:** #1–3, 4c)–e), 5c–e), 6c)–e), 7, 9–11, 13, 15, 18 **Extension/Enrichment:** #1, 6, 7, 9, 12, 14, 16–19	TR pages 172, 177 Chapter 5 Foldable, TR page 158	TR pages 174, 175, 177	
5.3 Factoring Trinomials • 120–180 min (TR page 178)	Students should be familiar with • algebra tiles • area of rectangles • distributive property • grouping algebraic terms	• algebra tiles	Master 5 Algebra Tiles (Positive Tiles) Master 6 Algebra Tiles (Negative Tiles) BLM 5–3 Chapter 5 Warm-Up BLM 5–4 Chapter 5 Unit 2 Project BLM 5–7 Section 5.3 Extra Practice	**Essential:** #1a), b), 2a), b), 3a), c), 4a), c), f), 5a), b), d), f), 6a), d), f), 7a), h), 8a), 9a), 10a), 13, 14, 21, 22 **Typical:** #1a)–c), 2a–c), 3a)–c), 4b), d), e), f), 5c), e), f), 6b), c), e), 7b), c), d), f), j), 10, 11b), 13–15, 21, 22 **Extension/Enrichment:** #5f), 7e), h), 10–12, 14, 16–22	TR pages 180, 186 Chapter 5 Foldable, TR page 158	TR pages 184, 186	
5.4 Factoring Special Trinomials • 100–120 min (TR page 187)	Students should be familiar with • using grids to find area • algebra tiles • multiplication of binomials • square roots • factoring using common factors • factoring trinomials • recognizing patterns	• centimetre grid paper • scissors	Master 5 Algebra Tiles (Positive Tiles) Master 6 Algebra Tiles (Negative Tiles) BLM 5–3 Chapter 5 Warm-Up BLM 5–4 Chapter 5 Unit 2 Project BLM 5–8 Section 5.4 Extra Practice	**Essential:** #1–3, 5, 6, 7a) –c), 8a), b), 9, 10, 12, 13, 16, 20, 24, 25 **Typical:** #1–7, 8a), b), 9–13, 16, 20, 22, 24, 25 **Extension/Enrichment:** #8, 10, 12, 14, 15, 17–19, 21, 23, 26	TR pages 190, 197 Chapter 5 Foldable, TR page 158	TR pages 193, 196, 197	
Chapter 5 Review • 60–90 min (TR page 198)		• algebra tiles • rulers	Master 5 Algebra Tiles (Positive Tiles) Master 6 Algebra Tiles (Negative Tiles) BLM 5–5 Section 5.1 Extra Practice BLM 5–6 Section 5.2 Extra Practice BLM 5–7 Section 5.3 Extra Practice BLM 5–8 Section 5.4 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 5 Foldable, TR page 158	TR page 199	
Chapter 5 Practice Test • 45–60 min (TR page 200)		• algebra tiles • rulers	Master 5 Algebra Tiles (Positive Tiles) Master 6 Algebra Tiles (Negative Tiles) BLM 5–9 Chapter 5 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1, 3–10, 12, and 13.	TR page 201		TR page 201 BLM 5–9 Chapter 5 Test
Unit 2 Project • 60–90 min (TR page 202)		• coloured pencils and markers, coloured paper, scissors, glue, and other materials for artwork	Master 1 Project Rubric BLM U2–3 Unit 2 Project Final Report				TR page 203 Master 1 Project Rubric
Unit 2 Review and Test • 60–90 min (TR page 204)		• algebra tiles • rulers	BLM 5–10 Chapter 5 BLM Answers	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapters 3, 4, and 5 Foldables	TR page 204	TR page 204

Polynomials

Opener

Mathematics 10, pages 202–203

Suggested Timing

45–60 min

Materials

- one sheet of 11× 17 paper
- three sheets of 8.5 × 11 paper
- scissors

Blackline Masters

BLM 5–2 Chapter 5 Prerequisite Skills
BLM 5–4 Chapter 5 Unit 2 Project
BLM U2–2 Unit 2 Project Checklist

Key Terms

binomial
trinomial
least common multiple
difference of squares
polynomial
distributive property
greatest common factor

What's Ahead

This chapter introduces students to multiplying and factoring polynomials. They will use algebra tiles to examine patterns when multiplying polynomials and use these patterns to better understand applying the distributive property to multiplying and factoring polynomials. Students will learn to recognize special products and use this knowledge to factor differences of squares and perfect square trinomials.

By the end of the chapter, students will be able to

- multiply polynomials involving
  - monomials and polynomials
  - binomials and binomials
  - binomials and polynomials
- factor polynomials involving
  - common factors
  - trinomials of the form $ax^2 + bx + c$, $a = 1$
  - trinomials of the form $ax^2 + bx + c$, $a \neq 1$
  - differences of squares
  - perfect square trinomials
- recognize and understand the importance of patterns in mathematics and use these patterns to
  - develop understanding of concepts
  - develop skills and processes
  - solve problems in everyday life

Planning Notes

Unit Project

You might take the opportunity to discuss the Unit 2 project described in the Unit 2 opener on TR page 107. Throughout the chapter, there are individual questions for the unit project. These questions are not mandatory but are recommended because they provide some of the work needed for the final report for the Unit 2 project assignment.

The Unit 2 project is integrated throughout the chapter. You will find questions related to the project in the Check Your Understanding in sections 5.1, 5.3, and 5.4.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 203 and how it might be used to summarize Chapter 5. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

Ensure that students understand the difference between multiplying and factoring, and point out that the labels for these two processes appear on opposite sides of the Foldable. On the left side of the Foldable, students could focus on methods of multiplying. On the right side, students could focus on the factoring of polynomials. The centre of the Foldable provides ample space for students to model concretely the multiplication and factoring of polynomials.

Suggest that students draw and colour an example of both multiplication and factoring. Encourage students to write in their own words what each term and process is and to generate examples different from those in the student resource.

The Foldable accommodates alternative methods of multiplying. If you introduce alternative methods, use the same question throughout so that students focus only on the difference in methodology. For example, for the product of $(2x + 1)(x + 2)$, have students complete the product using algebra-tile models in the centre of their Foldable, using the distribution method, and using one or more alternative methods.

As students progress through the chapter, provide time for them to keep track of what they need to work on. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 5–2 Chapter 5 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Post the student learning outcomes for the entire chapter for students to refer to.
- Hand out to students **BLM U2–2 Unit 2 Project Checklist**, which provides a list of all of the requirements for the Unit 2 project.
- **BLM 5–4 Chapter 5 Unit 5 Project** includes all of the unit project questions for this chapter. These questions provide a beginning for the Unit 2 project.
- Some students may benefit from completing all unit project questions.
- Read the chapter opener as a class and then have students restate the Chinese quotation, using a reference that is more familiar to them. For example, students in the North might say, "three good furs, two mediocre furs, and one poor fur are sold for $300."
- Have students create posters illustrating the Key Terms for the entire chapter. Have students research the definition for each Key Term and create a representation for each definition.
- With the class, read the information about the Recreation Facilities Manager and ask students questions like the following:
  – Who does this job in our community?
  – What is the job title in our community?
  – Is this a job you might like to have?
- Have a class discussion about how the career of Recreation Facilities Manager connects to the chapter.

Enrichment

- To get students thinking in terms of polynomials, challenge them to find a polynomial that describes the process of going from wholesale cost to final cost including taxes. Ask them how this polynomial might be used to develop a spreadsheet that a retailer might use.

Gifted

- After students have read the information about the Recreation Facilities Manager, ask them to think about formulas that might apply to the creation of recreational projects such as pools, ice rinks, and play areas. These formulas might include those for area, volume, and length. Ask students to speculate how an engineer might use these formulas during the design process. Encourage them also to explore the engineering of winter sport facilities such as bobsled tracks, ski jumps, and half pipes.

Career Connection

Invite students who are interested in a career in recreation to research the training and qualifications, employment opportunities, and career longevity of recreation workers. Have them explore newspapers, the Internet, and local town and city employment offices to see what recreation career opportunities exist in their local area. They should also research what other career choices would involve the same set of skills of scheduling facilities and staff, maintaining budget responsibilities, and improving the lifestyle of the community. How would strong math skills help in scheduling staff, maintaining the facility, controlling the budget, and scheduling community use of the recreation facility?

Web Link

For definitions of some of the terms in this chapter, go to www.mhrmath10.ca and follow the links.

Multiplying Polynomials

5.1

***Mathematics 10,* pages 204–213**

Suggested Timing

100–120 min

Materials

- algebra tiles

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–5 Section 5.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), d), 2, 3a)–d), 4a)–c), 5, 6a), 8–10, 12, 18
Typical	#1b), e), f), 3c), e), f), 4b)–d), 5, 6b), c), 7–9, 11, 12, 14, 18
Extension/Enrichment	#1f), 3f), 4e), f), 6d), e), f), 8, 9, 17, 19

Unit Project Note that #8 and 9 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

In this section, it is important that students have enough practice with models to make a connection between concrete models (such as algebra tiles) and the algebra they will now use to represent real ideas. They should use models long enough to gain an understanding of the patterns and rules for manipulating algebraic expressions. They should then make the transition to using algebraic expressions as soon as they are comfortable with these patterns and rules. Every now and then, have them revisit models to help with the connection between the concrete and the abstract.

Investigate Multiplying Polynomials

In this Investigate, students use their knowledge of rectangular area and patterns to understand the distributive property and to develop a technique for multiplying polynomials. It is important to give students an opportunity to explore with the tiles. It may be useful to have students work in small groups of two or three, depending on how many sets of algebra tiles you have available. If algebra tiles are not available, you may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)**.

Use leading questions to help students define what each tile should represent. For example, you might ask the following questions:
- Which tile do you think best represents the number 1?
- Which tiles represent a product of 1 and a number?
- Which tiles best demonstrate the product of a number and itself?
- How do you represent a negative value using the tiles?
- What happens when you add two tiles of the same size, one positive and one negative?
- Can you use algebra tiles to represent numbers? Explain.
- Which tile would you use to represent 1? 10? 100?

Now, allow students time to explore #1 to 3 of the Investigate. Remind them that their solution should form a rectangle. The following questions are some prompts you might use:
- What number does the large box represent?
- What product did you use to get each of the boxes?
- What value does each of the small boxes represent?
- What is the sum of all of the boxes?
- How does this compare with your original product?

Have students complete #4 of the investigation. They should use the remaining tiles to form a rectangle. Students need to be aware of the patterns that they see after completing a few questions using the algebra tiles. These patterns should lead them to understand the distributive property.

Have students complete #5. Then, enlarge the groups to up to five per group, and have students discuss and answer #6 in their groups and then share with the entire class.

Use prompts to lead students to a better understanding of the patterns of algebra tiles and the distributive property. For each of parts a) to c) of #5, ask the following questions:

- What product did you use to produce the large squares? Record that product.
- What product did you use to produce each of the small squares? Record those products.
- What is the sum of these products?
- How does this compare with the answer you obtained using algebra tiles?
- With the algebra tiles, which term of the answer represents the product used to get the large squares?
- Which term of the answer represents the product and sum of the rectangles?
- Which term of the answer represents the product used to get the small squares?
- Are these patterns useful for other products? Explain.

Have students create their own example of a product of two binomials and see if the pattern works with their example. Tell them to have a partner check it.

Meeting Student Needs

- Explain to students that the painting in the section opener is by Piet Mondrian. Ask students to identify ways in which elements of the painting are used in the design of the cups and building.
- Suggest that students watch for and record examples of designs in the style of Mondrian's paintings. While shopping or looking through magazines, they may find examples on clothing, furniture, art, wallpaper, etc. You might suggest that they cut out or sketch the examples, or take pictures with their cell phones or cameras. Then, you can have these pictures on display while the class works on the chapter.
- Have students research geometric designs in a culture that interests them. For example, see the Web Link that follows for sites that show how Aboriginal peoples have used geometric constructions. As an extension, have students find out about the mathematical/knowledge system that might have been used in the design of Navajo homes.
- Relate mathematical words to non-mathematical words so that students can make the connection. For example, to assist them with the term *binomial*, have them brainstorm words such as *bicycle*, *bilingual*, and *biweekly*, and discuss how *bi* means two. Similarly, to assist them with the term *trinomial*, have them brainstorm words such as *tricycle*, *tripod*, *triangle*, and *trimester*, and discuss how *tri* means three. These connections will help students to identify the new terms and their meaning more easily.

ELL

- Assist students with the terms *sum*, *product*, and *factor* before beginning the section by having a class brainstorming session to define these words. Write students' ideas on the board. Students can then refer to the brainstormed definitions when completing the Investigate.
- Students may be unfamiliar with the term *dimensions*. Explain to students that it refers to the measurements in a diagram, such as the length and width of a rectangle.

Enrichment

- As students work on the first part of the Investigate, ask them to expand the modelling of multiplication to include hundreds places; for example, (115)(100). Ask students how having three different place values affects the model. Also, ask if the model has an algebraic equivalent.

Gifted

- Encourage students to investigate polynomials that could be used to create designs similar to Mondrian's work. Have them look for patterns in polynomials that tend to produce designs that are pleasing to the eye. Ask them if they think it is possible to create great art mathematically.

Common Errors

- Some students may have difficulty determining the product if they cannot make the correct configuration with the algebra tiles.

R_x Work your way around the class giving help and asking leading questions:
- What edges of the tiles correspond?
- What shape will occur if you have a product of $(x)(1)$? $(x)(x)$? $(1)(1)$?
- What is the sum of all of the tiles in the solution?
- What do you think this sum represents?

Web Link

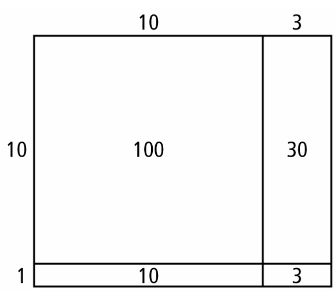
Students can research Aboriginal geometric designs by going to www.mhrmath10.ca and following the links.

Answers

Investigate Multiplying Binomials

1. 143

2. a)

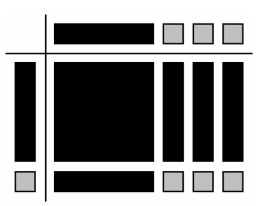


b) Example: The factor 10 + 3 is represented by the length of the rectangle, with a dividing line indicating the 10 and the 3. The factor 10 + 1 is represented by the width of the rectangle, with a dividing line indicating the 10 and the 1.

c) 143

3. Example: Differences: In #1, there are two different products, (1)(13) = 13 and (10)(13) = 130. In #2, there are four areas: (10)(10) = 100, (3)(10) = 30, (1)(10) = 10, and (1)(3) = 3. Similarities: In #1, we determine a sum: 13 + 130. In #2, we also determine a sum: 100 + 30 + 10 + 3. The answer to both is 143.

4. a)

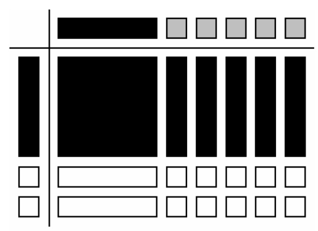


b) Example: The factor $x + 3$ is shown at the top of the diagram and represented by a rectangular x-tile and three small square 1-tiles. The factor $x + 1$ is shown at the left side of the diagram and represented by a rectangular x-tile and one small square 1-tile.

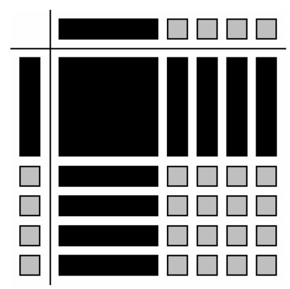
c) $x^2 + 4x + 3$

d) Example: Similarities: The models are similar in that the factors are represented by the dimensions of the rectangle and the product is represented by the area. The large square in #2 represents (10)(10), just as the large square in #3 represents $(x)(x)$.
Differences: In #2, (1)(3) and (1)(10) are each represented by just one rectangle. In #3, $(x)(1)$ is represented by one rectangle but $(3)(x)$ is represented by three rectangles, and (3)(1) is represented by three squares.

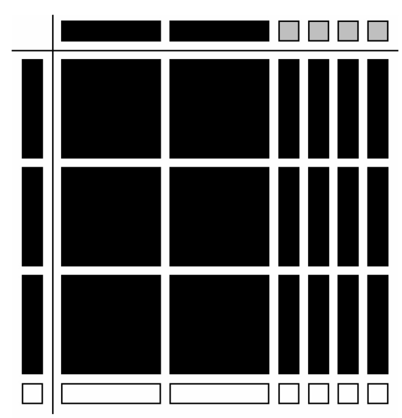
5. a)



b)



c)



Answers

Investigate Multiplying Binomials

6. a) Example: The factors are represented by the dimensions of the rectangle, and the product is represented by the area of the rectangle. An x^2-tile in the area has two corresponding x-tiles in the dimensions. An x-tile in the area has a corresponding x-tile and 1-tile in the dimensions. A 1-tile in the area has two corresponding 1-tiles in the dimensions.

b) Example: The product of the first terms of the factors produces the first term of the answer; this is represented by the sum of the x^2-tiles. The product of the last terms of the factors produces the last term of the answer; this is represented by the sum of the 1-tiles. The sum of the products of the first term of one factor and the second term of the other factor produces the middle term of the answer; this is represented by the sum of the x^2-tiles.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students work in pairs to generate possible patterns that they see. • If students are having difficulty getting started, direct them to look at the initial terms in the factors and then the first terms in the product for each question. Suggest a similar approach for the last terms in the product. • It would likely benefit the entire class to have a discussion about the patterns and how students would multiply the binomials without tiles. Their response here will help guide you to possible areas that require extra explanation as you proceed through the examples.

Link the Ideas

Use the product of 42 and 26 to demonstrate how the distributive property can be used in the multiplication of numbers.

A useful technique to demonstrate the distributive property is to use lines or arrows to indicate what expressions are being distributed.

$$(40 + 2)(20 + 6)$$

Discuss the definition of the distributive property as outlined in the margin of the student resource.

Example 1

These questions demonstrate the basic multiplication skills required of students at this level. Have students attempt part a), using the algebra tiles and the distributive property.

Make sure that they are comfortable and competent with both methods before moving on.

Discuss the addition of positive and negative tiles. In this example, there is a sum of $-5x$ instead of $7x$, which occurs because one of the x-tiles is negative and it makes a zero pair with one of the positive x-tiles. Ensure students understand that white tiles are negative. You may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

After multiplying using algebra tiles, share with students how this same operation can be completed using the distributive property as outlined in the student resource. Ask students to determine which expression will be distributed over the other expression. You should demonstrate that the order is not important and that the same solution will occur when you multiply $(2x + 1)(x - 3)$ as when you multiply $(x - 3)(2x + 1)$.

After completing part a) of Example 1 together, you may wish to have students do Your Turn part a) using both algebra tiles and the distributive property.

Part b) of Example 1 involves an algebraic expression with two variables. Students may need some help using algebra tiles to express the second variable with tiles. Prompt them with leading questions like the following:

- Using algebra tiles, how do you express $2y$? $-2y$? $-4y$?
- How will you set up the dimensions of a rectangle to solve this product?

After going over the solution, have students attempt the same question using the distributive property. You will begin to see that some students are ready to move away from algebra tiles and use only the distributive property. Direct students to use substitution as a technique to check their answers. Have students complete Your Turn part b).

Example 2

This example extends the binomial multiplication skills to a product of a binomial and a trinomial. Once students are comfortable with binomial products and the distributive property, distributing the binomial over the trinomial should be a straightforward step. You might want to remind students of how to distribute a monomial over a trinomial first. Have them multiply each of the following: $x(x^2 + 3x – 5)$, then $3x(x^2 + 3x – 5)$, and finally $–3x(x^2 + 3x – 5)$. Beginning this way may help students to make the transition to the next level.

After completing the example, have students do Your Turn part a), share their solution, and then do Your Turn part b).

Example 3

This example involves simplifying a more complex product of polynomials. For part a), remind students that they can multiply $3(2x + 4)(6x – 2)$ in any order. Students should decide on the order that they are most comfortable with when multiplying three or more factors. Once the multiplying is done, they should then collect like terms.

Have students complete Your Turn part a), share their solution, and then complete part b).

Example 4

This example demonstrates how to apply the skill of multiplying polynomials to concrete problems. Students should get in the habit of communicating exactly what the variable represents at the beginning of the problem before using an algebraic expression to represent a side or part of a problem. Use prompts to set up this understanding:

- What information are you given in the problem?
- What dimension is missing?
- How can you express the side of the painting?
- How do you find the area of a square?

Once students answer part a), use leading questions to direct students in solving part b):

- What are the dimensions of the red square if the total area of the square is 3600 cm^2?
- What operation can we use to find the length of a side?

Next, have students complete the Your Turn question and check their answer.

Key Ideas

Have students summarize the relationship between multiplying polynomials and the application of the distributive property. Encourage students to use arrows to indicate how each term of the first factor will distribute multiplication over each term of the second factor. Ask students to rewrite in their notebook the product listed in the Key Ideas and draw arrows to represent the distribution of $3x$ over the factor $4x + 5$ and $–2$ over the factor $4x + 5$. Have students do the same for the product $(c – 3)(4c^2 – c + 6)$. Ask them to indicate what operation the arrows signify.

Meeting Student Needs

- Have students work on the Investigate in pairs or groups of three. You may need to provide extra examples to give students more practice manipulating the algebra tiles to determine specific products.
- In Example 3, students learn to *simplify*. Students often struggle with this term because a completely different set of steps might be required with a different question. Take the time to discuss why the word *simplify* is used here.
- You might want to have students look at the examples and discuss whether the questions could be simplified in fewer steps. In particular, address when it is okay to drop brackets and when it is necessary to keep the brackets.
- For Example 4, you may wish to have students create a problem based on a story or legend relevant to their community. For example, they might write an equation using animals or characters from their favourite legend(s). You may wish to invite community Elders or Knowledge Keepers to tell appropriate stories for this purpose.
- You may wish to assist students in order to reactivate their skills in simplifying polynomials. Discuss that like terms contain identical variables but may have different numerical coefficients.
- You may need to remind students to combine like terms when simplifying the product.
- It may be helpful to highlight the terms to reinforce the method of multiplication. For example, highlight +1 in one colour:
  $(x – 3)(2x + 1) = x(2x + 1) – 3(2x + 1)$
  Similarly, x, –3, and $2x$ would each be highlighted in different colours. Students could even put the terms on different-coloured construction paper and manipulate them. This visual activity may help students to better understand the method.

Common Errors

- When working with more than one variable, some students may become confused about the sum of expressions like $-4xy$ and $-2yx$.

R_x Remind students to express variables in alphabetical order.

- When working with more complicated products of binomials and trinomials, some students may be confused about which factor to distribute over the other.

R_x Have them try both ways to see if the answer is different. This will empower them to solve the question their way. Given a choice, they can decide which method they are most comfortable with.

- Some students may find it challenging to collect like terms. For example, they may confuse cubed and squared variables and may add the coefficients without realizing that the exponents of the variables are different.

R_x Remind students of the definition of like terms and suggest that they consider putting a line through terms that have already been collected.

- Some students may find problem solving challenging.

R_x Discuss with students techniques for approaching a problem. For example: First, students should make sure they understand the problem.
  - Read once for general understanding.
  - Read a second time to determine what needs to be determined.
  - Read again to gather pertinent information.

  Then, they should put together a plan. Students should ask themselves the following questions:
  - Do I need a diagram to help?
  - Do I know what I am trying to find?
  - What will I use as a variable and what will it represent?
  - Can I use an algebraic expression to help me?
  - What operation will I use?
  - Can I solve the problem now?

Answers

Example 1: Your Turn

a) $x^2 - 5x - 3x + 15 = x^2 - 8x + 15$

b) $10m^2 - 30m - 2m + 6 = 10m^2 - 32m + 6$

Example 2: Your Turn

a) $3r^3 - 4r^2 - 38r + 24$ **b)** $10x^3 - 36x^2 + 78x - 36$

Example 3: Your Turn

a) $13x^2 + 25x - 26$ **b)** $-8x^2 + 12x + 31$

Example 4: Your Turn

$x^2 - 12x + 32$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may need to assist students having difficulty with the algebra-tile model by going over the significance of the colours of the tiles. Help students to understand that where different colours meet in the rectangle, you have a negative value. Where the same colours meet in the rectangle, you have a positive. • Remind students that a subtraction sign in front of a number also indicates that the number is negative. This is particularly important when students solve using the distribution method. • Encourage students to try both methods, or use one method and check with another. Have students who use different methods work in pairs. Have students talk through why they chose the method they did. • If students are having difficulty with the methods in Example 1, you could show them FOIL, the box method, or vertical multiplication. One of these approaches may be easier for them to understand.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students note that both binomials have a subtraction sign in them. • Alternative methods that you might show students are the box method or vertical multiplication.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may be overwhelmed by the length of the expansion. They may benefit from dividing up the question into two parts by doing $(x + 1)(5x + 3)$ first and then $+ 3(2x + 4)(6x – 2)$, then combining like terms. • Take extra time with the class to discuss and point out the importance of brackets and how multiple brackets are used in the questions. • Have students work in pairs. Once they have done the questions, have them talk through the thinking they went through as they worked on the solutions.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label their diagrams and compare them with a partner. • If students require assistance to complete the question, before moving on, have them determine the same question with new dimensions.

Check Your Understanding

Practise

For #1, students have an opportunity to practise basic binomial multiplication using algebra tiles. You may wish to have students work in pairs. Make sure that they set up the product as the dimensions of a rectangle and that they create a rectangle as the solution. Work your way around the room, making sure that the groups of students are confident in their work. Have students discuss the patterns that they see using the algebra tiles.

For #2, students must represent a given set of algebra tiles with an algebraic expression. They should be able to determine both what the product is and what the dimensions are from the given diagram.

For #3 and 4, students make the transition from multiplying polynomials using tiles to using the distributive property. Most students should be able to use the distributive property but some may have difficulty with the product of integers or the addition of integers. Provide these students with some integer remediation.

By completing #5, students will have an opportunity to see the patterns of binomial multiplication. They should start to recognize that the first term in the answer is the product of the first terms of the binomials and that the last term in the answer is the product of the last terms in the binomials.

For #6, you may wish to suggest that students use square brackets to separate the products into simpler expressions. They can then complete the product within each square bracket before collecting like terms.

Apply

For #10, encourage students to sketch a diagram with lines drawn to indicate what sections are being cut off of the picture. You may wish to inform students that the picture shows an historical elevator in McNabb, Alberta.

For #11, make sure students recognize that the area of a circle requires the radius and the question gives the diameter of the circle.

For #12, you might suggest that students complete the question themselves and then compare their answer to Bryan’s answer.

Students will find #13 and 14 to be similar to #10. Again, it might be helpful to have students sketch a diagram to get a visual image of what is going on. Make sure students set a variable for the unknown dimension and then apply polynomial multiplication to simplify the area of the new object.

Question #15 provides another real-world application. Given the diagram, all students who have tried at least one of #8, 13, or 14 should be able to complete it.

Though they are similar questions, students may find #16 a little more complicated than #12; develop an appropriate solution. It is an effective question for summarizing the ideas of finding errors and checking your answer.

Extend

The Extend question is targeted at students who fully understand all of the concepts in the section. You may wish to have students discuss it in small groups before completing it.

Create Connections

All students should be able to complete part a) of #18. For part b), some students may need coaching to make the connection between representing any number by *n* and writing algebraic expressions for the next three consecutive numbers. Ask leading questions:

- What number did you start with in part a)?
- What is the second number you chose? Why?
- How much larger was the second number than the first number?
- How much larger should the second number in part b) be than *n*?
- How can you represent this new second number using *n* and the difference between your first two numbers?
- How much larger was the third number you used in part a) than the first number?
- How can you represent this new third number using *n* and the difference between the first and third numbers?
- How much larger was the fourth number you used in part a) than the first number?
- How can you represent this new fourth number using *n* and the difference between the first and fourth numbers?

Part c) of #18 will require students to complete two products using their new expressions n, $n + 1$, $n + 2$, and $n + 3$. They could be coached to develop the connection between the product of the first and last numbers and the product of the middle two numbers:

- How did you represent the first number in part b)?
- How did you represent the last number in part b)?
- Write down and multiply these two expressions. What product did you get?
- How did you represent the second number using *n*?
- How did you represent the third number using *n*?
- What is the product of these two values?
- How does the product of the first and last values compare with the product of the middle two values?

For #19, have students represent any multiple of 10, using *t* as a replacement for 10. Have them use a coefficient and *t* to represent numbers like 60, 70, and 90. Then, using *t*, a coefficient, and a sum, they can represent numbers like 63, 72, and 95. When they are comfortable with using algebraic expressions to represent other numbers, have them tell you how they will represent the numbers 45 and 34 using t, a coefficient, and a sum. Then, have them write the product of the new factors to represent the multiplication algebraically. Ask them if they can use the distributive property to multiply these factors. Have them draw lines or arrows to show which terms are being multiplied.

Unit Project

The Unit 2 project questions, #8 and 9, provide an opportunity for students to solve problems involving combining like polynomial terms as well as to generate their own polynomials to model an area formula.

Students will approach #8 in a variety of ways. Have students clarify that they understand the meaning of each polynomial and how many terms each contains. If students are having difficulty generating their own polynomial, have them work with a partner before developing an individual response.

For #9, students need to demonstrate combining like polynomial terms, arrange them in an artistic design, and write the algebraic equation. Encourage students to check that each other's algebraic equation correctly summarizes the addition or subtraction.

You might have students use **BLM 5–4 Chapter 5 Unit 2 Project**.

Meeting Student Needs

- Provide **BLM 5–5 Section 5.1 Extra Practice** to students who would benefit from more practice.
- Allow students to continue to use algebra tiles as long as they need them.
- In connection with #7, you may wish to have students research Aboriginal art online. See the Web Link that follows in this resource.

ELL

- For #11, clarify that the word *inset* is another word for within or inside of.

Gifted

- As soon as students begin to see the connection between the middle term of the answer and the sum of the products of the remaining terms of the binomials, challenge these students to look for these patterns and to make generalizations about them.

Common Errors

- For #2, some students may not understand what each tile represents.

Rx Assist students by asking leading questions to elicit this information from them.

- For #6, some students may have difficulty with the subtraction of two products.

Rx Remind students to multiply by –1 each term in the polynomial that follows the subtraction sign.

- For #11, some students may have difficulty finding the product of π and the square of $3x + 2$.

Rx It may help students to write $(3x + 2)^2$ as the product $(3x + 2)(3x + 2)$ first.

Web Link

To view prints by Inuit artists, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a), b), d), 2, 3a)–d), 4a)–c), 5, 6a), 8–10, and 12. Students who have no problems with these questions can go on to the remaining questions.	• Students may need help recalling the values of the tiles and how to model the question. Remind them that the product of a positive and a negative will result in a white tile and the same signs will result in a shaded tile. • If students are having difficulty starting #3, have them verbalize how they divide the first binomial into two parts. Ensure they take the operational sign with the second term. • If students find #4 challenging, review the distribution of a monomial over a polynomial, and have students draw lines or arrows from the first factor (monomial) to each term in the second factor (polynomial). • Encourage students to use one of the methods that they feel most comfortable with to solve #5. It could be a method other than the distributive property or algebra tiles. Point out to students that answers are generally written in descending order in terms of the exponent of the variable. • Asking students to explain the order of operations may assist some learners to begin #6. • Encourage all students to draw and label a diagram for #10. Encourage visual learners to verify the multiplication with tiles. • Ask students who do not see an error in #12 to explain like terms. Refer them back to the solved steps to check whether like terms were used consistently.
Unit 2 Project If students complete #8 and 9, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 5–4 Chapter 5 Unit 2 Project**, and have them finalize their answers.
Assessment *as* Learning	
Create Connections Have all students complete #18.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • Students may require clarification and an example of *consecutive numbers*. You could have students generate sets of consecutive numbers to clarify understanding. • Have students compare their conclusions regarding the pattern before they go on to part b). • If students are unable to see a pattern, complete a sample problem using the numbers 1, 2, 3, and 4. Determine the pattern and an algebraic expression for the multiplication. Once students understand the process, ensure they select a different set of four numbers.

5.2 Common Factors

Mathematics 10, pages 214–223

Suggested Timing

100–150 min

Blackline Masters

BLM 5–3 Chapter 5 Warm-Up
BLM 5–6 Section 5.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:

- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), c), 2a), c), e), 3a), c), e), 4a), c), e), 5a), b), e), 6a), c), e), 7a), c), d), 8–10, 12, 14, 19
Typical	#1–3, 4c)–e), 5c)–e), 6c)–e), 7, 9–11, 13, 15, 18
Extension/Enrichment	#1, 6, 7, 9, 12, 14, 16–19

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter is directed toward developing the skill of factoring using common factors. Students are asked to use previous skills of finding factors of given values, writing numbers as a product of prime factors, and determining common factors of numbers. They also find the greatest common factor and the least common multiple for given numbers. These skills will be extended to writing an algebraic expression in factored form using the greatest common factor.

Have students discuss the sets of formulas in the opener and answer the listed questions. This introduction should reinforce at least one reason to express polynomials in factored form.

Investigate Common Factors

The purpose of this Investigate is to help students discover how to express algebraic expressions as a sum of products with one of the factors being the greatest common factor (GCF). This is a first step in factoring polynomials using common factors.

Students may need assistance with their skills in finding the greatest common factor and least common multiple (LCM) of whole numbers. Once confident, students will be ready to use these techniques to find the greatest common factor for algebraic expressions. Avoid giving a direct solution to any of the Investigate questions. Instead, ask leading questions that involve students and allow them ownership of the concept, idea, skill, or process. Then, they will have a connection to it, and it will become part of their learning base. Encourage students to share with the class their methods for finding GCF and LCM. This will also help students to take ownership of their knowledge and to attain the new skills.

In #1, students determine the prime factors of a whole number. Then, they decide whether or not it is possible to determine the prime factors of 1 and 0. Let this question lead to a discussion of the definition of *prime factors*. This discussion sets students up for #2, in which they identify the prime factors and GCF of whole numbers.

In #2 and 3, students find the GCF and LCM. Ask questions to help lead students to gaining these skills:

- Can you express this number as a product of smaller numbers? Explain.
- Are there some special numbers that you can use to express this product?

- How can you check to see that you have the right product of prime factors for each number?

In #4, students expand on their knowledge to include writing two numbers as a product of the GCF and another factor. Allow students to explore the questions. If necessary, coach them toward an answer by using prompts such as the following:
- What number would you multiply 24 by to get 72?
- Is there a way to find that number using addition, subtraction, multiplication and/or division?

Coach students to discover that the easiest way to find the second factor is to divide the original number by the GCF. Use the following leading questions:
- What number do you multiply the GCF of 24 and 72 by to get 48?
- What operation did you use to find that number?
- Can you make a "rule" for finding the second factor if you know the original number and the GCF? (e.g., Second factor = $\frac{\text{original number}}{\text{GCF}}$)

In #5 and 6, students determine the GCF for variables with exponents. They should discover that the GCF for two exponential expressions with common bases is the expression with the smaller exponent. Students experiencing difficulty should be directed toward finding the GCF with questions like the following:
- How can you write 6^2 as a product of factors?
- How can you write 6^3 as a product of factors?
- What is the GCF of these two numbers?

Once students see the pattern, they should be encouraged to extend it by making their own "rule" for finding the GCF of algebraic expressions. Hold a class discussion for students to share their rules, and derive a class rule for finding GCFs involving expressions with variables. In #6c), students once again determine the second factor by dividing the original expression by the GCF.

In #7, students use this new skill to write a polynomial as a sum of products. They should recognize at some point that only polynomials with common factors can be written as a sum of products, and that a sum of products can include the sum of positive and negative terms. You may wish to ask more advanced students to demonstrate how they can use the reverse of the distributive property to write out the sum of products in a different way: $(4x^3)(3x) + (4x^3)(2)$ can also be written as $(4x^3)(3x + 2)$. Share with students that this is known as the factored form of the expression.

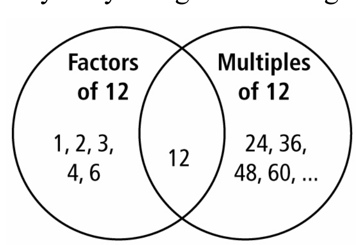
They should now be ready to discuss #8 in groups and develop their own method of factoring a polynomial using GCF.

Meeting Student Needs

- Provide to some students the three shapes in the section opener. Then, have them measure the needed dimensions and determine the surface area using both formulas. This allows students to confirm that both formulas work and assists them in developing an understanding of which formula they prefer to use. Plus, this hands-on activity may be beneficial to concrete and kinesthetic learners.
- You may have to demonstrate to some students how to write a number as a product of prime factors. Use a number, such as 72, that factors in many ways but ultimately results in the same set of prime factors. Also, demonstrate the prime factorization for an expression with variables, such as $12x^3y$, so students understand the skills they will need for the Investigate.
- Clarify to students that when a polynomial is factored, it is slightly different from when a number is factored. The number 24 could be factored as (3)(8) or (4)(6). However, when students must factor a polynomial, it means to factor out the GCF and to have a prime factor in the brackets. For discussion, write $8x - 12 = 2(4x - 6)$ on the board and ask students if $8x - 12$ has been factored.
- For the Investigate, students may need assistance in finding the GCF and LCM using the product of prime factors.

ELL

- Assist students with the term *factor*. Make sure students can differentiate it from *multiple*. One way is by using a Venn diagram:



As a class, fill in the circles and overlapping area. Discuss how not all of the multiples can be listed.

Enrichment

- Have students find and simplify the formulas for the surface areas of combinations of shapes, such as the sum of the surface areas of a right prism, cylinder, and cone.

Common Errors

- Some students may find it challenging to determine the factors of a number.

R_x Encourage students to use the Guess and Check strategy, using a calculator to check their guesses. They may need to be reminded that they are finding only the whole-number factors.

Answers

Investigate Common Factors

1. a) $30 = (2)(3)(5)$

b) No. You cannot write 1 as a product of prime factors because the only factor of 1 is 1, and the number 1 is not prime.

c) No. You cannot write 0 as a product of prime factors because a value of 0 results from the product of 0 and any number, and the number 0 is not prime.

2. a) $60 = (2)(2)(3)(5)$, $48 = (2)(2)(2)(2)(3)$, GCF: 12

b) $25 = (5)(5)$, $40 = (2)(2)(2)(5)$, GCF: 5

c) $16 = (2)(2)(2)(2)$, $24 = (2)(2)(2)(3)$, $36 = (2)(2)(3)(3)$, GCF = 4

3. a) 60 **b)** 100 **c)** 288

4. a) 24 **b)** $72 = 24(3)$, $48 = 24(2)$

c) Divide the original number by the GCF.

5. a) 6^2, 8^4, x^2

b) Example: Similarities: The GCF is the power with the smaller exponent.
Differences: The numeric examples have whole numbers as the bases of the powers whereas the algebraic expressions have variables as the bases of the powers.

6. a) x^5 **b)** $x^5(1)$ or $x^5(x^0)$, $x^5(x^2)$

c) Example: Divide the original expression by the GCF.

7. a) $4x^3$ **b)** $4x^3(3x) + 4x^3(2)$

c) Example: Divide each term in the original expression by the GCF.

8. Example:
- Find the GCF of every term in the polynomial
- Divide each term in the polynomial by the GCF to find the other factor.
- Rewrite the polynomial as a sum or difference of the products of the GCF and the second factors.
- Use the reverse of the distributive property to write the polynomial as a product of the GCF and the remaining sum or difference of the second term.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what a common factor is and how they would find it. Encourage them to try to find more than one way to identify a GCF.	• Students may need to have their skills reactivated in the various methods for finding factors. • Some visual learners may benefit from expanding the powers into repeated multiplication and circling groups of common numbers between the two given powers. This process may be very beneficial for questions with variables, such as the GCF for x^6 and x^9. You can then link this process to the exponent rules that would apply in dividing out a common factor.

Link the Ideas

The Link the Ideas statement prepares students for the skills that they will learn in the examples.

Example 1

In this example, students extend their GCF skills to factor a polynomial using common factors. Point out to students that they have just used the skills they need to do this in the Investigate. Use leading questions to coach students to make this connection.

Method 2 involves listing factors to find the GCF. Since this method may be new to some students, you may need to help them understand that the numbers listed are not products but a list of any factors of the given expression. Students should be reminded that in a list of factors, each factor appears only once.

You may wish to pose the following leading questions:
- What numbers and variables divide evenly into the expression?
- If 4 is a factor, is 2 also a factor? How do you know?
- What is the product of the circled factors? How is this product related to the GCF?

Have students complete the Your Turn questions. Discuss the solutions with the class.

Example 2

In this example, students take the next step in factoring by using prime factorization to determine the GCF. With the group, discuss the process of identifying the GCF of the numerical coefficient and then of the variables.

You may wish to discuss another approach to this question: The GCF of $7a^2b - 28ab + 14ab^2$ is $7ab$, so the expression can be written as $(7ab)(a) - (7ab)(4) + (7ab)(2b)$. By using the reverse of the distributive property, the GCF can be factored out:

$$\frac{7ab}{7ab}[(7ab)(a) - (7ab)(4) + (7ab)(2b)]$$

$$= 7ab\left[\frac{(7ab)(a)}{(7ab)} - \frac{(7ab)(4)}{(7ab)} + \frac{(7ab)(2b)}{(7ab)}\right]$$

$$= 7ab(a - 4 + 2b)$$

The above method may help students who are having difficulty seeing where the resulting factor $a - 4 + 2b$ comes from. Once again, use prompts to assist students in their understanding:

- What happens when you multiply any number by 1?
- What is $3 \div 3$ equal to? $x \div x$? $7ab \div 7ab$?
- If you multiply the original expression by 1, does its value change? Explain.
- If you multiply the original expression by $\frac{7ab}{7ab}$, does its value change? Explain.
- Is multiplying $7a^2b - 28ab + 14ab$ by $\frac{7ab}{7ab}$ the same as $7ab\left(\frac{7a^2b - 28ab + 14ab^2}{7ab}\right)$? How do you know?
- Is $\frac{7a^2b - 28ab + 14ab^2}{7ab}$ the same as $\frac{7a^2b}{7ab} - \frac{28ab}{7ab} + \frac{14ab^2}{7ab}$? How do you know?
- What do you get when you divide $7a^2b$ by $7ab$, $-28ab$ by $7ab$, and $14ab$ by $7ab$?

Leading questions such as those above can help student understanding; it is important that you adapt your questions to allow for individual students' previous knowledge.

Have students do the Your Turn questions. Then, discuss the solutions as a class. Ask students to explain what strategies they used.

Example 3

The purpose of this example is to develop a technique for factoring by grouping using the GCF. The first question should be straightforward for most students. Suggest that those having difficulty highlight the common factor in each term first: $3x(x - 4) + 5(x - 4)$. Ask leading questions:

- How can you use the reverse of the distributive property to factor out the common factor?
- What do you do with the common factor $x - 4$?
- How can you move it in front of the brackets?
- What operation would you use?

For part b) of Example 3, if students group the expression in a different order than the one given in the student resource, ask the following questions:

- Do you get the same set of factors?
- What is important in grouping the two terms being factored?

If students find this question difficult, move through the class helping students group properly before factoring.

Have students do the Your Turn questions with a partner, then discuss the solutions in groups or as a class.

Example 4

This example helps students to see an application for greatest common factors. Students need to make the connection between the concept of greatest common factor and the maximum number of groups shared by all objects.

Ask leading questions to help students understand what is meant by a group of objects:

- What does Paul mean by groups of coins?
- Is he grouping just the toonies, loonies, and quarters? Explain.
- Does each group have the same number of toonies, loonies, and quarters? Explain.
- What mathematical idea (GCF, LCM, or factoring) can you use to determine the greatest number of groups?
- What is the GCF and LCM of the number of coins?
- Which idea, GCF or LCM, best describes the maximum number of groups?
- How can you determine the number of toonies, loonies, and quarters in each group?
- How can you determine the value of the coins in each group?

Have students attempt the Your Turn question in pairs or a small group. Students may find the list of materials daunting. Help them to organize their thinking by asking questions such as:

- How can you clarify how much of each type of wood Mr. Noyle's class has?
- Take a look at your list, and identify the GCF of the numerical coefficient.
- Take a look at your list, and identify the GCF of the variables. (There is none.)

- How can you use the GCF of the numerical coefficient to help you determine how many groups Mr. Noyle's construction class can have?
- Now that you know the number of groups in the class, how can you determine how much material each group has if the material is shared evenly?

Key Ideas

You may wish to use the following prompts to promote a discussion about what students have learned during this section of the chapter:

- When you factor a polynomial, what operation do you use?
- How can you summarize the relationship between factoring and multiplying polynomials?
- When you factor a polynomial, how do you use the GCF of the terms of the polynomial?
- What kinds of expressions can be a common factor?

To reinforce their new skills, have students work in small groups and design a poster summarizing the steps they use to factor a polynomial. The class may want to choose the best poster to hang on the wall.

Meeting Student Needs

- It may be helpful to remind students that a factor divides evenly into the given value.
- Some students may find manipulatives useful as they work on Example 4.

Gifted

- Pose the following question: When you list the factors of numerical coefficients to find the GCF, is it better to start with the greatest numbers first? Ask students to show whether this is a good strategy and to explain their thinking. Then, ask students to investigate if it is a good strategy for finding the GCF of the variables.

Answers

Example 1: Your Turn

a) $5mn$ **b)** $12ab^2c$

Example 2: Your Turn

a) $9rs^2(3r - 2r^2 - 4s)$ **b)** $2np(2p + 5n^3 - 6n^2)$

Example 3: Your Turn

a) $(x + 5)(4 - 3x)$ **b)** $(a + 2)(a + 8b)$

Example 4: Your Turn

a) 8

b) $3(10' \times 1 \times 4)$, $4(8' \times 2 \times 4)$, (4×8)

c) 32' of 2 by 4s and 30' of 1 by 4s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students select the method they prefer to solve the problems. Some students may benefit from reviewing factor trees to find the prime factors of the lead coefficient. Some learners may benefit from circling values that are common so they see what the GCF looks like. It may also help them to expand variable powers and circle the common number of variables in each group. • You might wish to have students work in pairs. Suggest that one student use one method while the other uses a different method. Have them compare answers and discuss any differences. They may wish to talk through how each method works.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that all students know the difference between the meaning of *factoring* and *multiplying*. You may wish to have them generate a rule for themselves to help them remember the difference. The following is a possible example: "You get rid of brackets when you multiply. You put brackets back in when you factor." • Remind students that the GCF must divide out of every term in the polynomial. • Have students check each other's work using the method shown in the example. They then can discuss any errors and how those errors occurred.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • If students are having difficulty with part a), ask them what they see that is the same in both parts of the expression. If they are able to identify quickly, show them how to factor out the common factor and how the leftover terms form the remaining binomial. Check for understanding and ask whether they are like terms and can be simplified further. • For part b), you may wish to ask students if it matters how you group the terms. Suggest that they try grouping the first and third terms and second and fourth terms. Allow them to investigate.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from using manipulatives. They can choose a unique manipulative for each type of wood. Ask them to use these manipulatives to illustrate how much of each type of wood Mr. Noyle has for his construction class.

Check Your Understanding

Practise

Students complete a set of basic GCF questions in #1 and 2, listing all the factors of each number and circling the greatest numeric factors.

For #3, students practise their skill in finding the LCM. Once again, student should use methods that they feel comfortable with to do this question.

For #4, encourage students to find the GCF for the coefficients and then for each variable. The final answer will then be a product of these GCFs. They can also use the rules developed by the class as suggested previously in the Investigate.

For #5, students complete a set of basic common factor questions. Students should first find the GCF for all terms and then use either division or the reverse of the distributive property to factor each expression.

In #6, students are given one of the factors and need to find the remaining factor. Students having difficulty with this type of question should factor each expression and then compare their results with the given factor.

In #7, students get practice in factoring by grouping. All students should be able to factor parts a) and b). Parts c) to e) require students first to isolate two groups of two terms with common factors, and then factor out the common factor into a form similar to parts a) and b). Help students to choose their groups wisely by asking if both groups have common factors. Then, help them to manipulate the expression into a simpler form like in parts a) and b).

Apply

Point out to students that #8, 13, and 15 are similar to Example 4 in the student resource. Discuss strategies for solving these problems.

In #9, students summarize the difference between listing factors and multiples of numbers. Students should also be able to explain which of these lists is useful for finding GCF and which for finding LCM.

Question #10 provides another opportunity for students to move from concrete to abstract models for polynomials.

The open-ended question in #11 has many different correct answers. Prompt students with the following questions:
- If this is the GCF, what polynomials can you produce?
- If you have a polynomial and a GCF, what are possible remaining terms in the other factor?

This question may be quite abstract for some students and is a great opportunity to build understanding of factoring using common factors.

The higher-level questions in #12 require students to recognize errors in factoring and make corrections. Suggest that students unable to see the errors do the factoring and compare their answers with the given examples.

Question #14 requires students to recognize that the length and width of the rectangle will involve the variable r from the circles. Prompts might include the following:
- What is the width of the rectangle in terms of r?
- What is the length of the rectangle in terms of r?

- What is the total area of the large rectangle in terms of r?
- How many circles are there?
- What is the total area of all four circles in terms of r?
- How can you find the area of the plate around the circles?
- What algebraic expression represents this area?
- Can this expression be factored? How do you know?
- Is there a common factor? If so, what is it?
- Write the expression in factored form.

In #16, students use their new skill of factoring to solve an area problem algebraically.

Extend

Solving #17 requires a good understanding of the relationship between GCF and multiples of the GCF. Students may need to solve this question using Guess and Check; others will list multiples of 871 until they find the smallest two numbers that satisfy the given constraints.

The skills and processes learned to this point in the chapter are covered in #18. In addition, this question challenges students to go beyond the concepts of this topic.

Create Connections

Question #19 requires students to use the skills of factoring and substitution, and then to consider their thinking process and state a preference for a particular formula.

Meeting Student Needs

- Provide **BLM 5–6 Section 5.2 Extra Practice** to students who would benefit from more practice.
- Students who find these concepts challenging should be encouraged to complete all of the questions in #1 to 7. It may be helpful for them to draw diagrams or work with manipulatives.
- You may wish to pair students to work together on the more difficult problems. Encourage students to write how they determined their answers.
- You may wish to conclude the section by having students work in pairs or groups to create a PowerPoint presentation that will demonstrate their understanding of factoring polynomials.

ELL

- Make sure students understand the meaning of *surface area* as the total area of the surfaces that can be touched.

Common Errors

- Some students may find it challenging to determine all of the factors for each value given.

R_x Suggest that students use another method that they have learned, such as writing out the prime factors and circling the common factors for each group.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1a), c), 2a), c), e), 3a), c), e), 4a), c), e), 5a), b), e), 6a), c), e), 7a), c), d), 8–10, 12, and 14.
Students who have no problems with these questions can go on to the remaining questions.</td><td>For #1 and 2, students may find it easier to draw a factor tree to determine the GCF.
Some learners may benefit from writing out the multiples for #3 until they find one in common. Others may benefit from having a multiples chart in their Foldable for quick reference for both multiples and factors.
For #5, make sure students understand that the common factor must divide out from each term. Further, ensure students know that we always factor out the greatest common factor. You could use part d) as an example and factor out 2. Ask students if this is factored completely. This exercise would help with #6 if students are having difficulty finding the missing factor. For part a), ask them questions like, “What could you multiply $2ac$ by to get $6a^2bc$?” For some students, focusing on the numerical coefficient first will assist them in starting to find the missing factors.
It may be beneficial for you to have learners explain what #8 is asking for. Ensure they understand that they are finding the minimum common height. Encourage them to draw multiple heights until they are the same. Ask students how they could have found the answer without drawing. Suggest that they list the factors in numerical order.
Encourage students to share their response to #9 with a partner and then write their response in their Foldable. Suggest that they include a diagram or model to help explain their thinking.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #19.</td><td>This question allows students to think about whether factoring does or does not make calculating easier. Discuss with students which formula is easier to calculate.
Students should be able to determine whether or not the answers using both formulas should be the same. Listen to any discussion that occurs around this issue and clarify any misunderstandings.</td></tr>
</table>

5.3 Factoring Trinomials

Mathematics 10, pages 224–237

Suggested Timing

120–180 min

Materials

- algebra tiles

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–7 Section 5.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcome

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), 2a), b), 3a), c), 4a), c), f), 5a), b), d), f), 6a), d), f), 7a), h), 8a), 9a), 10a), 13, 14, 21, 22
Typical	#1a)–c), 2a)–c), 3a)–c), 4b), d), e), f), 5c), e), f), 6b), c), e), 7b), c), d), f), j) 10, 11b), 13–15, 21, 22
Extension/Enrichment	#5f), 7e), h), 10–12, 14, 16–22

Unit Project Note that #14 and 22 are Unit 2 project questions.

Planning Notes

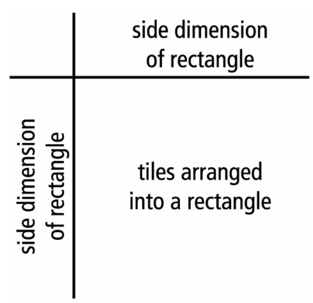
Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce prerequisite skills needed for this section.

In this section, students build an understanding of the concepts, skills, and processes required to factor trinomials. Have students use manipulatives to build an understanding that factoring is the opposite of multiplication. Students should use manipulatives until they have developed a full understanding of the patterns and processes involved in factoring trinomials. They will work toward developing a process to factor trinomials of the form $x^2 + bx + c$. Students' factoring knowledge will extend to incorporate trinomials of the form $ax^2 + bx + c$. Students will also be introduced to factoring bi-variable trinomials of the type $ax^2 + bxy + cy^2$. They will progress from using an algebra-tile method to an algebraic method of factoring.

Allow students time to understand this skill set fully as it is the foundation of many subsequent math skills.

Investigate Factoring Trinomials

It may be best to have students work in pairs for this activity. The algebra tiles will allow students to explore kinesthetically. If algebra tiles are not available, provide them with **Master 5 Algebra Tiles (Positive Tiles)**. You may wish to show students a template like the following to help them understand how to set up tiles.



Use leading questions to direct students' understanding regarding the length and width of the rectangle in #2b):

- What are the dimensions of the sides of your new rectangle?
- What do you think these dimensions represent?

It would be a good time to discuss the questions asked in #2 with the class. Make sure that students feel confident in determining the dimensions and have a process developed that works for them. Lead the class, using appropriate questioning, to see the connection between the side dimensions of the rectangle and the factors of the given polynomial.

Students should start to see relationships between the product of the terms of the factors and the first and last terms of the trinomial. They should also begin to wonder what relationship exists between the factors and the middle term of the trinomial. While students are practising factoring using the tiles in #3, ask questions that may help them to make this connection. As they work either individually or as a whole class, ask students leading questions such as the following:

- What value do you get if you multiply the first terms of the factors? the last terms of the factors?
- Do these values show up in the original polynomial? Explain.
- What value do you get if you add the last terms of the factors?
- Does this value show up in the original trinomial? Explain.
- Can you make a summary statement about the product of the first terms of the factors and the first term of the original trinomial? If so, what is it?
- Can you make a summary statement about the product of the last terms of the factors and the last term of the original trinomial? Is so, what is it?
- Can you make a summary statement about the sum of the factors and the middle term of the original polynomial? If so, what is it?

Using the solutions to #3, reinforce these summary statements and have students answer #4. They should now be ready to test these observations with the questions in #5. Go over the solutions to #5, and then have students work in groups to develop their own personal method for factoring trinomials of the form $x^2 + bx + c$ as outlined in the Reflect and Respond.

Meeting Student Needs

- To write the product of binomials, explain that notation is very important; specifically, the placement of brackets. For example, $x + 2(x + 3)$ means that only 2 and $x + 3$ are being multiplied. The product of these two binomials should be written as $(x + 2)(x + 3)$.
- Be sure to emphasize that the method in the Investigate works for trinomials of the form $ax^2 + bx + c$, $a = 1$.
- As a class, you may wish to produce a poster that illustrates FOIL and then discuss how factoring is the reverse.
- You may wish to have students use poster paper to display their group method for factoring trinomials of the form $x^2 + bx + c$. You could hang them in class for students to refer to while doing the Practise questions.

ESL

- Before beginning the Investigate, ensure students know the difference between *binomials* and *trinomials*.

Common Errors

- Some students may make errors in determining the factors of the rectangle.

R_x Once again, a template may help students to see what they are trying to find. Also, have students start at the top of the grid and use rectangular tiles to fit over the large squares and use small squares to fit over the width of the rectangular tiles. Then, have students determine the side dimension.

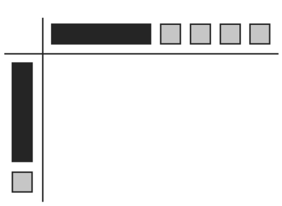
- Some students may not find it easy to see the relationships between the factors and the middle terms of the polynomial.

R_x Have students work in groups. By allowing students to create and draw their own group poster, all students will gain ownership for the process they develop.

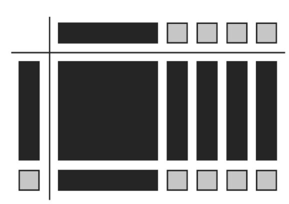
Answers

Investigate Factoring Trinomials

1. a)



b)



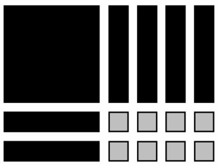
c) Example: The rectangle includes a large square that represents x^2, five rectangles that represent $5x$, and four small squares that represent 4.

Answers

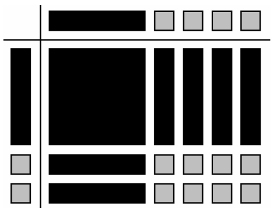
Investigate Factoring Trinomials

d) Example: The factors are shown at the top and left sides of the rectangle. At the top, the rectangle represents x and the four small squares represent 4. At the left side, the rectangle represents x and the small square represents 1.

2. a)



b)

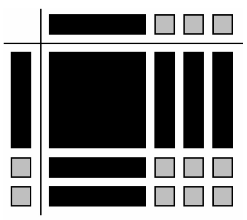


$(x + 4)$ and $(x + 2)$

c) $(x + 2)(x + 4)$. Example: The product is equivalent to the area of the rectangle.

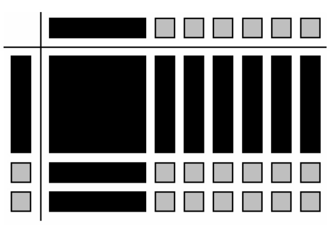
d) $x^2 + 6x + 8$; yes

3. a)



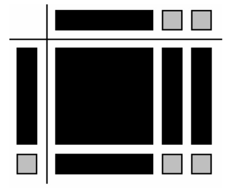
$(x + 2)$ and $(x + 3)$, $(x + 2)(x + 3)$, $x^2 + 5x + 6$, yes

b)



$(x + 2)$ and $(x + 6)$, $(x + 2)(x + 6)$, $x^2 + 8x + 12$, yes

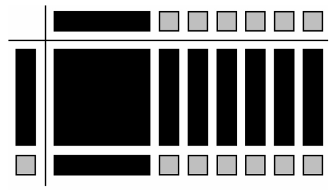
c)



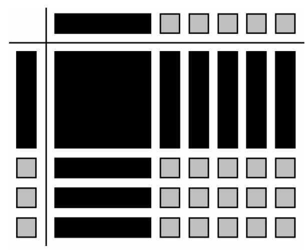
$(x + 2)$ and $(x + 1)$, $(x + 2)(x + 1)$, $x^2 + 3x + 2$, yes

4. Example: b = the sum of the integer terms of the binomial, c = the product of the integer terms of the binomial

5. a) $7 = 6 + 1$ and $6 = (6)(1)$



b) $8 = 5 + 3$ and $15 = (5)(3)$



6. Example: Find two numbers that multiply to c and add to b. If the two numbers are represented by m and n, the factors will be $(x + m)(x + n)$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss their process for factoring trinomials.	• If students find it challenging to generate a process for #6, redirect them to what they did in #3. Ask them to look at the result of their binomial product before they added like terms. In each case, is there any relationship between the two like terms and the value of c? Have them add the like terms and then return to #6.

Link the Ideas

The generalization in the student resource for factoring trinomials of the form $x^2 + bx + c$ may or may not correspond with what students discovered during the Investigate. Students should use the method that they feel most comfortable with or most attached to.

Support the following statement from the student resource by reviewing the distributive property:

$$(x+2)(x+3) = x^2 + 3x + 2x + (2)(3)$$

You may wish to draw arrows to help the visual students make the connection. Some students may need you to group the like terms in brackets $(3x + 2x)$ and then discuss factoring out x from both terms to get $(3 + 2)x$.

You may also wish to use an example other than the one in the student resource for trinomials of the form $ax^2 + bx + c$, when $a \neq 1$. Students can even use their own binomials to multiply. Any product of similar binomials will work as long as all terms are positive. Using negative terms at this point may confuse students with weaker integer skills. Have students use algebra tiles to expand the product and determine the polynomial and then do the same algebraically. Use leading questions to develop the relationship for factoring trinomials of the form $ax^2 + bx + c$, when $a \neq 1$:

- What is the product of the first terms of the binomials?
- What is the product of the last terms of the binomials?
- What value do you get if you multiply these two products together?
- What is the product of the first and last terms of the trinomial?
- What is the product of the outside terms of the binomial?
- What is the product of the inside terms of the binomial?
- What is the sum of these two products?
- How does this compare with the middle term of the trinomial?
- What is the product of these two products?
- How does this compare with the product of the first and last terms of the trinomials?
- What are the factors of this product?
- Are there any factors that add together to form the middle term of the trinomial? Explain.

This is a challenging relationship for some students to grasp. You may need to spend time with some students individually to go over the process.

Example 1

Example 1 involves a progression of questions. Encourage students to make the transition from algebra tiles to an algebraic process for factoring. The example includes a table method for listing all of the possible factors of the last term of the trinomial and the sum of the factors. Coach students by asking them to determine which two numbers multiply to give a product of 4 and add together to give a sum of 5. This may help students to determine the factors in a more direct manner. It is important that all students see the relationship between terms of the trinomial and terms of the binomial factors, no matter what method is used to factor the trinomial. Note that if algebra tiles are not available for Example 1, you may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

Since the trinomial in part b) cannot be factored over the integers, hold a discussion about how many of all possible trinomials can be factored and why factorable trinomials are important. Have students share how they know that a trinomial is factorable. Some students may benefit from the following prompts:

- What two numbers multiply to 6 and add together to 4?
- Are there any two whole numbers that can do this? Explain.
- Is this trinomial factorable using whole numbers or integers? Explain.

Part c) involves a trinomial with positive and negative terms. Some students will require that you show them examples involving all of the following situations: $(x + m)(x - n)$, $(x - m)(x + n)$, and $(x - m)(x - n)$.

Some students will benefit from a discussion about the signs of the middle and last terms of the trinomial. When you discuss c as a positive value, you may wish to ask the following questions:

- What is the sign of the product of two positive numbers?
- What is the sign of the product of two negative numbers?
- What signs might the factors have if c has a positive value?
- Does the middle term, b, help you to determine if both factors are positive or negative? Explain.

When you discuss c as a negative value, you may wish to ask the following questions:

- What is the sign of the product of a positive number and a negative number?
- What sign or signs do the factors of a negative number have?
- What does the sign of the middle term, b, tell you about the values of the factors?

Have students work through a progression of factoring questions involving the same positive last term and middle terms with different signs, such as $x^2 + 5x + 6$ and $x^2 - 5x + 6$. Have students factor with algebra tiles, and then generalize the sign rules for trinomials with

- positive last terms and positive middle terms
- positive last terms and negative middle terms

Then, use examples like $x^2 + 2x - 8$ and $x^2 - 2x - 8$. Again, have students factor with algebra tiles, and then generalize the sign rules for trinomials with
- negative last terms and positive middle terms
- negative last terms and negative middle terms

Part d) introduces the concept of factoring trinomials using two variables. Since this may be confusing for some students, consider having them factor $x^2 + 3x - 18$ first. Have them compare the factors in their solution and the factors for $x^2 + 3xy - 18y^2$ as given in the student resource, and share with you similarities and differences. Coach using the following questions:
- Do the factors have the same first terms? Explain.
- Do the factors have the same last terms? Explain.
- How are they the same?
- How are they different?
- Can we use the same process to factor both trinomials? Explain.
- What do you have to do differently to factor trinomials with two variables?

Make sure all students are confident factoring trinomials before they move on to Your Turn parts a) and b). You may wish to have students work in pairs on these questions and talk through their thinking.

Example 2

This example requires that students have strong number skills and be able to complete a multi-step process. Algebra tiles will allow concrete learners to make the connection between the polynomial given and the factors. Making the transition to using algebraic methods may be beyond some students and they should be encouraged to continue to use tiles.

In all questions of this type, encourage students to check for a GCF first.

Part a), Method 1, may require the use of a template. Students rearrange the tiles into a rectangle and determine the factors using the side dimensions of the rectangle.

Method 2 is similar to Method 2 in Example 1, except that the product is not the value of c but the product of a and c from the trinomial. You can introduce this idea through observation of the algebra-tile model. Ask leading questions like the following:
- How many large squares do you have?
- How many small squares do you have?
- How many rectangles do you have?
- Are there any numbers that multiply to 3 and add to 8? Explain.
- Are there any numbers that multiply to 4 and add to 8? Explain.

Discuss with students that since the last two answers are no, they will have to use a different process to factor this type of trinomial. Move on to such prompts as the following:
- When you multiply $3x$ and 2, how many rectangular tiles do you get?
- When you multiply x and 2, how many rectangular tiles do you get?
- What is the total number of rectangles?
- What is the product of the large and small squares?
- Are there two integers that multiply to 12 and add to 8?
- Do we see these numbers represented by any type of tile? If so, which tile?
- What relationship do you observe between the number of rectangular tiles and the product of the first and last terms of the trinomial?
- Which term of the trinomial do the rectangular tiles represent?
- What relationship can you see between the product of the first and last terms of the trinomial and the middle term of the trinomial?

It is important that students have an understanding of this connection between the factors of the product of the first and last terms and the sum of these factors forming the middle term. This understanding is essential for them to factor trinomials of this type.

For this example, note that a suggestion is given to replace $8x$ by the sum $2x + 6x$. The order in which these are substituted will not affect the outcome of factoring and students should play with this idea, factoring the examples using both orders of the sum.

Part b) includes a second variable in the trinomial as well as positive and negative coefficients. This type of trinomial can be challenging for some students. Some students will factor using an algebra-tile method and some using an algebraic method.

Part c) demonstrates to students that not all trinomials of this type are factorable and that the determining measure is the ability to find two factors of ac that add to b. In this case, there are no two integers that multiply to 12 and add to 2, as indicated in the student resource. Ask students how they know whether a trinomial can or cannot be factored.

Part d) introduces the greatest common factor. Suggest that students always look for the greatest common factor first when factoring. They then test the remaining trinomial to see if it can be factored.

Have students complete the Your Turn questions, and then go over these questions with them.

Example 3

This example shows how to apply factoring skills to solve word problems. It requires students to factor given polynomials in the form of an equation. They then evaluate the equation by substituting into either the original polynomial or the factored form. Have students talk through what is happening in the Example.

It may be beneficial to begin a flowchart with the class, either on the wall or in their Foldable, that outlines some of the possible routes for factoring. For example, at the top of the chart would be the GCF. The next level could break off into the number of terms in the polynomial and how each gets factored. This could be a progressive chart to work on as a class.

Have students work in pairs on the Your Turn. Encourage them to discuss the strategy they are going to use, to separately determine the answer, and then to discuss what they did.

Key Ideas

To summarize the skills learned in this section, coach students with questions such as the following:

- Given trinomials of the form $ax^2 + bx + c$, where $a = 1$, and its factors are $x + m$ and $x + n$, what relationship exists between terms m and n and terms b and c?
- Given trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and its factors are $rx + m$ and $tx + n$
  - what relationship exists between terms r and t and terms a?
  - what relationship exists between the product of m and n and the product of a and c?
  - what relationship exists between terms m and n and term b?
  - if $m + n = b$, how can you rewrite the trinomial to form a polynomial with four terms?
  - How can you factor a polynomial of four terms?
- Compare factoring the trinomials $2x^2 + 5x + 3$ and $2x^2 + 5xy + 3y^2$. How is the process of factoring these trinomials similar? different?

Consider having students form groups to design three posters that summarize the process of factoring trinomials: one poster for trinomials of the form $ax^2 + \text{bx} + c$, where $a = 1$, one for trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and one to summarize the difference between factoring trinomials with one variable and factoring trinomials with two variables. Hang the posters in the room as a reminder of how to factor trinomials.

Meeting Student Needs

- It may help some students if you break down the process and have visual steps for them to follow. For example they can perform the following steps:
  - Factor out the GCF.
  - Look at the number of terms:
    Two Terms: Look for the difference of squares.
    Three Terms: Factor the trinomial.
    Four Terms: Factor by grouping
  - Factor completely and check by multiplying.

  Explain and have visuals for the rules for determining the signs in each factor.
- It may be helpful for you to provide additional polynomials for students to solve in pairs or groups.
- Some students will find it difficult to distinguish between the two types of trinomials, $ax^2 + bx + c$, $a = 1$ and $a \neq 1$. Ensure they are aware that the method used for trinomials with $a \neq 1$ will work for all trinomials in the form $ax^2 + bx + c$.
- A visual can be a helpful tool for students finding it challenging to determine the integers they need to factor a trinomial. For example, when factoring $x^2 - 8x - 20$, set up the following for students:

  $\square \times \triangle = -20$

  $\square + \triangle = -8$

  This visual shows them that the same two integers must multiply to –20 and add to –8.
- You may wish to discuss what is meant when you cannot factor a trinomial *over the integers*. Inform students that this means that no integers can be used to factor a given trinomial. Refer to and discuss the Did You Know! on page 228.
- For Example 3, be prepared to discuss why the formulas are factored first. Make sure students see that they get the same answer if the time is substituted into the original formula or into the factored form.

Web Link

For more information and practice with factoring trinomials, have students go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

1. a) $(x + 2)(x + 5)$ **b)** $(r - 9s)(r - s)$

Example 2: Your Turn

2. a) $(2x - 1)(x + 4)$ **b)** $-3(s^2 + 17s + 10)$ **c)** $(2y + x)(y + 3x)$

Example 3: Your Turn

3. a) $h = -16(t - 10)(t + 1)$ **b)** 464.64 ft

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Allow students who need it extra time for practice and exploration. • Some students may need to use the algebra tiles longer to ensure that they see the patterns of factoring. • If students are using the table, encourage them to identify the *b* value and *c* value for each question and the pairs of factors they will use. Developing this habit will assist them in quickly reviewing for incorrect signs on factors in later problems. • If students do not have strong number skills, provide a multiplication table that could be inserted into their Foldable for quick reference.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that the operation sign in front of a term also identifies whether it is positive or negative. This understanding is especially important when students find the value for *c*. • For students who may get confused about whether or not to multiply *a* and *c*, show them that all factorable polynomials can be factored using the method in this example; therefore, they need learn only one method. • Identify for students that when using grouping, the signs of each factor must be enclosed within the brackets. Initially, the operation sign between the brackets will be positive. • Encourage students to experiment with different methods as they work on these questions. They may factor with one method and check their answer using a second method.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students to check polynomials for a GCF before proceeding to finding the factors of *c*. • Discuss with students whether or not it matters which formula they use to evaluate. Their response will serve as a good assessment of their understanding of factoring. • You may wish to have students work in pairs to solve this problem, then join with a second pair and share what they did. Encourage them to talk through the different methods they used, and why.

Check Your Understanding

Practise

For #1, students are given algebra-tile models. If students have difficulty with this question, coach them on the use of algebra tiles to represent trinomials. For #2, students do basic factoring questions using tiles. For #3, students practise number skills required for factoring trinomials.

For #4, students complete basic $x^2 + bx + c$ factoring questions that involve trinomials with only positive terms. It is useful to reinforce the understanding of the relationship between the last term of the trinomial and the last terms of the binomial factors.

In #5, positive and negative terms are introduced. This question will help you to identify which students need coaching on integer skills. These students may need extra practice on this type of question.

In #6 and 7, students build from basic trinomials with positive terms, to trinomials with two variables, to trinomials involving positive and negative terms. Note that some trinomials may not be factorable.

Apply

For #8, students make a connection between algebra tiles and a geometric area with algebraic dimensions.

In #9 to 12, students have an opportunity to use generalizations about factoring to develop factorable trinomials. Some students will need encouragement to apply the concept of factoring trinomials and then use their number skills to decide on the possible solutions.

For #13, students demonstrate much of the same understanding as #9 to 12 but are using their knowledge to create a trinomial expression that cannot be factored.

For #15 and 16, students apply their factoring skills to real-life contexts. All students who can factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, should be able to complete #15. For #16, students go beyond factoring to making a connection between the factors and an expression of the number of jackets sold and a price per jacket.

Extend

Question #17 is open-ended. Challenge students to try to come up with different answers for this question, and then to compare answers.

You may wish to have students draw a diagram for #18, discuss the strategies they can use, and then have them solve the question and compare their answers. Challenge students to verify that their side length is correct by multiplying to determine the area.

You may wish to have students work together on #19.

Have students compare the polynomial for #20 to the polynomial for #18 and discuss whether or not they are working with a square. Ask them to verify their thinking mathematically. Challenge them to consider whether more than one shape could provide this area.

Create Connections

For #21, students summarize their understanding of the connection between multiplying binomials and factoring trinomials.

Unit Project

The Unit 2 project questions, #14 and 22, give students an opportunity to model their understanding of the link between multiplying and factoring as well as their ability to model a trinomial. They then incorporate these models into an abstract piece of art of their choice. This activity relates to the overall goals of the unit project.

Meeting Student Needs

- Provide **BLM 5–7 Section 5.3 Extra Practice** to students who would benefit from more practice.
- Remind students that the GCF should always be removed first before they factor any trinomial.
- Before students begin #8 to 12, remind them of the importance of sum and product when factoring trinomials.

Enrichment

- Ask students to use “reverse engineering” to create trinomials that can be factored by first creating binomials and then multiplying. Encourage the use of challenging coefficients. Have them switch trinomials with a classmate to practise factoring.

Gifted

- The formula for finding the distance travelled by a falling object is $d = \frac{1}{2}at^2 + vt$, where d is distance, in metres, a is acceleration due to gravity, t is time, in seconds, and v is the starting velocity of the object that is falling. Ask students to explore the formula, including factoring it and entering various values for time, starting velocity, etc., in order to get a sense of the effect of gravity on objects. Students might consider graphing the formula, using technology. Ask them how this type of formula might be used to create video games such as a golf game.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1a), b), 2a), b), 3a), c), 4a)–c), f), 5a), b), d), f), 6a), d), f), 7a), h), 8a), 9a), 10a), 13, and 14.
Students who have no problems with these questions can go on to the remaining questions.</td><td>• In #1 and 2, students have an opportunity to demonstrate that they can determine an algebra-tile model given a polynomial and determine the factors from an algebra-tile model. Allow students to use algebra tiles, if necessary. Although the requested approach may vary in each question, students can use the tiles to verify their thinking.
• Caution students to watch for the signs on numbers in #3 as they will affect the signs on their factors. This activity relates directly to the questions in #4 and 5, in which they find factors.
• Have students verbalize the factoring process to be used in #6. It may be beneficial to have them complete the first question as they verbalize the steps. Ensure students understand the process and are successful in #6 where all values are positive, before moving to #7 where factors will have different signs.
• If students are confused about the role of 15 cm in #8, have them explain the dimensions of a rectangle and how to determine area. Then, ask how this links 15 cm.
• In #9 and 10, students must demonstrate that they understand the role of b and c in the factoring process. If students are unsure, use $x^2 + bx + 16$ as an example. Have students generate as many possible factors for 16 as they can. Encourage students to use two negative numbers as well.</td></tr>
<tr><td>Unit 2 Project
If students complete #14 and 22, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• You may wish to provide students with BLM 5–4 Chapter 5 Unit 2 Project and have them finalize their answers.
• For students who are unsure of how to begin their designs for #22, encourage them to make several models, lay them out on their desk, and slide them around until they create a design.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have students complete #21 and 22.</td><td>• In #21, students demonstrate their understanding that binomial multiplication can result in a trinomial. Encourage students to use whatever method makes most sense for them in their description.
• Make sure algebra tiles are readily available.
• Encourage students to generate their own examples. They may wish to review Examples 1 or 2 for ideas. Show students that they can choose their own factors, e.g., 4 and 5, and write their own binomials. From there, they can generate a trinomial. This will help them to personalize their responses.</td></tr>
</table>

Factoring Special Trinomials

5.4

***Mathematics 10*, pages 238–251**

Suggested Timing

100–120 min

Materials

- centimetre grid paper
- scissors

Blackline Masters

Master 2 Centimetre Grid Paper
Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–8 Section 5.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcome

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 6, 7a)–c), 8a), b), 9, 10, 12, 13, 16, 20, 24, 25
Typical	#1–7, 8a), b), 9–13, 16, 20, 22, 24, 25
Extension/Enrichment	#8, 10, 12, 14, 15, 17–19, 21, 23, 26

Unit Project Note that #12 and 13 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter helps students develop an understanding of why differences of squares and perfect square trinomials are special. It also leads them to use this understanding to develop a process of factoring these trinomials algebraically. Do not move too quickly through this section. Ensure students have sufficient time to process the concepts and develop their own models.

Direct students' attention to the photo of the patchwork quilt. This visual shows how geometric shapes (small squares) can form the area of a rectangle, just as algebra tiles form the area of a rectangle. Students should share with each other examples of items made up of geometric shapes that they have seen in their everyday life. They should be given the goal by the end of the section of connecting these examples with polynomials and factors.

Investigate Factoring Differences of Squares

This Investigate is a hands-on (kinesthetic) activity and with the right coaching will lead students to develop an intuitive proof for the difference of squares factoring relationship $a^2 - b^2 = (a + b)(a - b)$. You may wish to give students **Master 2 Centimetre Grid Paper** for the Investigate.

Allow students to work with others or to complete the Investigate by themselves, with some discussion with classmates. Have students complete #1 to 3, leading them to see that the area of a square is a product of two sides like a rectangle, $A = l \times w$, but that it is also the square of a side, $A = s^2$. It is the latter form that students need to note. If students are using only the rectangular method, prompt their thinking:

- Is there another way to find the area of a square? If so, what is it?
- What do you notice about the lengths of the sides of a square?
- Is there another way to express the product of a number and itself? If so, what is it?
- What do we call the operation when we multiply a number by itself?

Once students have made the observation that the area of a square is the square of the sides, help them to discover a method for finding the area of the new irregular shape. They can find the area of the remaining shape by using one of the following methods:

- counting the squares
- cutting it into two separate rectangles and finding the sum of those areas
- subtracting the area of the small square from the area of the large square

After students have explored one of the above methods of their choice, direct them to the last of the above methods. Remember that they will make a stronger connection through discovery than by being told the relationship. Prompt students to make a connection to the last method:

- What is the area of the large square?
- What is the area of the small square?
- What mathematical operation (addition, subtraction, multiplication, or division) do you think is represented by cutting the small square from the large square?
- Can you apply the same operation to the areas of the large and small squares to determine the area of the new shape? Explain.
- What area did you find?
- How does this compare to the area you found using another method?

In #4, students find another physical shape by cutting off part of the irregular shape and connecting the two rectangles to form a large rectangle. If students do not see how to fit the two pieces together, ask them leading questions:

- Are there any sides that have the same length?
- How can you join them together along that side?
- What dimensions will this rectangle have?
- How do we find the area of a rectangle?

Before moving on to #5, it is important that students see the area of the new shape as being determined either by taking the difference of the areas of the large square and the small area or by finding the area of the large rectangle. They must also understand that the dimensions of the rectangle can be found by the following product: (side length of large square + side length of small square) × (side length of large square – side length of small square).

Students who find this relationship challenging may need leading questions to make the discovery:

- What is the length of the new large rectangle?
- What is the sum of the sides of the large and small squares?
- How does this sum compare with the length of the large rectangle?
- Write the length as a sum of the sides of the squares.
- What is the width of the new large rectangle?
- What is the difference of the sides of the large and small squares?
- How does this difference compare with the width of the large rectangle?
- Write the width as a difference of the sides of the large and small squares.

For #5, give students enough time to play and explore with the grid paper and to try at least two other examples of their choosing. Have them write down the answers to the questions in #5 based on their exploration of at least two examples. Discuss as a class what relationships they discovered.

In #6, students use this same technique to find the area of similar shapes with one unknown side. Most students should be able to make the connection between areas of the rectangle and the difference of squares. Now, they learn to use x as the side length of the large square. Some students may need coaching to remember to find the side length of a square by taking the square root of the area. Use the following leading questions:

- What is the area of the small square?
- What number multiplied by itself equals 25?
- What is the side length of the small square?
- What expression multiplied by itself equals x^2?
- What is the side length of the large square?

Once students make the connection between x and the side length of the large square, have them determine the side lengths of the large rectangle.

In #7, students extend their understanding from a concrete model to an abstract relationship. Use leading questions to coach students to see that the area can be expressed as $a^2 - b^2$, which represents the difference of areas of the squares:

- What is the area of the large square if a is its side length?
- What is the area of the small square if b is its side length?
- What is the area of the shape after the small square is cut off?
- What expression, using a and b, best represents the length of the large rectangle?
- What expression, using a and b, best represents the width of the large rectangle?

In #8, students describe in their own words what patterns and relationships they observe between areas of squares.

Meeting Student Needs

- Some students may need assistance in recalling their understanding of square roots. Consider preparing a pre-assessment of students' familiarity with squares from 1 to 169. You could also discuss $\sqrt{x^2} = x$, $\sqrt{x^4} = x^2$, etc.
- Produce posters illustrating the patterns for differences of squares and perfect trinomial squares and post them in the classroom. For example, you may wish to include the following patterns:
  $a^2 - b^2 = (a - b)(a + b)$
  $(ax)^2 + 2abx + b^2 = (ax + b)^2$
  $(ax)^2 - 2abx + b^2 = (ax - b)^2$
- It may be beneficial to your class for you to split this section into two parts and teach them separately. Begin with factoring differences of squares, and have students complete the related questions. Then, move on to factoring perfect square trinomials and have students complete those related questions. Once students understand the two processes, discuss and explore the relationship between the two forms.
- Ensure students see the connection between factoring and multiplication as this will help them in their understanding of factoring.
- In #4c) of the Investigate, students must explain how one area is related to another. Before beginning the Investigate, give a few examples. For example, draw a rectangle and find its area. Draw its diagonal and find the area of one of the triangles. Write on the board a sentence that relates the area of the rectangle to the area of the triangle. Allow students to offer suggestions. Leave their examples on the board for them to refer back to.
- The Investigate presents an opportunity for students to learn through doing. Ensure that students have a chance to work through three different squares. The key to understanding differences of squares is found in #7 and 8. Students need to make the connection to $a^2 - b^2 = (a - b)(a + b)$.

Common Errors

- Some students may find it challenging to rearrange the cut sections into a larger rectangle.

R$_x$ Ask students to tape together the sides of the smaller rectangles that are the same length and then to describe the dimensions of the new large rectangle.

- Some students may not be able to find the side lengths of the squares when given the area.

R$_x$ They should be asked to describe what operation occurs when you multiply a number by itself and what operation is the opposite of multiplying a number by itself. Ask them if they can use these operations to find the area of a square, and given the area of a square, find its side length.

- Some students may not know how to use abstract variables to represent the sides of a square.

R$_x$ They should be coached to replace the length of the large square with x in #6 and replace the sides of the large and small squares with a and b for #7. Have them use centimetre grid paper to find the areas, replacing the side length of the large square with x or a and the side length of the small square with 5 or b, depending on the step that they are working on.

- Some students may find it challenging to write equations to summarize the difference of squares relationship.

R$_x$ Ask students to compare the areas of the irregular shape and the large rectangle. Use leading questions:

  - What does the term *equals* mean?
  - If you know that two expressions are equal, how can you write an equation to express this relationship?
  - How can you use the variables a and b to describe each of these areas?
  - Since the area of the irregular shape is equal to the area of the large rectangle, how can you write an equation to describe this relationship?

- Some students may not be clear on how to recognize a difference of squares.

R$_x$ Prompt students with the following questions:

  – How many terms are in a difference of squares?
  – What type of expression is the first term?
  – What type of expression is the last term?
  – Why do we call these polynomials *differences of squares*?
  – When you see a polynomial made up of two terms, how will you decide if it is a difference of squares?
  – Once you determine that the polynomial is a difference of squares, how can you factor it?

Answers

Investigate Factoring Differences of Squares

1. a) 100 cm^2

b) Example: Determine the product of the side lengths (10 × 10) or the square of the side lengths (10^2).

2. a) 16 cm^2

b) Example: Determine the product of the side lengths (4 × 4) or the square of the side lengths (4^2).

3. a) 84 cm^2

b) Example: Area of large square – area of small square

c) Example: Yes. Count the number of centimetre squares.

4. a) 6 cm × 14 cm **b)** 84 cm^2 **c)** The areas are the same.

6. a) Example: $x^2 - 25$

b) $x + 5$ and $x - 5$

c) Example: The product of the dimensions $x + 5$ and $x - 5$ is equal to the area $x^2 - 25$.

d) $x^2 - 25 = (x + 5)(x - 5)$

7. a) $a^2 - b^2$ **b)** $a + b$ and $a - b$

c) $(a + b)(a - b$ **d)** $a^2 - b^2 = (a + b)(a - b)$

8. a) Example: The rectangle has dimensions of (sum of the side lengths the large square and small square) × (difference of the side lengths of the large square and small square). The area of the rectangle is the same as the area of the irregular shapes. The area can be found by finding the side lengths of the rectangle.

b) Example: The area you get from subtracting the area of the smaller square from the area of the larger square is equal to the area of the large rectangle with side lengths listed in a) above.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond It may be beneficial to complete the responses to #7 and 8 as a class.	• Have students work in pairs to complete the Investigate. Have the pairs share their answers to #7 and 8 with the class. • As a class, generate a pattern and then ask students to create their own pattern to put into their Foldable. This activity sets a strong foundation for students' conceptual understanding of the difference of squares.

Link the Ideas

The Link the Ideas section summarizes the discovery from the Investigate and introduces students to another special product that they will be factoring later, perfect square trinomials. Discuss with the class if u and v represent a relationship different from or similar to the one in the Investigate involving a and b. Have students describe both special products. Ask students what pattern or connection they see between the factors and the product.

Most students should understand the difference of squares example but may have difficulty with the development of the perfect square trinomials example. Prompt them with the following questions:

- What relationship exists between the first terms of the factors and the first term?
- What relationship exists between the last terms of the factors and the last term?
- What special relationship exists between the square roots of the first and last terms and the middle term?
- Is this relationship always true? Explain.
- Do the equations $(ax)^2 + 2abx + b^2 = (ax + b)^2$ and $(ax)^2 - 2abx + b^2 = (ax - b)^2$ describe the relationship that exists between the square of a binomial and the resulting trinomials? Explain.
- What do we call trinomials formed by squaring binomial expressions?
- How can you use this relationship to factor trinomials of this type?

Encourage students to write in their Foldable anything from the Link the Ideas section that may assist them.

Example 1

Example 1 demonstrates various forms of trinomials involving differences of squares that students may see. It also shows students that they can factor these types of polynomials in more than one way. You may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

In Method 1, students may not recognize the reasoning behind adding the positive and negative rectangles. Remind them that the product of binomials will always form a rectangle and ask what parts of the rectangle are missing in the first diagram. Prompt their thinking:

- What happens when you add a positive and negative rectangular tile?
- How many positive tiles are needed to form a rectangle?
- How many negative tiles are required if the sum is zero?
- What binomial represents the top of the large rectangle?
- What binomial represents the side of the large rectangle?
- What relationship exists between the binomials $x - 3$ and $x + 3$ and the product $x^2 - 9$?
- How can you check that the factors are correct?

In Method 2, students may wonder why the terms $-3x$ and $+3x$ are inserted as middle terms in the first step. Prompt them with questions like the following:
- What is the sum of $-3x$ and $+3x$?
- Where did the value 3 come from?

Students should see Method 3 as similar to the method developed in the Investigate. If they do not, refer them back to #6 of the Investigate and ask if they see similarities in the methods.

Part b) requires students to factor a difference of squares involving two variables.
- When have you factored polynomials with two variables before?
- How did you change your factoring method from factoring single-variable polynomials?
- Can you use the same technique with differences of squares? Explain.

Part c) involves a polynomial that cannot be factored over the integers. Students should be able to describe in their own words why $m^2 + 16$ cannot be factored using a difference of squares method. Ask students what is meant by *the binomial cannot be factored over the integers*. Discuss as a class having only integer coefficients in the factors.

Part d) introduces a common factor and requires students to recognize that the expression $7g^3h^2 - 28g^5$ is not a difference of squares but that one of its factors is a difference of squares. Prompt their thinking with the following questions:
- Do the terms $7g^3h^2$ and $28g^5$ have a common factor? What is it?
- What expression do you get when you factor out the common factor?
- Is this expression a difference of squares? How do you know?
- What are the factors of $h^2 - 4g^2$?
- What three factors need to be multiplied together to produce the polynomial $7g^3h^2 - 28g^5$?

Have students complete the Your Turn questions. Suggest that they use the following strategy:
- Is this polynomial a difference of squares?

↓ YES ↓ Factor it.

↓ NO ↓ Is there a common factor?

↓ YES ↓ Factor it out. Then, ask: Is the remaining factor a difference of squares?

↓ YES ↓ Factor it.

↓ NO ↓ It is not factorable.

↓ NO ↓ It is not factorable.

Example 2

This example demonstrates three methods of factoring perfect square trinomials and then shows examples of types of polynomials that can be factored using one of these methods.

In Method 1, have students look for patterns in the tiles to determine what is special about perfect square trinomials. Ask questions like the following:
- What do you notice about the first term?
- What do you notice about the last term?
- What do you notice about the number of rectangles to the right of the large square and the number of rectangles below the large square?
- What relationship exists between the number of rectangles to the right of the large square and the middle term of the trinomial?
- What relationship exists between the number of rectangles below the large square and the middle term of the trinomial?
- How can you determine if a polynomial is a perfect square trinomial?

In Method 2, if students are unsure of how to factor by grouping, prompt them with leading questions:
- What two numbers have a product of 9 and a sum of 6?
- Why is $6x$ replaced by the sum of $3x$ and $3x$?

- Why is the trinomial written as $(x^2 + 3x) + (3x + 9)$?
- How is it possible to write $(x^2 + 3x)$ as $x(x + 3)$?
- What is the common factor in the expression $x(x + 3) + 3(x + 3)$?

For Method 3, prompt students with the following questions:

- How do we know that $x^2 + 6x + 9$ is a perfect square?
- What is $\sqrt{x^2}$?
- What is $\sqrt{9}$?
- How can you express 9 as a square?
- Why is it important that the middle term is twice the product of the square root of the first term and the square root of the last term?

In part b), the trinomial includes a common factor. Some students may not see that the polynomial can be factored using a perfect square trinomial method. Assist them with the following prompts:

- Is $2x^2 – 44x + 242$ a perfect square trinomial? How do you know?
- Does the polynomial $2x^2 – 44x + 242$ have a common factor? What is it?
- What expression do you get after factoring out the common factor 2?
- Is the remaining polynomial a perfect square?
- Can you factor $x^2 – 22x + 121$ using a perfect square trinomial method?
- How can you check that $2(x – 11)(x – 11)$ is the factored form of $2x^2 – 44x + 242$?

Part c) challenges students to determine if a given polynomial is a perfect square trinomial. Ask students how they know that $c^2 – 12c – 36$ is not a perfect square trinomial. If students cannot answer, ask the following questions:

- What is the square of +6?
- What is the square of –6?
- What do you notice about the signs of the squares of each number?
- Can a perfect square term have a negative value?

Have students complete the Your Turn questions. Suggest that they ask themselves the following questions:

- Is the polynomial a perfect square trinomial?
- If no, does the polynomial have a common factor?
- If yes, is the remaining factor a perfect square trinomial?

These questions should help them determine if they can use the perfect square trinomial method to factor the polynomial.

Key Ideas

The Key Ideas points summarize the methods used to factor differences of squares and perfect square trinomials. Have students write these summaries in their own words in their notebook and refer to them when factoring polynomials. Suggest that they include in their notes how to determine if a polynomial is a special product as well as how to determine if a polynomial is factorable.

Meeting Student Needs

- You may wish to use whole numbers to show students the pattern for differences of squares. For example:
  $$\begin{aligned}(5+3)(5-3) &= (5)(5)+(5)(-3)+(3)(5)+(3)(-3)\\ &= 25-15+15-9\\ &= 25-9\end{aligned}$$
  Ask students to discuss the relationship between the numbers found in the original question and the numbers in the bottom line.
- For perfect square trinomials, ensure that students make the connection to the middle term. Show several examples and highlight the two middle terms so that students know why the middle term is twice the product of the square root of the first term and the square root of the last term.
- Encourage students to do more questions similar to Example 1, Method 1. The visual representation will assist many students.
- Ensure students know that if the last term is negative, the trinomial is not a perfect square trinomial and that any number squared is always positive. Also, ensure that they understand the reasoning behind these concepts.
- The perfect square trinomial may be a challenge for some students to recognize. Have them do the multiplication of perfect squares until they start to see a pattern. Then, have them write this pattern on an index card to refer to as they do the Check Your Understanding questions.
- For some students, it will be very important that they experience a good deal of modelling using diagrams or manipulatives before they begin the Check Your Understanding questions.

Gifted

- For Example 1, have students explore whether or not $m^2 + 16$ can be factored using a set of numbers other than integers, such as rational numbers, irrational numbers, or real numbers.

Common Errors

- Some students may not recognize that a given polynomial is a difference of squares or a perfect square trinomial.

R$_x$ Coach students to remember what is special about each of these two types of products, and as a class, brainstorm a lists of characteristics that make them special. Students could work in groups and design posters to describe the special relationship between terms of differences of squares and perfect square trinomials. The posters could also summarize the skills of factoring each type of special product. Display the posters in class.

Web Link

To give students an opportunity to practise factoring special products, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $(7a + 5)(7a - 5)$ **b)** $5(25x^2 - 8y^2)$ **c)** $(3pq + 5)(3pq - 5)$

Example 2: Your Turn

a) $(x - 12)(x - 12)$ or $(x - 12)^2$

b) not factorable

c) $3(b + 4)(b + 4)$ or $3(b + 4)^2$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Review to ensure students can explain what *difference of squares* means. Ask students how they would be able to distinguish a difference of squares question from a trinomial question. Ask them to consider whether the answer would be the same if the question were $x^2 + 25$. • Initially, when students are factoring a difference of squares, allow them to use a method of their choice. However, encourage them to find a second method by which they can factor. This will help them to identify different presentations of the differences of squares. • Some students may still be at the concrete stage. Encourage them to use manipulatives, as needed.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students often confuse the difference of squares and perfect squares because they focus solely on the constants and do not recognize the significance of the different signs. Point out how the factors differ between the two types of questions. • Allow students to use a method of their choice for factoring. Point out that the methods used in the previous lessons will be used here. Encourage students to review their factoring method of choice from the previous section.

Check Your Understanding

Practise

For #1, students complete basic algebra tiles factoring questions. Most students should be able to complete #2, which is a review of multiplying polynomials. If they are not confident, coach them on multiplying polynomials and give some remedial questions as practice before they complete #2. For #3, students also multiply polynomials, this time determining the squares of binomials. Ask students if they can write the question out as a product of two binomials instead of binomials squared.

Since #4 is a higher-level question, students should work with a partner if they find it challenging. You may need to work individually with some students, asking them to identify the type of special product each question represents and then leading them to use the appropriate method of factoring to find the missing expressions.

For #5, students factor differences of squares. Note that some are not factorable. Students should indicate which questions cannot be factored and state why.

For #6, students factor trinomials. Again, some are not factorable. The questions require students first to decide if the trinomial is factorable and then to factor it as a perfect square trinomial. Have students explain why a polynomial cannot be factored.

For #7, students factor special product polynomials with common factors. Parts a) and c) involve factoring by using only common factors. All other questions require students to factor out the common factor and then use a second step to factor using a method for differences of squares or perfect square trinomials.

Apply

In #8, students have a chance to reinforce their understanding of the relationship between the middle term of a perfect square trinomial and the first and last terms of the trinomial. Discuss that perfect square trinomials can be either in the form $(ax)^2 + 2abx + b^2$ or in the form $(ax)^2 - 2abx + b^2$. To help them in their thinking, ask the following questions:

- Using either $2abx$ or $-2abx$ from the definition, what expression is equal to n in the question?
- How can you find the value of a?
- How can you find the value of b?
- What value of n will satisfy the relationship if the middle term is $2abx$ or $-2abx$?

For #9, students explain why a given polynomial is not a difference of squares or not a perfect square trinomial. Ask students the following questions:

- What do you notice about the first term in a product involving a difference of squares and a perfect square trinomial?
- What do you notice about the last term in a product involving a difference of squares and a perfect square trinomial?
- What do you notice about the number of terms in a difference of squares?
- Why are they called differences of squares?
- What relationship exists between the first and last terms and the middle term of a perfect square trinomial?

In #11, students apply the patterns they have observed using differences of squares to understand how some number tricks might work. This question can be used as an optional activity to challenge and entertain interested students.

For #13, students apply the concept of difference of squares to the area of a property. It allows students to make the connection between a concrete problem and an algebraic expression to model the situation.

Students may find #14 to be a higher-level question since it involves a formula with difference of squares and binomial perfect square terms.

Before students begin #15, you may wish to inform them that Gena LaCoste, the artist who created the original watercolour painting, is a southern Alberta native and has studied and painted across Western Canada and in parts of the southwestern United States. See the Web Link that follows in this resource for more information on Gena LaCoste.

For #15, students must recognize that if a print is enlarged by a factor of 3, all sides increase by a factor of 3. They may need some coaching to make this connection:

- How can you express the area of the original print if its side is $(2x - 3)$ cm long?
- If the original print is $(2x - 3)$ cm long, how can you express the side if it is 3 times longer?
- If you express the length of a side as $3(2x - 3)$ cm, how would you express the area of the large print?
- How can you express the difference of these areas using a difference of squares?

Once students understand these expressions, they should be able to expand and simplify, and then check using substitution. Make sure that they use a small enough value for substitution to keep the check reasonable.

For #17 and 18, students use the concept of perfect squares to solve area problems. In #17, students use this concept to determine the side length of a square when given its area, and then use the side length to find the perimeter of the square.

In #18, students use the concept to find the radius of a drum given its area. Some students may need coaching to recognize that $r^2 = 9x^2 + 30x + 25$. They should then be able to factor this perfect square trinomial to find the radius and then the diameter.

Question #19 requires students to verify statements using their understanding of differences of squares and perfect square trinomials. Part b) may provide a challenge, since if $b = 0$, the statement is only sometimes true. This question is useful to develop students' ability to think in the abstract.

In #20, students must find the error. This type of question is useful for students to self-assess their work and be aware of common errors made by students. This is a good class discussion topic and should lead to a better recognition of common errors in factoring special products.

Since #21 is a volume question, to determine the answer, students must express the product as three factors. The most challenging step in this question is recognizing that the original polynomial can be grouped and then factored using common factors. Coach students to see this relationship using the following prompts:

- How many terms do you see in the volume expression?
- How can we factor polynomials with four terms?
- How can you group the terms into two sets of binomials with common factors?
- Is there more than one way to group these terms into two sets of binomials with common factors? Explain.
- Factor these two groups.
- Do you see another common factor in your solution? If so, what is it?
- How can you factor it out of the expression?
- Can one of the factors be factored further? Explain.
- How many factors do you have in total?
- How many dimensions do you require to describe the volume of a rectangular prism?
- How can you check that you have found the correct dimensions?

Extend

In #22, students apply the concepts developed in the Investigate to solve the problem. Some students may need some coaching to recognize that one of the factors of the perfect square trinomial represents the side of the square. If they can make that connection, they should be able to determine the dimensions of the rectangle and then its area.

For #23, students apply to integers their understanding of the relationship between the terms of a difference of squares and the sum of the square roots of the terms. For students who are not sure how to begin, tell them to use values between 1 and 5 and use Guess and Check to find one set of integers whose difference of squares equals the sum of the integers. Once they have found one set, have them find two more. They should then be able to make a generalization about integers that satisfy these conditions. Part b) will require the application of the generalization made in part a).

Create Connections

For #24, students summarize their understanding of the relationship between the terms a, b, and c of a perfect square trinomial.

For #25, students summarize their understanding of the relationship $a^2 - b^2 = (a + b)(a - b)$, using two methods, likely a concrete visual model and an algebraic model, to prove the equality between a difference of squares and its factors.

In #26, students show their understanding of the patterns they have discovered when factoring different trinomials.

In #27, students apply their understanding of how a difference of squares relationship can be used to simplify the product of any two numbers that differ by 2. Then, they extend their understanding to develop a technique for multiplying any two numbers with an even number difference.

Unit Project

As Unit 2 project questions, #10 and 12 allow students to apply their skills from the section and create some artwork. In particular, students with a more concrete approach to learning will benefit from these activities.

Meeting Student Needs

- Provide **BLM 5–8 Section 5.4 Extra Practice** to students who would benefit from more practice.
- Before assigning questions, refer students to the student learning outcomes for this section.
- It may benefit your students for you to assign one part from each Practise question and have students work on them in small groups.
- Encourage students to look for the patterns as they complete the questions, but also allow some students to use either algebra tiles or multiplication to determine each product.
- For #7, explain to students that *factor completely* likely means the question involves a common factor plus another type of factoring. For example, $5x^2 + 25x + 30 = 5(x^2 + 5x + 6)$, which factors further to become $5(x + 2)(x + 3)$.
- For #11, you may wish to challenge students: While they complete the questions using a calculator, you complete the questions mentally. Afterward, demonstrate how you were able to find the answers so quickly. Encourage them to challenge someone at home.
- For #14, ensure students understand that concentric circles are circles that share the same centre, with one circle lying within the other.

- Associated with #18, you may wish to consult Elders and Knowledge Keepers about someone who could visit the class to talk about the traditional drums used or created locally.
- Remind students that when substituting a value for x, they may choose whatever value they wish for x. In this section, they check that the factored form matches the expanded form. Ensure that students understand to substitute their chosen value into each form and that the objective is to end up with the same result from both forms.

Common Errors

- Some students may not be able to recognize that a trinomial is a special product.

R_x Coach students to see that they can factor perfect square trinomials using other methods developed for factoring trinomials, but that it is much easier to factor if they recognize the special product. Assist students in seeing that a difference of squares has a form unique from other polynomials and that binomials can only be factored using common factors or a difference of squares.

Web Link

For students to see other examples of Gena Lacoste's work and to learn more about the artist, have them go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, 6, 7a)–c), 8a), b), 9, 10, 12, 13, 16, and 20. Students who have no problems with these questions can go on to the remaining questions.	• In #2 and 3, students have an opportunity to demonstrate their multiplying ability. Encourage them to look back in section 5.1 and review possible methods of finding products. Ask them to identify which are the factors of a difference of squares and which are factors of a perfect square. Have them explain their reasoning. You could also ask students how many terms they expect in their product. • In #5, 6, and 7, students are required to factor. Review what the first steps are in factoring. Some students may have developed their own form of inspection to factor. Encourage them to use whatever method is easiest for them. If students still find a question challenging, ask them to identify whether the question is a difference of squares or perfect square trinomial. This will help narrow the possibilities for the solution. • Students requiring help with #8 should be coached in the meaning of *b* and *c* in the trinomial. You could prompt them by asking the following questions: – What is a perfect square? – Are 25 and 100 perfect squares? How do you know? – How could the square roots help you find *n*? • Since #9 is an excellent question for assessment *for* learning, use this question to identify any weaknesses in students' understanding of the concepts. • Use the discussion from #10 to assist in prompting for #16. Students should be able to see the visual link between the two questions. If not, reviewing the Investigate may be beneficial.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 2 Project If students complete #10 and 12, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 5–4 Chapter 5 Unit 2 Project** and have them finalize their answers. • For #10, encourage students to choose a difference of squares that is not too large for modelling. Visual learners will likely benefit from using algebra tiles. Some students may prefer grid paper. • In #12, students model the squaring of a binomial. Some students may find it easier to square a binomial algebraically first and then design their model based on their answer. If this is the case, ask them how they know their product is correct. Have students do several examples before generating their rule for squaring. Discuss this rule with the class. Have them write their rule, with examples, into their Foldable for future reference.
Assessment *as* Learning	
Create Connections Have all students complete #24 and 25.	• Both #24 and 25 allow students to summarize, in their own words, the key concepts presented in this lesson. Again, encourage students to include their own specific examples, models, and diagrams to help in the completion of their explanation.

5 Chapter 5 Review

***Mathematics 10*, pages 252–253**

Suggested Timing

60–90 min

Materials

- algebra tiles
- rulers

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–5 Section 5.1 Extra Practice
BLM 5–6 Section 5.2 Extra Practice
BLM 5–7 Section 5.3 Extra Practice
BLM 5–8 Section 5.4 Extra Practice

Planning Notes

Allow students to use algebra tiles as they complete the review. Encourage them to work independently on solving each question, but after each section, suggest that they check and discuss their solutions with a classmate.

Have students who are not confident discuss strategies with you or another classmate. Encourage them to refer to their chapter Foldable, summary notes, classroom-developed posters, worked examples, and previously completed questions in the related sections of the student resource.

Have students make a list of questions that they found easy, medium, and difficult. Tell them to use this list to help them prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 5–5 Section 5.1 Extra Practice**, **BLM 5–6 Section 5.2 Extra Practice**, **BLM 5–7 Section 5.3 Extra Practice**, and **BLM 5–8 Section 5.4 Extra Practice**.
- You may wish to hand out a copy of the student outcomes. Have students circle the outcomes that they still find challenging. Have student use their responses to direct them to various questions within the review.
- Before students begin the chapter review, have students create a question to illustrate each outcome. Encourage them to share their questions with a classmate.
- Encourage students to use algebra tiles or diagrams to help them see patterns and possible solutions for multiplying or factoring questions.
- Have students sketch and label diagrams involving area to help them visualize the problem and possible solutions.
- You may wish to allow students to use calculators for finding GCF and LCM.
- Encourage students to use tables, number factor lists, or calculators to find possible number relationships involved in factoring polynomials.
- Ensure that students check their answers to factoring by either substituting for the variable or multiplying the factors.
- Remind students that *factoring* means to find all factors of the given polynomial and writing it in factored form as a product of all factors.

Gifted

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

WWW Web Link

For a site that reviews factoring polynomials and gives a summary with examples of all algebraic methods for factoring polynomials, go to www.mhrmath10.ca.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 5 Review The Chapter 5 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the student resource.	• Encourage students to express their thinking by using algebra tiles or by drawing diagrams. You may wish to provide them with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**. • You may wish to have students work with a partner. • Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test. • For students who would like a broad selection of questions, have them complete two questions from each of #2, 3, 9–11, 13, and 14.

5 Chapter 5 Practice Test

Mathematics 10, **pages 254–255**

Suggested Timing

45–60 min

Materials

- algebra tiles
- rulers

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–9 Chapter 5 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions. Tell them that this test is for practice, and identifying which questions are most challenging serves to show them what concepts they need to revisit.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1, 3–10, 12, and 13.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1, 6	5.1	Example 1	✓ multiply binomials
#6, 7	5.1	Example 3	✓ perform operations on products of polynomials
#5	5.2	Example 1	✓ determine greatest common factors and least common multiples
#9	5.2	Example 2	✓ factor a polynomial using common factors
#9	5.2	Example 3	✓ factor polynomials using grouping
#8	5.3	Example 1	✓ factor trinomials of the form $ax^2 + bx + c$, $a = 1$
#3, 8, 10	5.3	Example 2	✓ factor trinomials of the form $ax^2 + bx + c$, $a \neq 1$
#10	5.3	Example 3	✓ apply factoring to solve problems
#2, 4, 8, 11, 13	5.4	Example 1	✓ factor a difference of squares
#8, 12	5.4	Example 2	✓ factor a perfect square trinomial

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 5 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Make algebra tiles available to students, or provide them with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**. • Have students use their responses on the practice test and work they have completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 5 Test After students complete the practice test, you may wish to use **BLM 5-9 Chapter 5 Test** as a summative assessment.	• Consider allowing students to use algebra tiles or area models. • Consider allowing students to use their Foldable.

UNIT

2

Unit 2 Project

Mathematics 10, **page 256**

Suggested Timing

60–90 min

Materials

- coloured pencils and markers, coloured paper, scissors, glue, and other materials for artwork

Blackline Masters

Master 1 Project Rubric
BLM U2–3 Unit 2 Project Final Report

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:

- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN2 Demonstrate an understanding of irrational numbers by:

- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Planning Notes

Begin the class by having students discuss examples throughout the unit of how artists use mathematics in their work. Prompt them to discuss how they might use their understanding of irrational numbers, exponents, and polynomials to design their own artwork. Emphasize that the artwork can be historical or contemporary. It might involve nature, stylized models, or another idea of their choice that relates to the unit. List and post ideas from the class discussion and encourage students to use the ideas as a springboard to develop their own artwork.

Encourage students to think about what format they might use to present their work. Students who are interested in using multimedia (e.g., images and text, images and audio) to publish their work online may find the related Web Link at the end of this section helpful.

Have students complete their unit project by creating their artwork and developing a presentation. Review the expectations for the presentation outlined on page 256 in the student resource. Clarify with the class that the final part of the project involves a brief report that describes how the mathematics from Unit 2 specifically relates to the work of art.

Emphasize that students are being encouraged to use their imagination to create their artwork, and then to describe how the mathematics skills they have acquired relate to their artwork.

Give students time to review the contents of their project portfolio and ensure that they have completed all required components for their final report and presentation.

WWW Web Link

For information about how to develop a multimedia presentation that can be created in a web tool using images, audio, and/or video, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 2 Project This unit project gives students an opportunity to apply and demonstrate the concepts, skills, and processes learned in Unit 2. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on the Unit 2 project.	• You may wish to have students use **BLM U2–3 Unit 2 Project Final Report**, which provides a checklist for students to identify where in their project they demonstrate the skills, concepts, and processes explored in Unit 2. • Reviewing **Master 1 Project Rubric** with students will help clarify the expectations and the scoring. It is recommended to review the scoring rubric at the beginning of the unit, as well as intermittently throughout the unit, to refresh students about the project assessment.

The Specific Level Notes below provide suggestions for using **Master 1 Project Rubric** to assess student work on the Unit 2 project.

Score/Level	Specific Level Notes
5 (Standard of Excellence)	• provides a complete and correct response with clear and concise communication; may include a minor error that does not affect the understanding of the overall project; may include weak communication in no more than one calculation
4 (Above Acceptable)	Demonstrates *one* of the following: • shows a thorough understanding of exponents and polynomials by providing a complete response to all parts of the project, with possibly weak or missing justification in no more than two calculations; includes good communication that addresses the relationships among radicals, irrational numbers, exponents, and polynomials; demonstrates a clear understanding of radicals to analyse the golden ratio • provides a complete response with one error that is carried through correctly (e.g., incorrectly reads the dimensions from a polynomial model but carries through the multiplication of the factors correctly); includes good communication that addresses how the concepts relate to art • provides a response that addresses all parts of the project but that is difficult to follow and that lacks organization; does not provide support for how mathematics is incorporated into art; includes good communication
3 (Meets Acceptable)	Demonstrates *one* of the following: • makes a correct start to all sections of the project • correctly completes square roots and cube roots and explains their connection to geometric representations; with some errors, demonstrates a basic understanding of the golden ration and how it applies to a rectangle and related problems; uses models and algebraic expressions for combining like terms of polynomials; models the relationship between multiplication and factoring; with some possible errors, models and explains the difference of squares and squaring a binomial; includes good communication with some connections • provides answers to all parts without supporting work or justification
2 (Below Acceptable)	• makes a start to various sections of the project; provides some correct links • is able to calculate square roots and cube roots with some difficulty; links them to geometric representations • draws and determines the ratio for the golden rectangle; has some success in applying or explaining whether other objects model the golden ratio • demonstrates the ability to solve radical numbers; is able to solve irrational numbers with limited success • demonstrates combining like terms of polynomials with a model/diagram and algebraically • models multiplication of polynomials with relative accuracy • attempts to use models to explain the difference of squares or squaring a binomial, with some success • includes some communication
1 (Beginning)	• makes a start to various sections of the project but is unable to carry through or link concepts together • calculates square roots or cube roots but does not link them to geometric representations • draws and determines the ratio for the golden rectangle; is unable to apply or explain whether other objects model the golden ratio • demonstrates the ability to solve radical numbers but has difficulty with irrational numbers • when combining like terms, demonstrates addition *or* subtraction of polynomials with limited success • attempts to model multiplication of polynomials with limited success; may not attempt multiplication • attempts to explain the difference of squares or squaring a binomial, with little or no success • includes little or no communication

UNIT 2

Review and Test

Mathematics 10, pages 256–261

Suggested Timing

60–90 min

Materials

- algebra tiles
- ruler

Planning Notes

Have students work independently to complete the review and then compare their solutions with a classmate. Alternatively, assign the Unit 2 Review to reinforce the concepts, skills, and processes learned so far. If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their notes in each chapter Foldable and then to the specific section in the student resource and/or their notebook. Once they have found a suitable strategy, have students add it to the appropriate section of their chapter Foldable. Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the unit test.

Meeting Student Needs

- Encourage students to draw and label diagrams, when appropriate.
- Encourage students to use their chapter Foldables and to add new notes if they wish.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 2 Review The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students review their notes from each Foldable and the tests from each chapter to identify items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section they are having difficulty with.
Assessment *of* Learning	
Unit 2 Test After students complete the unit review, you may wish to use the unit test on pages 260 and 261 as a summative assessment.	• Consider allowing students to use their Foldables.

UNIT 3

Relations and Functions

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF2 Demonstrate an understanding of relations and functions.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF4 Describe and represent linear relations, using:
- words
- ordered pairs
- tables of values
- graphs
- equations.

RF8 Represent a linear function, using function notation.

General Outcome

Develop algebraic reasoning and number sense.

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determing the:
- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Mathematics 10, pages 262–265

Suggested Timing

30–40 min

Blackline Masters

BLM U3–1 Unit 3 Project
BLM U3–2 Unit 3 Project Checklist

What's Ahead

In Unit 3, students graph functions and relations in a variety of ways, including with technology. They learn to distinguish between a function and a relation and how to determine the domain and range. They also study the concept of the slope of a line. This unit also introduces students to three different forms of writing linear equations: slope-intercept form, general form, and slope-point form. Students explore y-intercepts, slopes, and x-intercepts, and use them to graph linear equations. They convert linear equations among the three forms presented and use the slope-point form to write the equation of a line given a graph. Finally, students determine the equation of a line that is parallel or perpendicular to a given line using a specified point that lies on the line.

Planning Notes

Introduce Unit 3 by pointing out the relations and functions organizer on page 262 of the student resource. This organizer shows how the topics in this unit—relations and functions, linear relations, and linear equations and graphs—are related. The organizer is repeated at the beginning of each chapter and is shaded to show which topics are covered in that particular chapter.

The Looking Ahead box at the bottom of page 263 identifies the types of problems students will solve throughout the unit. You may wish to reactivate students' knowledge of these topics.

Unit 3 Project

The Unit 3 project focuses on the application of relations and funtions to forensic archaeology. The project is continuous in nature and is divided between Chapters 6 and 7.

Introduce the Unit 3 project by reading and discussing the introductory notes on page 264 of the student resource as a class. The project is related to a fictional occurrence during the Klondike Gold Rush. When discussing the gold rush with students, please point out that, as any economic boom, there were advantages and disadvantages for all groups of people.

Consider distributing **BLM U3–1 Unit 3 Project** to inform students about how the project develops throughout the unit. This master provides an overview of the project as well as the requirements for completing the Unit 3 project.

You may wish to point out the questions related to the Unit 3 project that are indicated throughout Chapters 6 and 7 with a project logo. Note that these questions are not mandatory but are recommended because they provide some of the background and research needed to complete the Unit 3 project. The questions are also available on masters, one for each chapter. You may decide to use these masters to create a student booklet and have students record their finalized answers in the booklet either after they have completed their in-class work, during assigned project work time, or in conjunction with chapter assignments. Alternatively, you may wish to provide students with **BLM U3–2 Unit 3 Project Checklist**, which lists all of the related questions for each chapter. Students can use the checklist to monitor their progress and prepare their presentation and report. Have students store all the work for the Unit 3 project in a portfolio.

Career Connection

Direct a discussion about careers that are related to relations and functions. You may wish to have students read the Unit 3 introduction on page 262 to give them some ideas. Invite them to speculate in which careers it would be beneficial to have an understanding of relationships between quantities. Also, have them discuss in which jobs it might be helpful to use graphs. An example might be a city planner, who works with community groups, government agencies, and others to make plans for the growth and development of a city. These plans may include schools, housing, roads, malls, and office buildings. A city planner must consider factors such as noise, traffic congestion, environmental protection, and local laws and regulations. Being a city planner requires the ability to consider many variables in order to best meet the projected needs of a growing population. You may wish make a list of all career ideas that students contribute and then add to the list as you work through the unit.

6 Linear Relations and Functions

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF2 Demonstrate an understanding of relations and functions.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF4 Describe and represent linear relations, using:
- words
- ordered pairs
- tables of values
- graphs
- equations.

RF8 Represent a linear function, using function notation.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
6.1	✓ describe a possible situation for a graph
	✓ sketch a graph for a given situation
6.2	✓ determine if a relation is linear
	✓ represent linear relations in a variety of ways
	✓ explain why data points should or should not be connected
	✓ identify the dependent and independent variables in a relation
6.3	✓ understand the meaning of domain and range
	✓ express domain and range in a variety of ways
6.4	✓ sort relations into functions and non-functions
	✓ use notation specifically designed for functions
	✓ graph linear functions
6.5	✓ determine the slope of a line
	✓ use slope to draw lines
	✓ understand slope as a rate of change
	✓ solve problems involving slope

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 6–1 Chapter 6 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 266 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about linear relations. You might provide the following prompts: • How might linear relations apply to real life? • What are some examples of linear relations? • What are some ways you can represent linear relations? • How do you show a linear relation graphically?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 6–2 Chapter 6 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 6 Foldable As students work on each section in Chapter 6, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter. • Encourage students to write examples of their own into their notebooks or math portfolios. They should have at least one example for each method that is covered in the chapter.
Assessment *for* Learning	
BLM 6–3 Chapter 6 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 6 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 30–40 min (TR page 211)			BLM 6–1 Chapter 6 Self-Assessment BLM 6–2 Chapter 6 Prerequisite Skills BLM 6–4 Chapter 6 Unit 3 Project BLM U3–1 Unit 3 Project BLM U3–2 Unit 3 Project Checklist		TR page 210 Chapter 6 Foldable, TR page 210	TR page 210	
6.1 Graphs of Relations • 100–120 min (TR page 213)	Students should be familiar with • reading information from a graph • the relationship between speed, distance, and time • describing the relationship between the variables of a graph	• ruler • grid paper • CBL interface with a motion detector • computer or graphing calculator with appropriate software	BLM 6–3 Chapter 6 Warm-Up BLM 6–5 Section 6.1 Extra Practice	**Essential:** #1–3, 6, 8, 15–17 **Typical:** #1–3, 5, 8–10 **Extension/Enrichment:** #3, 4, 11–15, 17	TR pages 215, 220 Chapter 6 Foldable, TR page 210	TR pages 217, 218, 220	
6.2 Linear Relations • 100–120 min (TR page 221)	Students should be familiar with • creating a table of values • plotting points and ordered pairs • graphing linear relations • describing patterns in a graph • writing linear relations • interpolation and extrapolation • substituting values into an equation • determining the degree of a polynomial	• measuring tape • ruler • grid paper or graphing technology	BLM 6–3 Chapter 6 Warm-Up BLM 6–6 Section 6.2 Extra Practice	**Essential:** #1–3, 5–8, 13 **Typical:** #1–3, 5, 7, 9, 13, 14 **Extension/Enrichment:** #4, 10–14	TR pages 223, 227 Chapter 6 Foldable, TR page 210	TR pages 225, 227	
6.3 Domain and Range • 120–180 min (TR page 228)	Students should be familiar with • identifying independent and dependent variables • describing the relationship between variables • analysing and interpreting information from a graph • describing a pattern • working with number lines • using technology to create a graph	• ruler • graphing calculator or spreadsheet software • grid paper	BLM 6–3 Chapter 6 Warm-Up BLM 6–7 Section 6.3 Extra Practice TM 6–1 How to Do Page 300 Example 4 Using TI-83/84 TM 6–2 How to Do Page 300 Example 4 Using Microsoft® Excel	**Essential:** #1–4, 6–8, 12 **Typical:** #1–6, 7 or 8, 12, 13 **Extension/Enrichment:** #8–13	TR pages 230, 234 Chapter 6 Foldable, TR page 210	TR pages 233, 234	
6.4 Functions • 180–240 min (TR page 235)	Students should be familiar with • reading information from a graph • substituting values into an equation • solving linear equations • adding, subtracting, and multiplying polynomials • interpolation and extrapolation	• grid paper • ruler	BLM 6–3 Chapter 6 Warm-Up BLM 6–4 Chapter 6 Unit 3 Project BLM 6–8 Section 6.4 Extra Practice	**Essential:** #1–4, 6–8, 9, 16, 17 **Typical:** #1–7, 9, 10, 16, 17 **Extension/Enrichment:** #9, 11–15, 18	TR pages 236, 241 Chapter 6 Foldable, TR page 210	TR pages 238, 241	
6.5 Slope • 180–240 min (TR page 242)	Students should be familiar with • ratios • solving proportions • equivalent fractions • operations with integers • dividing zero by an integer and dividing an integer by zero • plotting points and ordered pairs • the relationship between speed, distance, and time	• grid paper • plastic transparent ruler • toothpick • tape • ruler • one or more small boxes	BLM 6–3 Chapter 6 Warm-Up BLM 6–9 Section 6.5 Extra Practice	**Essential:** #1, 2, 3a)–d), 4, 5, 7, 8, 17, 19 **Typical:** #1–5, two of 8–12, 17–19 **Extension/Enrichment:** #9, 13–19	TR pages 243, 249 Chapter 6 Foldable, TR page 210	TR pages 246, 247, 249	
Chapter 6 Review • 60–90 min (TR page 250)		• grid paper • ruler • graphing calculator	BLM 6–5 Section 6.1 Extra Practice BLM 6–6 Section 6.2 Extra Practice BLM 6–7 Section 6.3 Extra Practice BLM 6–8 Section 6.4 Extra Practice BLM 6–9 Section 6.5 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 6 Foldable, TR page 210	TR page 250	
Chapter 6 Practice Test • 50–60 min (TR page 251)		• grid paper • ruler • graphing calculator	BLM 6–10 Chapter 6 Test BLM 6–11 Chapter 6 BLM Answers	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1–8, 10	TR page 252		TR page 252 BLM 6–10 Chapter 6 Test

Linear Relations and Functions

Opener

***Mathematics 10*, pages 266–267**

Suggested Timing

30–40 min

Materials

- sheets of 11 × 17 paper
- stapler

Blackline Masters

BLM 6–2 Chapter 6 Prerequisite Skills
BLM 6–4 Chapter 6 Unit 3 Project
BLM U3–2 Unit 3 Project Checklist

Key Terms

relation	linear relation
non-linear relation	discrete data
continuous data	independent variable
dependent variable	domain
range	function
function notation	vertical line test
slope	

What's Ahead

In this chapter, students learn about linear functions and relations. Students graph functions and relations in a variety of ways, including with technology. They learn to distinguish between a function and a relation and how to determine the domain and range of a function or relation. Students express domains and ranges in different formats, including interval notation. For linear functions, students use function notation to express and work with the function. Also, for linear functions and relations, they explore the concept of the slope of a line, including lines that are horizontal or vertical.

Planning Notes

The chapter opener asks students to consider the interpretation and analysis of data. You could begin this chapter by having students consider the amount of data available to them and ways to effectively find, use, and organize data. Ask them to consider their mathematical experience so far and to brainstorm the methods that they know to organize data, represent data, and use data. In particular, you may wish to have them articulate how numerical data can be used to make predictions or decisions in a situation.

Unit Project

The Unit 3 Project focuses on forensics, and how mathematics might be useful in forensic science. You might take the opportunity to discuss the Unit 4 project described in the Unit 4 opener if you have not done so already. See TR page 206. In this chapter, there are individual questions related to the unit project that will help students as they work on the project. You will find these related questions in section 6.4 of the student resource. While students may not need to complete these questions to successfully complete the project, the questions are intended to be helpful and it is recommended that students complete them when possible.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 267 and how it might be used to summarize Chapter 6. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

The Foldable is designed to provide students a place to write notes as well as collect, diagram, and model different lines, slopes, and function applications. Each section of the Foldable has a blank page facing a page with a pocket. There are exactly enough pages for each of the five sections in the chapter. Encourage students to write terms and explanations on the blank page and insert examples, quizzes, and any information related to each section into the pocket that faces it. The middle section for section 6.3 will be the only page to have two facing pockets. Encourage students to put the title What I Need to Work On on the back page to keep track of difficulties they have. As students progress through the chapter, provide time for them to keep track of what they need to work on, which they can record on the back of the Foldable. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 6–2 Chapter 6 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- You may wish to post the student learning outcomes and discuss the entire chapter prior to beginning. Involve students in a discussion about the learning outcome terms they do not understand.
- Post three or four linear graphs found on the Internet, withholding what the graphs represent. Ask students to suggest what the graphs might illustrate. At the end of the discussion, reveal what the graphs actually represent. Discuss how close students' ideas were.
- Encourage students to create a bookmark that lists all the Key Terms of the chapter. Discuss the terms prior to beginning the chapter.
- Relate the topic of this chapter to something students are familiar with, such as global warming. You might ask the following questions:
  - What would happen if all the polar ice melted?
  - How does this affect Northern communities?
  - Do you need math to understand these changes?
- Ask students which TV programs about forensics they watch. Ask them to describe what they do.
- Consider having students staple a copy of **BLM U3–2 Unit 3 Project Checklist** to the back of the Foldable. This master provides a list of all the related questions for the Unit 3 project. Students can use it to keep track of the questions they have completed.
- Some students may benefit from completing all unit project questions.
- If you have students do the Unit 3 project questions, consider offering them the option of working on these alone or with a partner.
- **BLM 6–4 Chapter 6 Unit 3 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 3 project analysis.

ELL

- Encourage students to create their own vocabulary dictionary for the Key Terms using written descriptions, examples, and diagrams.

Enrichment

- Challenge students to create a real-life context for a graph of a vertical line involving two variables. Have them explain what a vertical line implies on a time-versus-distance graph for a car. (Examples: On a price-versus-sales graph, a vertical line means that the price of an item remained constant while the sales increased. A vertical line on a time-versus-distance graph means that the car was in more than one place at the same time.)

Gifted

- Present students with the following scenario: Suppose a population of endangered falcons is declining according to a linear relation. Ask students the following questions:
  - What might a scientist predict from the graph?
  - How might a scientist use the graph to find the cause of the decline?

  (Example: Scientists might predict when extinction might occur. Working backward to the beginning of the decline and seeing what might have occurred in that time frame could identify the cause of the decline.)

Career Connection

The career connection in this chapter is forensics. Many students will be familiar with forensic science, especially as it is represented in the media and popular culture. If you have Internet access, you may wish to have students spend a few minutes searching to see how mathematics can be an important part of the work of a forensic scientist. If Internet access is not available, choose a few mathematical topics and ask students to suggest how they could be used in forensics. For instance, you could ask the following questions:

- What are some ways that geometry could be important to a forensic specialist?
- What is a situation in which a forensic scientist would use a graph?
- When would an equation be used in an investigation?

Graphs of Relations

6.1

***Mathematics 10*, pages 268–278**

Suggested Timing

100–120 min

Materials

- ruler
- grid paper
- CBL interface with a motion detector
- computer or graphing calculator with appropriate software

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–5 Section 6.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

RF1 Interpret and explain the relationships among data, graphs and situations.
RF4 Describe and represent linear relations, using:
- words
- ordered pairs
- tables of values
- graphs
- equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 6, 8, 15–17
Typical	#1–3, 5, 8–10
Extension/Enrichment	#3, 4, 11–15, 17

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Students get a sense for reading and interpreting general graphs by comparing the goalie stats. Since there is no scale given on each axis, students will need to realize that, whatever the scale, it is the same for each goalie, so they should look at the “trend.” Ask questions to prompt their thinking:
- Why are the points in each graph not connected? (Do not use the word *discrete*. Just get students thinking about something they will explore further in section 6.2.)
- Who lets in more goals per game?
- Who faces more shots per game?
- Can you determine which goalie has the better goals-against average? Explain.
- Can you determine which goalie has the better save percent? Explain.

Encourage a debate among students by trying to get them to decide who the better goalie is.

Investigate Describing and Sketching Graphs

You may wish to discuss the results of #1 and 2 before students proceed with the remainder of the investigation. After the pairs of students have finished #1, select a small number of students to report on their responses. Ask if other pairs of students have different responses. In a similar fashion, after pairs have finished #2, you may wish to choose a small number of students to report on and defend their results.

As the groups work on #3, monitor their progress. Watch to see that the correct quantities are placed on the x-axis and y-axis. You may want to ask the following questions:
- Why is time placed on the horizontal axis?
- Does the placement of the quantities on the axes fit with what you have done in other classes, for example, science?

You may want to ask them to review their graph and make sure that it reflects each part of the written description. Allow time for all groups of students to finish #3 before they pass their work to another group. When they examine the work of others, you may want to ask if the scenario they have been given provides enough detail that they can tell which piece of the graph each part of the description pertains to. You can use students' scenarios to check for understanding.

In #4, it is inevitable that groups will have different graphs. As students find discrepancies, ask the following questions:

- Are the differences in the graphs evidence that one group made an error?
- Is it possible that both graphs are correct? If so, how can they be different?
- Is it all right for a different group's graph to show a different speed or length of time than yours for the same part of Connor's trip?

As students are already working in small groups, you may wish to have them remain in those groups for the Reflect and Respond in #5. After each group has had a few minutes to decide on their responses, choose a representative from each group to report to the whole class.

Meeting Student Needs

- Have students create two other scenarios that could be represented by the graphs in the opener.
- Have students complete #1 and 2, assigning one or the other to pairs of students. Each pair can make a brief presentation outlining their responses to the question assigned. Students might complete #3 in small groups and then work as an entire class on #4 and 5. Ensure that all students have a chance to write, sketch, and present.
- For #3, if your community does not have paved roads, skateboarding may not be an activity your students engage in. Have students go online to research what an ollie looks like. You may wish to have them look at other aspects of skateboarding as well. See the related Web Link that follows in this Teacher's Resource.
- If you discussed global warming in the chapter opener, relate back to it in the Reflect and Respond by asking how students would graph ice melt and temperature, or ice melt and water level.

ELL

- Some students may not be familiar with the term *goalie* or perhaps even the sport of hockey. Show them pictures or a video of the game and indicate the goalie while you describe what the goalie's role is in the game. Alternatively, have a volunteer describe the game and what a goalie does.
- Have students who are not familiar with skateboarding view a video of skateboarding and how to do an ollie. See the related Web Link that follows in this Teacher's Resource.
- For the Investigate, you may wish to have students pair up with another student who can assist them with the language and explain the steps.

Common Errors

- Some students may create graphs that do not completely agree with the scenario presented. Often one or more of the statements in the scenario is not represented on the graph.

Rx Have students assign labels A, B, C, etc., to each written statement or phrase, then place that label on the corresponding segment of their graph to ensure that the graph is complete.

- Students may not obey all the conventions for making graphs.

Rx As you watch them work, remind students to give their graph a title and label the axes with titles, correct units, and variables.

Web Link

To find out more about skateboarding and ollies, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Describing and Sketching Graphs

1. a) Example: From 0 to A: Climbing; the distance from the base camp increases. AB: Resting; the distance from base camp does not change. BC: Climbing; the distance from the base camp increases. CD: Descending; the distance from the base camp decreases. DE: Resting; the distance from base camp does not change. EF: Climbing; the distance from the base camp increases. FG: Resting; the distance from base camp does not change.

b) Example: Yes. The climber may have been travelling across a flat plateau during the times indicated by the segments AB, DE, and FG, and the climber may have slid downward during the time indicated by the segment CD.

c) Use a steeper slope. A steeper slope indicates that more metres are climbed each minute.

d) Example: Add a line that goes from point G down to $d = 0$.

Answers

Investigate Describing and Sketching Graphs

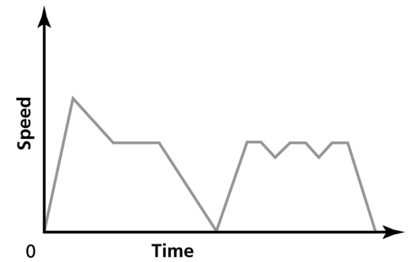
2. a) Graph C; the hot chocolate gradually cools. Vertical axis: temperature, horizontal axis: time

b) Graph B; the car speeds up until it reaches a specific speed, and then it maintains that speed. Vertical axis: speed, horizontal axis: time

c) Graph D; as time passes, the amount of distance the hiker covers also increases. Vertical axis: distance, horizontal axis: time

d) Graph A; the height increases as the ball goes up, until it reaches a maximum height, and then it goes back down. Vertical axis: height above ground, horizontal axis: time

3. Example:



5. a) a straight line with a slope

b) a straight line with a steep slope

c) a horizontal or vertical line

d) a curved line

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to complete this activity by having students work in pairs, as it may promote more discussion among students, especially when the graphs differ. • It would benefit all students to discuss #2 and 3 as a class and have students label their examples to identify what is happening in each segment. • Have students who are having difficulty with #5 use their previous labels as examples to compare to. This should make it easier for them to see the similarities between the existing graphs and what is asked for in #5. • Some students may find it difficult to identify changing quantities without a specific reference. In this case, suggest that for #5 they use a distance-versus-time graph to represent each part of #5.

Link the Ideas

You may wish to ask students to look at the graph they created in light of the information in Link the Ideas. Ask the following questions:

- Are there places on your graph that have constant rates of change?
- Which section of your graph has the highest rate of change? the lowest?
- How can you tell when your graph shows a non-constant rate of change?
- Are there sections of your graph that show no change at all? How can you tell?

Example 1

Explain to students that the letters represent where the various line segments on the graph connect. Also, ensure students note that the graph has a smooth curve. This will help them to understand the rate of change for CD, for example, which is described as being slow at first.

When students consider the example and solution, be sure that they address the questions in the margin. The second question is likely the most challenging. To help students see the answer, you could ask them to draw a point, then pick a distance and draw all the points that are that distance from the original point. They will see that the result is a circle. Ask students guiding questions:

- Why does the distance increase slowly at first?
- What kind of path would allow this?
- How do you know the rate is constant?

You may wish to have students complete the Your Turn by first writing a description, then exchanging with another student to compare. After all students have had an opportunity to complete and edit their work, you could select a number of students to present their descriptions to the whole class. Remind students that the vertical axis is the speed of the boat. To assist students in their thinking, you may wish to ask students to think of another way to describe increasing speed (acceleration).

Example 2

Students are likely to know that bacteria grow by doubling and thus increase in number quite quickly. Before students look at the solution, you may wish to ask a few questions to focus their thinking:

- How do bacteria grow?
- Is bacterial growth an example of a constant rate of change?
- Is it important to your response that the food supply is limited?

You might also wish to ask the following questions:

- If bacteria are given unlimited space and a continuous food supply, what do you think their growth would be?
- Is there a graph above that matches this scenario?

For the Your Turn, you might wish to ask the following questions:

- Are there periods in a person's life when they gain height more rapidly?
- Are there times in a person's life when they do not gain height (they stop growing)?
- Why does each graph dip back down after a certain number of years?

You may wish to have students write their answer on a small piece of paper and hold it up so that only you can see it. This will enable you to check the understanding of the class quickly. If you have a student response system with your interactive whiteboard, you could use it for the same purpose. If time allows, you might ask students to write a scenario that fits each of the incorrect responses for the Your Turn. This activity should help to solidify understanding.

Example 3

Lead students to see that each part of the description of Josaphee's trip is reflected in a segment on the graph. Students should understand that each time there is a change in the situation, the graph must change as well. Ask students why line segments appear on the graph instead of curves.

You may wish to approach the Your Turn as a class. Have a volunteer sketch a graph on the board that represents the given situation. Then, have the class decide if the graph could be improved. If so, another student comes forward to make the change. The class again decides if the graph could be improved, and so on. In this activity, it is important that students understand that the mathematics is being critiqued, not the individuals presenting it. Before they begin, encourage students to think through the scenario and determine appropriate slopes for lines and appropriate time frames while applying their understanding of how to show quantities changing.

Key Ideas

The Key Ideas represent the concepts that students need to understand upon completion of this section. Allow time for students to incorporate the Key Ideas into the Foldable they created for this chapter or in their math journal. In either case, students should put the Key Ideas in their own words. As horizontal lines are a special case, you may wish to have students place some emphasis on explaining why a horizontal line represents a quantity that is not changing, or a rate of change of zero. Encourage students to give an example, either from the section or of their own, for each type of change.

Meeting Student Needs

- Depending on the makeup of your class, you may wish to change the order in which the Examples in this section are covered.
- Consider discussing with students why the endpoints of each interval in Example 1 are curves rather than points.
- Students may need to be reminded of the meaning of *constant rate of change*. Along a straight line, the rate of change is constant. The moment the line changes steepness, the rate of change also changes. The steeper the line, the greater the rate of change is.
- Explain to students that *constant distance* means the same distance. If you are moving and positioned a constant distance away from your school, you are perhaps walking around the school but staying the same distance away from the school.
- Request that students find two graphs in newspapers, magazines, or online. For each graph, have students identify and interpret what is represented. Ask them the following questions:
  - Are there any discrepancies in the graph?
  - Is any of the information distorted by the choice of units used on the axes?

  Have students mount the two graphs and their interpretation on construction paper. Then, display them in the classroom or hallway. You may choose to give students two or three days to complete the activity at home.
- Some students may not be familiar with wakeboarding. To show students what

wakeboarding looks like, have them research the sport. See the related Web Link that follows in this Teacher's Resource.

- Encourage students to study distance-versus-time graphs in detail, as this will be a concept studied again in physics. Display the graph from Example 1. Trace the graph as you discuss each segment. Some students may need another example before working through the Your Turn.
- You may want to sketch a model of the scenario in Example 1. Sketch a lake, put an X where the boarder starts, and sketch a sample path on the lake based on the distance and time information given on the graph.
- Refer to the list of definitions created at the beginning of the chapter. You may need to refresh students' understanding of terms such as *domain* and *range*.
- Discuss the step-by-step instructions for creating the distance-versus-time graph for Example 3.
- With the class, make a poster outlining the Key Ideas for this section. A small illustration for each situation would be helpful for students.

ELL

- You may need to explain such terms as *steepness*, *gradual*, *bacteria*, *limited*, *continued*, *initially*, *rapid*, *jogs*, and *curves*. Use a combination of words, visuals, and examples.
- Encourage students to add *constant* and *rate of change* to their vocabulary dictionary, Foldable, or other graphic organizer that they may be using.
- For students unfamiliar with wakeboarding, have them watch some video footage of the sport. See the related Web Link that follows.

Common Errors

- In Example 1, some students may think the horizontal line segments mean that the boat has stopped.

R_x Remind them that the quantities being compared are speed and time.

WWW Web Link

To see some online video footage of wakeboarding, go to www.mhrmath10.ca and follow the links.

Answers

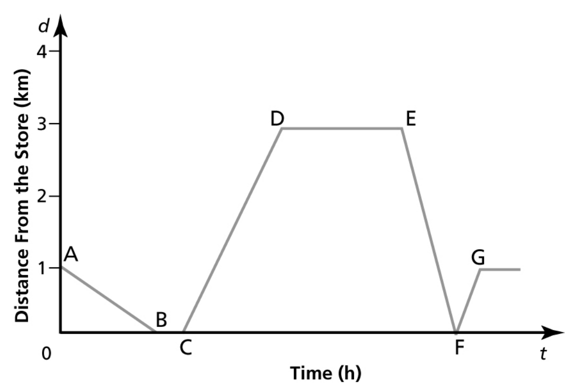
Example 1: Your Turn

The boat accelerates at a constant rate. It then travels at a constant speed. The boat quickly slows down to a stop. It accelerates, slightly slower than before, at a constant rate. The boat travels at a constant speed, the same speed at which it travelled earlier. Finally, it gradually decelerates to a stop.

Example 2: Your Turn

Example: Graph B because height increases as one gets older. Then, when one reaches physical maturity, height remains the same until the senior years when height decreases slightly.

Example 3: Your Turn



AB: Josaphee walks from her house to the store; BC: Josaphee is at the store; CD: Josaphee walks to her friend's house; DE: Josaphee is at her friend's house; EF: Josaphee runs from her friend's house to the store; FG: Josaphee continues running to her house.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner of similar ability. • Some students may need to be coached through the first three segments of the graph. It may help them to describe the segments if they are labelled individually. • Use the last three segments for students to work on alone and confirm they have grasped the concept. • Some students may benefit from a mock scenario that they can use in their description.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to have a discussion of the physiological changes that occur with aging. • It may benefit some learners to first describe what is happening in the graph orally. This would provide an opportunity to clarify any misunderstandings in their interpretations of segments of the graph. • Encourage students to always refer back to the labels on the axes when they are describing a graph.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner of similar ability. • Listen to the discussion that students have regarding faster/slower speeds as well as distance covered. Ensure students understand the difference between a positively sloped segment and a negatively sloped one. • If several students are having difficulty, as a class write on the board a description of what Josaphee is doing in relation to the store. Take each description individually and ask students for feedback on what that segment might look like on the graph. Work on a few together and then allow partners to complete the description.

Check Your Understanding

Practise

For #1, without using slope terminology, students state whether the slope of a segment of the graph is positive, negative, or zero. Students might refer to Example 1 if they need help with this question.

In #2, students match graphs to given scenarios, similar to their work in Example 2.

As students work on #4, you may wish to ask the following questions:
- Does your scenario contain a description for each portion of the graph?
- Does your scenario correctly explain the changes and rates of change in the graph?

Apply

In #5, students create a graph to fit a scenario. As usual, there is some ambiguity in the scenario, in particular about how long Paul must wait while he transfers trains. You may wish to ask students to stipulate any assumptions they make in sketching the graph, especially about this waiting time. This is an opportunity for students to see that many correct graphs can exist to model this situation, depending on the assumptions made.

For #8, students create a graph to show their own daily water consumption. Students will need to think about the problem before beginning the graph. You may want to direct their thinking with a couple of guiding questions:
- What will be the scale for your vertical axis?
- How many of these activities are typical for you?

These prompts should help students determine the maximum value necessary on the vertical axis. Since the topic of #8 is related to the Unit 4 project, you may want to have students refer back to this question when they work on the project.

Answers may vary for #9, as students may overlook a particular section of the ride. You may want to ask students to consider the various rates of change experienced on the roller coaster, when those changes should be represented by line segments, and when they should be represented by curves.

For #10, some students may make the mistake of sketching the graphs independently of each other. Students need to understand that for the parts of the journey where more than one person is present, those portions of the respective graphs must be identical.

For #11, ask students if they expect each graph that they sketch to be identical. Discuss that despite the times and speeds specified, there may be room for small variances in students' work, depending on the assumptions that are made about the nature of travel. For example,
- Are the changes in speed uniform?
- Does the skydiver's body position remain the same?

Extend

For #12, you may need to emphasize that these are rates of change, not numbers of births and deaths, so the changes indicated by the graph are more complicated than students may first think.

In #13, the term radioactive half-life refers to the time required for half the atomic nuclei of a radioactive sample to decay (change to a stable nuclear state).

In #14, students encounter changes in a graph with an undefined slope. Note that students are not expected to use this terminology at this point.

Create Connections

Students will need some time to complete #16. You might have them work with a partner.

Question #17 is a Mini Lab. The size of the group for the activity will depend on your access to the CBL technology. It is possible for students to complete the activity in groups as small as three and as large as the whole class. Note that students will gain most from this activity if given the opportunity to walk in front of the CBL to try to produce the desired graph.

Meeting Student Needs

- Provide **BLM 6–5 Section 6.1 Extra Practice** to students who would benefit from more practice.
- Ask students for at least five main ideas from the section. Encourage them to use the terminology and concepts from the section. Allow them to include diagrams to illustrate specific information.
- Some students may benefit from a discussion about the distances described in #10. One student could create a table of values while another student could move manipulatives from one location to the next on a diagram similar to the one given.
- For #14, explain to students that for them to describe the *rate scheme*, they need to determine how much a particular item or service costs for a specific time (e.g., 1 h, 1 day). Also, discuss the meaning of open and closed circles on the graph.
- For #15a), discuss "negative time" with students. Many will not realize that the graph shows time moving backward.

ELL

- Some students may not be familiar with terms such as *scenario*, *profit*, *distributing*, *recorded*, *complete*, *daycare*, *skydiver*, *parachute*, *demography*, *mortality, half life*, *substance*, *decay*, *vehicle*, *rate scheme*, and *impossible*. Assist them to understand by using descriptions, examples, and visuals.
- For #2, it may be helpful to provide diagrams or photos to students in addition to the descriptions of the different scenarios in part a).
- For #5, have students work with a partner who can guide them through the question by indicating Paul's journey on the diagram and explaining what happens during the journey.
- For #6, make available pictures that will show the meaning of *vinyl albums*, *cassette tapes*, *compact discs*, and *digital downloads*.
- For #7, some students may not be familiar with *snowmobiling*. Show pictures while you describe this activity.
- For #8, students may not be familiar with some of the activities listed under Water Use in the table. Invite them to pair up with another student and search for each of these activities on the Internet.
- For #9, invite students to share their experiences with roller coasters. This will help English language learners to learn the term and may encourage them to share their own stories of riding roller coasters or other similar rides, if they have done so. Also, have students watch a video of a roller coaster ride. See the Web Link in the student resource.
- For #17, have students work in a group in which others can help them with the language and can explain the steps.

Enrichment

- Give students the following scenario: When people consume food and drink that contain sugars, their blood sugar level rises. When they exercise, their muscles burn sugar, and so their blood sugar falls. Encourage students to track for 24 h their consumption of food and drink that may increase their blood sugar and also their physical activities that may burn sugar. Have them create a time-versus–blood sugar level graph. Ask students to comment on the slope of the blood sugar level line. (Graphs should show a positive slope soon after consuming sugars, and a negative slope as the result of physical activity.)

Gifted

- Present students with the following information: A golf ball follows a curve as it travels through the air. The graph of height versus distance for the ball is the same as the curve. Suppose the ball travelled 20 yd horizontally every second and reached a maximum height of 50 yd. Have students sketch a graph of height versus distance for a golf ball that hits the ground 100 yd from where it was struck. Ask students to place discrete points every second of travel and then join those points with straight lines. Have them comment on the slopes of those lines. Then, ask them to speculate on the meaning of those slopes relative to the way the ball travels. (Example: The slope of the travel up to maximum height is positive. At the maximum, the slope is zero. The slope becomes negative after that until the ball hits the ground.)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 6, and 8. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 should be encouraged to review Example 1 as well as their response to the Your Turn for that example. • Students having difficulty with #2 should be encouraged to review Example 2 as well as their response to the Your Turn for that question. The description and diagram for part b) would be useful to include in students' Foldable. If students are struggling with matching the graph, ask them to verbally describe what is happening in each graph. You will find that they will substitute their own label for the vertical axis. Based on their description, coach them to find one from the graphs given. • For #3, you may wish to discuss what each of the choices means and possible pairings. Based on the suggestions, have students see if any of the pairings are found in the graphs given. • You may need to review the different recorded music formats for #6, as not all students will know what they are. • To prompt students for #8, as a class, write out a brief description of water use in a 24-h period. Without using exact values of litres, identify where more or less or no water use at all might be likely. Then, pair students up to complete the question.
Assessment *as* Learning	
Create Connections Have all students complete #15–17.	• Both #15 and 16 are useful Assessment *as* Learning questions that students should include in their Foldable. Encourage students to use unique stories for #16. Have a partner review the story for accuracy. • You may wish to have students work in groups of three or four to complete the Mini Lab, #17. Have students share their results on the board so they can compare and determine whether their graphs are similar. Suggest that students identify each graph with a quick description of the action that resulted in the graph.

Linear Relations

6.2

***Mathematics 10*, pages 279–291**

Suggested Timing

100–120 min

Materials

- measuring tape
- ruler
- grid paper or graphing technology

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–6 Section 6.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

RF2 Demonstrate an understanding of relations and functions.
RF8 Represent a linear function, using function notation.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5–8, 13
Typical	#1–3, 5, 7, 9, 13, 14
Extension/Enrichment	#4, 10–14

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Relationships in the Human Body

You may want to discuss with the class real-world data, which is often imperfect, versus the mathematical models that are used for making predictions.

As you monitor students' work through the investigation, ensure that they make measurements in centimetres and that in #2 they answer the question "How many of your feet does it take to make your height?" If students need to round this value, have them round to the nearest tenth of a centimetre for convenience.

Since the class needs to have access to each student's data, you may wish to set up a convenient way to share the information. For example, each student could record his or her information on the board, overhead, or interactive whiteboard after taking measurements and making calculations. You may wish to have the class pause after #3 and compare to see that there is agreement on the mean number of feet in the height of a high-school student.

After students have completed #4 to 6, you may wish to have another quick debriefing. You could have volunteers show their graphs and share their answers to #5 and 6. Alternatively, you could have each pair of students pass their graph and answers to another pair for comment.

For #6, discuss whether the graph is a straight line or a curve, and how this relates to whether or not the relationship is linear.

As students answer #7 with their partners, circulate and listen to their answers to assess understanding. Answers to part a) should include tables, graphs, equations, and written descriptions, based on this Investigate, but students may have additional methods. Part b) may be more challenging and may require a little more discussion. You may wish to have some tables of values available for some simple linear functions, so that students can test their hypotheses.

You may wish to encourage students to research Leonardo da Vinci and his study of the proportions of the human body to help them make a connection between da Vinci's work and the relationships. See the related Web Link that follows in this Teacher's Resource. You might wish to have students take the investigation further by answering these questions:

- Look at the illustration of the Vitruvian man. What relation is da Vinci suggesting by inscribing a man, with outstretched arms, in a square?

- By using a person's height only, how would you determine the area of the square the person would fit into? Write the relation as an equation: Area = ____________
- Complete a table of values for the relation between a person’s height and the area of the square. Start at a height of 1 ft and use increments of 0.5 ft. Use your equation to determine the corresponding area.
- Plot the results from your table of values to see a graph of this relation. Place height on the horizontal axis and area on the vertical axis.
- Should you draw a line or curve through the points on the graph? Why or why not?
- Would you describe this relation as linear or non-linear? Why?
- Determine the difference between each area value in your table. What do you notice?
- What changes would you need to make to the height values if you wanted the graph to reflect this relation for grade 10 students only?

Students may be interested in the following facts about average body proportions. The ratio of

- hand to foot is 7 : 9
- arm span to height is 1 : 1
- leg to height is 1 : 2
- torso and head to height is 1 : 2
- shoulder width to height is 1 : 4
- waist width to height is 1 : 6
- width of top of leg to height is 1 : 12
- head to height is 1 : 8
- neck to height is 1 : 12
- foot to forearm is 1 : 1

Meeting Student Needs

- Have students identify the root word in *linear* to guide them to understand that all linear relationships will form a straight *line* when graphed. Then, engage students in a discussion about the characteristics of a straight line.
- The investigation lends itself well to partner or small-group work.
- Explain to students that a *model relation* is a general relationship between two variables created from data that has been collected experimentally.
- Students may need help setting up the scale on the axes for their graph.

ELL

- Have students work with a partner who can help them with the language and can explain the steps.

Common Errors

- Some students may measure in inches, or may know their height in feet and inches and attempt to use that value.

R_x Remind students to read the instructions carefully, and circulate through the classroom to ensure that students measure in centimetres.

Web Link

To research Leonardo da Vinci’s study of human proportions, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Relationships in the Human Body

1. Examples:

a) heights: 183 cm and 158 cm

b) foot lengths: 24 cm and 21 cm

2. Example: approximately 7.6 and approximately 7.5

3. Examples:

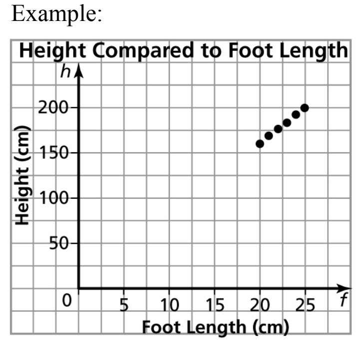
a) 7.5; 8 foot lengths to height

b) $h = 8f$, where h is height, in centimetres, and f is foot length, in centimetres

4. Example: shortest: 20 cm; longest: 25 cm

Foot Length (cm)	Estimated Height (cm)
20	160
21	168
22	176
23	184
24	192
25	200

6. a) Example:



Answers

Investigate Relationships in the Human Body

6. b) Example: Drawing a line is feasible because you can interpolate values. For example, if a foot measures 21.5 cm, the height can be estimated to be 172 cm.

c) The graph is a straight line. Example: This is a linear relation. In a linear relation, each integral increase on the x-axis results in a constant increase on the y-axis.

d) The differences are a constant of 8 cm.

7. a) Example: words, an equation, a table of values, ordered pairs, a graph

b) Example: Determine the differences between values; if the y-value changes by the same amount for each integral change in the x-value, the relation is linear. The relationship between height and shoe size may be linear since shoe size typically increases with height. However, it is more likely that various people of the same height wear different shoe sizes.

c) Height can vary by any fraction of a centimetre, whereas shoe sizes are only in whole or half sizes, so the points should not be connected.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner in answering #7a), or you may wish to encourage individual personal strategies and have them compare with a partner afterward. • For #7b), individual results in groups may not show the pattern in their table of values that will lead them to a correct solution. Use several examples and obtain suggestions regarding whether each one is linear. It may be beneficial to model some of the results on the board. Use several examples from groups to prompt a class discussion around whether the results should be linear and how students could prove it.

Link the Ideas

You may wish to take a few minutes to discuss this section. Have students decide if all of the methods for representing relations were discovered by the class during the Investigate and if the class had any representations that are not given in the student resource. Ask students if the margin definitions agree with the ones they found as they worked through the Investigate. You may wish to emphasize that there are alternative ways to express these concepts.

Many students will not be familiar with discrete functions and may even resist the concept. You may wish to ask students if they can think of situations in which a discrete relation or graph makes sense. A common example is one that involves a number of people; for example, the number of people attending a movie over a period of time.

Example 1

It may be beneficial to have students first discuss the answers to these questions without looking at the solution given. For instance, you could have students work in groups of three to five. Have students write their answers to the questions, and then have students in each group rotate their answers clockwise so that a peer can read them. Students could discuss any questions on which they disagree. You could have each group quickly report their results to the whole class.

To assess the understanding of the class, you could have each student complete the Your Turn individually, writing each answer on an index card or small piece of paper. Then, as you ask each question aloud, students could hold up the relevant response. In this way, they are able to answer anonymously, yet you can quickly judge the progress of the class. This is also easily accomplished using a student response system with an interactive whiteboard, if that technology is available.

Example 2

Two solution methods are given for this example. As students discuss the example, you can ask the following questions:

- Do you prefer one of these methods to the other? Why?
- Does the method you choose depend on the question? How?
- Can you think of a different method that could be used to solve the problem? How do you know that method is valid?

Students who are visual learners may prefer to draw the graph of the relation and then determine whether or not it is linear.

In the Your Turn, students have the opportunity to use any of these methods to determine the linearity of a number of relations. You may wish to challenge students by asking how many different methods they can use to answer the questions.

Example 3

Have students write on an index card or small piece of paper the letter of the representations that match each relation described in the example, and then hold up the index card when you ask. This will highlight any areas of misunderstanding and indicate the necessity to discuss those issues.

You may wish to have students answer the Your Turn in the same manner. In both cases, if there is any disagreement, students could be assigned to look back at prior work and material in the student resource in order to adjudicate. A small reward could be offered for the first student to find a way to support the correct answer.

Key Ideas

You may wish to review what students learned by asking them to create a list of all the different ways to represent a relation. Have students write each representation on the overhead, board, or interactive whiteboard. When all the ideas have been exhausted, have students look back through the section in the student resource to identify which ones have been missed and also which ones the class has identified that are not in the student resource, if any.

Meeting Student Needs

- Remind students that the *degree of an equation* is the highest exponent value found in any single term of a polynomial equation. For example, $x^2 - 5x + 7$ has a degree of 2. However, if a single term has more than one variable in it, the degree of the term is determined by adding the exponent values of the variables. For example, the term $5xy$ would have degree 2 since $1 + 1 = 2$.
- For Example 1, have students research Les Folies Grenouilles, which is held in the community of St-Pierre-Jolys, near Winnipeg. See the related Web Link that follows in this Teacher's Resource.
- For Example 1, students could make origami frogs and have their own frog-jumping competition. They could measure ten leaps for their frog, determine the mean distance per leap, and then complete the questions.
- For the Example 3 Your Turn, you may wish to change the animal in the question to one that students are more familiar with, for example, moose, deer, horses, or dogs.
- With the class, create posters illustrating the ways a relation can be represented. Use the same relationship for all ways so that students may study the connections between words, an equation, a table of values, a mapping diagram, and a graph.
- As they work through this section, students could create a chart containing information about how to identify linear relations:
  - From a table of values (or mapping diagram): Each unit increment in the domain corresponds to a constant change in the range.
  - From an equation: The equation contains one or two variables. The degree of the equation is 1.
  - From a graph: The graph is a straight line.
- Ensure that students have a chance to learn the vocabulary terms as they are presented.

ELL

- Assist students with such terms as *exceptions*, *unconnected*, *infinite*, *champion*, *consecutive*, *fireworks*, *shells* (in relation to fireworks), *duration*, *radioactive isotope*, *dance hall*, *scuba diver*, and *caribou*. Use visuals, descriptions, and examples.
- Ensure students understand the terms *relation*, *linear relation*, *non-linear relation*, *discrete data*, *continuous data*, *independent variable*, and *dependent variable*. Encourage them to add them to their vocabulary dictionary, Foldable, or other organizer they may be using.

WWW Web Link

For more information about Les Folies Grenouilles, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

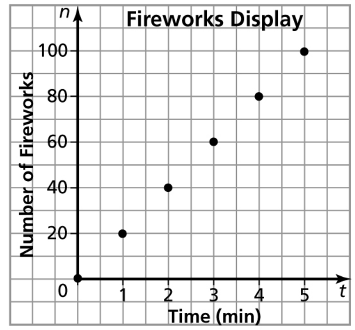
a) The relationship is linear. The total number of fireworks that have been sent off increases by 20 each time, and the time increases by one minute each time.

b) The variable n, for the number of fireworks, is the dependent variable, and the variable t, for time, is the independent variable.

c)

Time, t (min)	Number of Fireworks, n
0	0
1	20
2	40
3	60
4	80
5	100

d)



The data are discrete.

Example 2: Your Turn

a) The relationship is linear. Each time the number of people attending the dance changes by 1, the cost changes by \$5.

b) The degree is 2, so the relation is not linear.

c) The relation is linear. With each increase of 5 in the independent variable, the dependent variable decreases by 8.

Example 3: Your Turn

A, B, and E

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may see that some students find it easier to create a table of values and graph first and then respond to the linearity question. Some students may use patterning to solve the table of values. • Strongly encourage students to use the correct terminology for the question. • For students who confuse *independent* and *dependent*, ask them what an independent person is like. Ask them what a dependent person is like.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Example 2 reinforces the different methods used to determine whether a relation is linear. Some students may prefer to join the points on a graph to obtain a visual. Although it may be a correct strategy, encourage them to look at alternative approaches and determine linearity through dependent and independent variables or a table of values. • For part b), you may wish to suggest some values for x that students could use in order to get ordered-pair values. As this graph takes the shape of a circle, students will likely need some assistance with points. You could start by suggesting x = 5, 4, 3, 0, –5. Watch that they do not end up with a rhombus. • It may benefit some of the visual learners to have grid paper available.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Suggest to students who are having difficulty deciding which equation is representative of the caribou that they set up a table of values to determine a pattern first. Prompt them by asking them how many legs one caribou has, then two caribou, then three, and so on. Ask them to verbalize the pattern. Have them compare this pattern to the choices given.

Check Your Understanding

Practise

For #1, students might look back to Example 1 if they need to reacquaint themselves with the various types of representations.

For students to answer #2, you may wish to suggest that they look at Example 2 and ask them what their favourite method was for determining if a relation is linear. This discussion will emphasize that there are many different ways to determine the answer to this question.

In #4, students do not need to make calculations, but they may choose to change representations so that it is clear to them which graph matches the equation.

Apply

Before students attempt #5, have them read and discuss the related Did You Know? What experience do they have with bushels as a unit? Have students use a measuring stick to develop an approximation of 0.036 m^3. This could be a cube 0.4 m by 0.3 m by 0.3 m.

You may want to point out to students that the parts of #5 do not necessarily need to be completed in the order given. For example, they may wish to determine whether the relation is linear and whether it is discrete once they have drawn the graph.

In #6, students create a relation that satisfies given criteria. This is a higher-order skill, and for many students it would be best to complete this question with a partner or in a small group. If students have difficulty, you may wish to refer them to Examples 1 and 2 and have students recall their preferred methods for determining linearity.

In #7, students complete a problem-solving question that recaps the important ideas of the section. You may wish to ask students to work with a partner and alternate explaining their reasoning and justification. Students should be able to explain how they know that this is an example of a continuous relation.

Have students read the Did You Know? related to #8. The killer whale or orca is the largest member of the dolphin family. Killer whales are voracious eaters that consume a large quantity of different types of prey, including fish, squid, sharks, octopi, birds, and other marine mammals.

Did You Know?

Sgáan is pronounced suh-haan, with the accent on the second syllable. According to some sources, this word is derived from the Haida word for supernatural. A Web Link with audio of several Haida terms is referenced at the end of this section.

Question #10 presents an enrichment opportunity for students. For part b), you may wish to suggest that they begin their set of ordered pairs by choosing an arbitrary magnitude for their first ordered pair.

Extend

For #11, ask students to determine whether a relation is linear, given a table. You may wish to refer them to Example 2 in this section.

Create Connections

In #13, students' answers may require qualification. You may wish to ask the following questions:

- Does your preferred representation change depending on the context?
- In which situations would you prefer each of the most common representations: graph, equation, words, or table?

Meeting Student Needs

- Provide **BLM 6–6 Section 6.2 Extra Practice** to students who would benefit from more practice.
- Encourage students to refer to the graphics placed in the classroom during the section and to the list of Key Terms they developed. Allow students time to discuss the terms with a partner. Have students consult with another pair of students when there is a discrepancy in the information shared within a pair.
- Post large sheets of paper around the classroom, each labelled with one question from #5 to 10. Each pair or small group of students can start at one question. In 5 min, they should answer as much as possible on the sheet before moving to the next sheet. If there is information on the sheet that the group does not agree with, they should add their comments or suggestions to the sheet. By the time each group has rotated through all of the questions, the final solution should be available. Students have the advantage of looking through information suggested by other groups along with seeing the final solution. Manipulatives and graphing calculators may be placed at each station, and students may request to use a manipulative other than what is provided.
- For #5, some students may not know what a bushel is. Explain and make comparisons to mass in SI

units. Note that one bushel is approximately equal to 0.02722 tonne (t).

- For #8, you may wish to read to students a couple of Aboriginal legends related to the killer whale and discuss killer whales in Aboriginal art. See the related Web Link that follows in this Teacher's Resource.
- For #10, encourage students to find out more about earthquakes in Canada. See the related Web Link that follows in this Teacher's Resource.

ELL

- Students may need assistance with terms such as *convert*, *current*, and *characteristic*.
- For some questions that are more language based, such as #5, 7, 9, 10, and 12, you may wish to have students pair up with another student who could assist with the vocabulary.

Enrichment

- Tell students to suppose $a = \frac{b}{c}$ where b and c are positive integers. Ask them to predict what happens to a if b is doubled and c is tripled. Have them explain their thinking. (Example: When b and c are positive integers, a decreases because the change in c, the denominator, is greater than the change in b, the numerator.)

Gifted

- Give students the following information: The International Space Station orbits Earth, travelling in a curve with its distance from Earth remaining constant. The force of gravity pulls the station toward Earth. Ask students what stops the station from crashing to Earth. Also, ask why the station's time-versus-speed graph remains linear. (The centripetal force acting on the International Space Station corresponds to the Earth's gravitational force. The station's inertia keeps it in orbit.)

WWW Web Link

To find Aboriginal legends about killer whales and information about killer whales in Aboriginal art, go to www.mhrmath10.ca and follow the links.

For a recording of how to pronounce words in the Haida language, go to www.mhrmath10.ca and follow the links.

To research earthquakes in Canada, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3 and 5–7. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 should refer back to Example 1 or their Foldable for the representative models they learned. Encourage them to use as many of the methods as possible. • In #2, students can use their own personal strategy. Again, they can refer back to Example 2 and the Your Turn, but encourage them to link the strategies from #1 to solving #2. Challenge students to be comfortable using three strategies to solve this question. • Remind students that the variable for which they can select any value to substitute into is the independent variable and the answer that results is the dependent variable. Alternatively, you may wish to discuss independent and dependent variables in terms of horizontal and vertical axes. • Some students may prefer to complete #3c) first and then return to parts a) and b). Remind students that variables should be appropriate for the question so they are easy to remember. • If students have difficulty with #6, encourage them to graph the given context first and then generate a table of values. Using their table of values, prompt them to verbalize how they could have achieved the answer without graphing first. • For #7, students are given the equation. If they do not recognize a linear graph, ask them to create a table of values and then use their own strategy to explain whether it is a linear relation or not.
Assessment *as* Learning	
Create Connections Have all students complete #13.	• In #13, students have an opportunity to assess the methods they find easiest when representing a relation. Encourage them to identify as many methods as they can confidently complete. They could also provide an example of their own to model each method. This would make an excellent Assessment *as* Learning question for the Foldable or to collect from each student. • Encourage students who complete #13 quickly to try #14.

6.3 Domain and Range

Mathematics 10, pages 292–304

Suggested Timing

120–180 minutes

Materials

- ruler
- graphing calculator or spreadsheet software
- grid paper

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–7 Section 6.3 Extra Practice
TM 6–1 How to Do Page 300 Example 4 Using TI-83/84
TM 6–2 How to Do Page 300 Example 4 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6–8, 12
Typical	#1–6, 7 or 8, 12, 13
Extension/Enrichment	#8–13

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Reasonable Values for the Dependent and Independent Variables

As students begin the Investigate, you may wish to have them recall prior learning. Ask the following questions concerning the two graphs given:

- Are the graphs linear or non-linear? How do you know?
- What is the independent variable?
- What is the dependent variable?
- Are the graphs discrete or continuous? How do you know?

This last question, in particular, can help serve as a connection between the prior section and this section concerning domain and range.

As students discuss the six graphs given in #3, you may want to guide their discussions with specific questions:
For the first graph in the top row:

- What does this graph say about the climate of the area?
- How would a graph of our winter climate compare?

For the second graph in the top row:

- What time frame does the story suggest this graph represents?
- Do you need to include a specific year? Explain.
- By how much does the population of caribou change?
- How could the graph be changed to allow more specific answers to these questions?

For the third graph in the first row:

- Which line represents the bear and which represents the hunters? How do you know?
- Can you tell how long a journey the bear and the hunters took? Explain.

For the first graph in the second row:

- Why are the points on this graph not connected?
- What does this graph tell you about the fight?
- What does this graph not tell you about the fight?

For the second graph in the second row:

- What units might the amount of snow be measured in?
- What can you say about the rate of change in this graph?
- Can you tell how long it took for the snow to melt? Why or why not?

- Why are the points on this graph shown as a solid line as opposed to dots?

For the third graph in the second row:

- What does the y-intercept of this graph represent?
- Can you tell the greatest and smallest points in the region? Explain.
- Why is the graph shown as a continuous curve?

For #4, since it is possible for each pair of students to choose a different set of graphs, it may be difficult for pairs to conveniently compare answers. A possible strategy to allow comparisons is to give a number of sticky notes to each pair. Have a recorder write the pair's response for each of the four chosen graphs, anonymously. Then, designate six spaces on the board, one for each of the six graphs. Have each pair place their sticky notes in the appropriate place. Then, students can circulate and read all the responses, without knowing who wrote each one.

For #4, students should focus on determining values that are appropriate for the situation. Allow some flexibility, since students may have limited experience with some situations. They should reread each section of the story before choosing their values for the dependent and independent variables, since sometimes the information is quite subtle.

The Reflect and Respond question should be answered by each pair of students. You may choose to repeat the sticky-note process described above to allow for the comparison of answers, or you may wish to have a reporter for each pair give a response. The last question, concerning axes labels, should help students understand that a graph is not complete without a domain and range, and thus should reinforce the need for students to understand these new concepts.

Meeting Student Needs

- Give students two situations in which the domain and range would be significantly different. Have them discuss the following points:
  - What size of values would need to be included?
  - What type of numbers would be useful (whole, integer, rational, real)?

  Discuss with students that different relationships require different scales on a graph. These values represent the domain and range.
- Post the student learning outcomes for the section as well as the key terms: *domain* and *range*.
- Extend the discussion in the opener about the necessity of well-labelled, accurate axes. Some students still may not understand that the scale on an axis must be similar to that found on a ruler or metre stick: each increment must represent the same amount.
- The six graphs in #3 could be enlarged and posted to allow students to write comments and responses directly on the poster. As a class, discuss each poster.

ELL

- Have students work in pairs or a group so that other group members can explain the vocabulary and steps for completing the Investigate.

Common Errors

- For the first graph in the second row of #3, some students may think that there should be only four dots, since there were four hunters.

R$_x$ It may help to ask students what the two points on the axes represent.

Answers

Investigate Appropriate Values for the Dependent and Independent Variables

1. a) Example: For a number of years, the body mass for all four tyrannosaurs does not change significantly, then it begins to increase at a steady rate, and then it levels off. The body mass for the tyrannosaurus increases at a significantly faster rate and by a greater amount than the body mass for the other species.

b) The amount of time and the tyrannosaurs' age for each stage of growth is established.

2. Temperature versus Day: winter for twelve months; Population versus Year: caribou; Distance versus Time: four of the village's skilled hunters to follow the bear home; Number of Hunters versus Time: a fierce fight killed three of the hunters and mortally wounded the fourth; Amount of Snow versus Time: the snow melted; Altitude versus Distance: hills and valleys

3. a) Example: Temperature versus Day: day values from 0 to 365 and temperature values from 0 °C to –30 °C. Distance versus Time: distance values from 0 km to 2 km and time values from 0 min to 30 min. Number of Hunters versus Time: values for the number of hunters from 0 to 4 and time values from 0 h to 3 h. Amount of Snow versus Time: time values 0 days to 30 days and amount of snow from 0 cm to 200 cm

4. a) Example: Population versus Year: The story does not indicate what the population of caribou was originally and by how much it decreased. Altitude versus Distance: The story does not indicate the height and depths of the hills and valleys.

c) Example: Values define the quantity involved. For example, instead of just noting that there was a change because of the steepness of the graph, the rate of change can be determined.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• It would be beneficial to all learners to begin this Investigate in pairs and then complete it as a class. • Encourage students to compare their answers in #5 with their partner's and to agree on a final version of their work. Suggest that they give reasons or justification for their answers. For #5b), open up a class discussion and ensure students support their answers with reasons. • To answer #5c), prompt students to look back at section 6.1 #2 and 4 for graphs without units.

Link the Ideas

You may want to allow students some time to assimilate the variety of ways to represent the domain and range. For instance, if students are using Foldables or math journals, you might have them include these concepts.

Example 1

You may want to have students attempt to determine the domain and range of each graph independently before looking at the solution. They should be free to choose their preferred representation for the domain and range. You might have them write a response on an index card and hold it up so that you can assess the understanding of the class. In this way, the discussion of the examples can be directed toward any misunderstandings.

Ensure students understand that, for example, $-6 < x < 3$ means the same thing as $x > -6$ and $x < 3$. You may wish to give students some examples of combinations of inequalities and have them practise writing them as double inequalities.

After students have answered the Your Turn, you may want them to again hold up their responses on an index card. Alternatively, they could quickly exchange responses with a partner and discuss any issues that arise. If you choose the partner approach, have one of the partners report to the whole class.

Example 2

In this example, students determine the domain and range from a situation and from ordered pairs. You may wish to have them briefly refer back to the Link the Ideas to revisit how domain and range can be obtained using ordered pairs. Focus their learning by asking which coordinate they look at when analysing domain. If students prefer certain representations, you may wish to help them switch between representations. For example, if students prefer a graph, you could ask the following questions:

- Which coordinate in an ordered pair refers to the y-axis on the graph?
- Does this help us determine the domain or the range?

It may also be helpful to ask why the graph does not begin at (0, 0).

To answer the Your Turn, students may prefer to work with a partner. You might want to remind them that the representations for domain and range need not be given in the order specified but that they may begin with the representation they find most accessible.

Example 3

When students consider the list, you may want to ask the following questions:

- Why does this method include set notation, but not the other notation, like inequality symbols?
- Would it be convenient to write a statement using set notation that is more like the previous examples? Explain.

Example 4

Many students will see a natural connection between domain and range and calculator settings. You may wish to ask students to graph the function given, without further instruction, and then poll them to see how many changed the window settings as a matter of course. Even if students do not automatically change the window, they will see that the default window is not suitable and they will also see the need for understanding domain and range in order to complete the question.

There are many ways for students to determine the answer to the margin question, "Which variable is the independent variable?" For example, they may refer to the fact that age is the independent variable, or they may observe that since the equation is written with height isolated, it is the dependent variable.

After students have completed the Your Turn, you may want to have them hold up their calculators so that you can quickly check their understanding. This will help you decide if further class discussion is needed and will give a jumping-off point for that discussion.

Key Ideas

As students consider the Key Ideas, they should state whether there are representations for domain and range that they feel more comfortable with. Alternatively, they may identify situations in which one representation seems more natural than the others. They should be able to use all of the representations, but if they are cognizant of their own preferences, they may use that knowledge as a starting point when solving problems.

Meeting Student Needs

- Take time to reaquaint students with the difference between an open and a closed circle when used on a number line. Spend time discussing interval notation and set notation. Emphasize when to use square brackets or parentheses within the notation. Also, have students discuss what they remember about the various sets of numbers, such as integers and real, rational, irrational, natural, and whole numbers.
- As a class, create a poster listing the five methods of displaying domain and range: words, number line, interval notation, set notation, and list.
- You may wish to give students different representations of the same domain or range and have them group the like ones. Also, you might set up a matching game in which there are two cards with the same domain or range but they are given in different forms.
- Example 3 discusses music written in 4/4 time, where there are four beats per measure (shown by vertical lines in the music). Divide the class into two groups. Play eight bars of 4/4 music. Have one group clap on the first beat of each measure and count how many times they clapped. Have the other group tap their foot with each beat and determine how many times they tapped. Students could then compare their data with the table of values provided. Ask students how this data would change if they counted the number of steps taken while dancing a two-step for the eight bars of music. Invite students to demonstrate.
- Explain to students that a *restricted domain* means the dependent variables are limited to certain values. In this section, the domain is restricted when using technology to graph a situation. Another way to help students understand the meaning of restricting the domain is to describe it as "limiting the values of the domain." This way, they will see that other values are permitted when graphing; however, to see the particular part of the graph that needs to be looked at closely, the domain can be restricted so only that part is displayed.
- For Example 4, have students enter the equation into their graphing calculator without a discussion about the window. Ensure that the graph is not visible in the standard window. Students will soon see that an understanding of domain and range is necessary to accurately represent mathematical relationships.
- As a class or in small groups, have students create a summary of the content of this section. Post their responses.

ELL

- Students may need assistance with terms such as *permitted*, *element*, *leftmost, rightmost*, *Ferris wheel*, *revolution*, *inclusive*, *motorized*, and *bar* (in relation to music).
- You may need to show a number of examples of *interval notation* and *set notation* to reinforce the name for this way of representing domain and range.
- Ensure students add the terms *domain*, *range*, *interval notation*, and *set notation* to their vocabulary dictionary, Foldable, or other organizer that they might be using.
- For students unfamiliar with *corn* and *cornstalks*, show pictures of each while repeating each word.

Enrichment

- Encourage students to make a list of real-life situations where the domain and range of a relation must be integers, whole numbers, and rational numbers. Challenge them to create a Venn diagram that displays their response. Ensure that they include areas of overlap. (Students might choose counting livestock for whole numbers, loans and savings for integers, and ratios of males to females for rational numbers. An overlap might be a historical situation where livestock was counted as payment for debt.)

Gifted

- Challenge students to create questions to test for understanding of the restrictions of domain and range. Suggest that they create expressions in which, without restrictions, the denominator could be zero or in which there could be a negative sign under a square root. Have them write the question and explain the answer. (Example: What is the domain for the expression $\frac{1}{1-x}$? Answer: The domain is all numbers greater than or less than 1, since there cannot be a denominator of zero.)

Common Errors

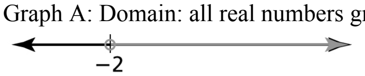
- Students may develop the misconception that graphs always begin at (0, 0).

Rx Direct them to the side bar in Example 2 that asks, "Why does the graph not begin at (0, 0)?" To help them see the answer to the question, you might ask the following questions:
  - How much time has passed when a person starts a ride on the Ferris wheel?
  - When a person boards a Ferris wheel, do they do so at ground level?

Answers

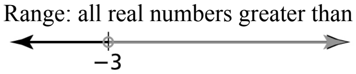
Example 1: Your Turn

Graph A: Domain: all real numbers greater than –2



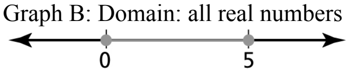
(–2, ∞), $\{x \mid x < -2, x \in R\}$;

Range: all real numbers greater than –3



(–3, ∞), $\{y \mid y > -3, x \in R\}$;

Graph B: Domain: all real numbers from 0 to 5, inclusive



[0, 5], $\{x \mid 10 \le x \le 5, x \in R\}$;

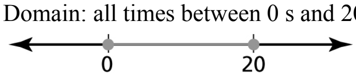
Range: all real numbers between 2 and 7, including 2 but not including 7

[2, 7), $\{y \mid 12 \le y < 7, y \in R\}$

Example 2: Your Turn

a) Point A represents the largest point on the Ferris wheel, 47 cm. Point B represents the starting point of the chair at 3 cm above the base. Point C is the origin and represents base level on the vertical axis and the starting time on the horizontal axis. Point D represents the time it takes for two complete revolutions. Its value is 20 s.

b) Domain: all times between 0 s and 20 s, inclusive



[0, 20], $\{x \mid 10 \le x \le 20, x \in R\}$;

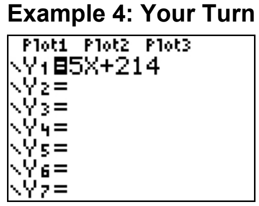
Range: the Ferris wheel's height above the base is between 3 cm and 47 cm, inclusive

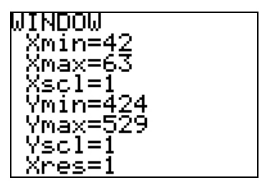
[3, 47], $\{y \mid 13 \le y \le 47, y \in R\}$

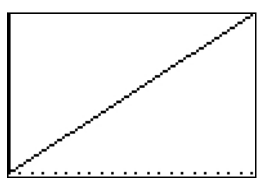
Example 3: Your Turn

Domain: $\{x \mid -3 \le x \le 2, x \in I\}$, {–3, –2, –1, 0, 1, 2};
Range: $\{y \mid 15 \le y \le 10, y \in I\}$, {5, 6, 7, 8, 9, 10}

Example 4: Your Turn







Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students identify which of the four representations they feel least comfortable with and coach them through it. Check for understanding by providing another graph. • Some students have difficulty remembering domain and range and which variable in the ordered pair each represents. Coach students to think alphabetically: *d* (domain) comes before *r* (range) and *x* comes before *y*. • Check that students are clear on how to do set notation. Provide additional examples, if needed.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to start with value B if students are having difficulty. • Encourage students to copy the graph into their books, label the distance B and have them verbalize how far up from the ground the wheel is. • From the information given, ask what role the 22-cm radius plays. • Encourage students to label ordered pairs for values on their graph, for example, (0, 3). Ask how this might assist in determining the domain and range. • Alternatively, you could have students act out the motion of the Ferris wheel.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Example 3 reinforces the notion of domain and range. Some students might find it easier to complete the Example 3 Your Turn and then go back and do the example. • Remind students of the link between the domain and values of *x*, and between the range and values of *y*. • Coach students to explain what the values mean beyond the link with values of *x* and *y*.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • It is important for students to understand how the domain can help with determining the values for the range, which provide the window settings in a graphing calculator. You may wish to complete the example by first putting the equation on the board and asking students to graph it. For many, they will go back and change the window setting until it provides a solution. Guide students to see the efficiency in using the domain.

Check Your Understanding

Practise

For #2, students may refer to Example 2 if they need help getting started or if they experience difficulty.

For #3, students can revisit Example 3 for help with this question if needed.

In #4, remind students that the parts need not be answered in the order given. You may wish to have students explain their method for answering the question.

Apply

As students work on #5, you may want to ask them if the points on the graph should be joined. While the answer may seem obvious, students may note that graphs of this type frequently have points joined when presented in the media. If you have access to several days' newspapers, you may be able to locate an example and have students discuss why this is mathematically incorrect.

For #6, you may want to ask students if they notice any connection between interval and set notation.

In #7, students encounter a problem-solving situation from which they obtain the domain and range. You may wish to ask the following questions:
- To make your graph, would it be more convenient to write the relation in a different representation first?
- If so, which representation would you choose?

Since #8 is similar to Example 2, you might refer students there if necessary.

For the problem-solving context in #9, you may want to ask students which representation they could use to express the relation that would make it more convenient to graph.

Extend

In #10, students graph a discrete function to determine missing elements of the domain and range. You may want to ask students if graphing the points that they know both coordinates for makes a pattern appear.

In #11, some students may be able to write the domain and range from the description given, while others may wish to first write an equation or draw a graph.

Create Connections

Students will be more likely to remember the definitions for domain and range if they write them in their own words. They are given this opportunity in #14. You may wish to have students include these definitions in their math journal or Foldable.

In #15, assist students by asking them if it is necessary to have an equation or graph in order to know that a domain or range is restricted. Refer them to their work on #9 to see that it is not possible to use many representations of the relation to answer this question.

Meeting Student Needs

- Provide **BLM 6–7 Section 6.3 Extra Practice** to students who would benefit from more practice.
- When students are looking at the graphs, they will need to interpret the value represented by each line of the grid. In most instances, the value is one unit. Some students may need labels in increments of one unit. Also, discuss the symbols on the end of a graph to ensure that all students understand their meaning and how these symbols affect the domain and range.
- Students could have teacher-prepared integer number lines to view while deciding on the domain and range for each graph in #2 and 3.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4 and 6–8. Students who have no problems with these questions can go on to the remaining questions.	• If students find #1 and 2 challenging, have them review Examples 1 and 2. Ask them the following questions: – Can you explain how you know when a number is and is not included on a number line? – What type of inequality signs can you identify? Ensure they are not confusing greater than and less than with interval notation. • For #3 and 4, coach students to verbalize what they know about domain and range. Prompt them by asking what you look for in the domain and range of ordered pairs, of a graph, and of a table of values. Responses to these questions should help clarify the work in #4. • Some students may find it easier to use a ruler to find the exact values for domain and range. • Students must be familiar with set and interval notations for this question. Coach students as needed. • Students may wish to use the same technique of graphing and labelling segments as was completed in Example 2 and the Your Turn. Prompt students to label the coordinates of points A and B. This will help to determine the domain and range. Ensure students do not simply state values for the domain and range without being able to interpret what the values represent.
Assessment *as* Learning	
Create Connections Have all students complete #12. If students successfully answer this question, have them complete #13.	• In #12, students articulate their own personal understanding of domain and range. Encourage students to write their response in their Foldable along with their examples and explanations. This is an excellent Assessment *as* Learning question. • For students who have little difficulty, you may wish them to combine #12 and 13 into one question and base it on a topic of their own choice.

Functions

6.4

***Mathematics 10*, pages 305–314**

Suggested Timing

180–240 min

Materials

- grid paper
- ruler

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–4 Chapter 6 Unit 3 Project
BLM 6–8 Section 6.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6–8, 9, 16, 17
Typical	#1–7, 9, 10, 16, 17
Extension/Enrichment	#9, 11–15, 18

Unit Project Note that #9 and 13 are Unit 3 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

After you read the introduction as a class, ask students the following questions:

- Are there any items that you use daily that require an input from you and then produce one output?
- Are there other examples from music?

Investigate Functions

The Investigate takes the form of a concept-attainment exercise. You may wish to emphasize to students that the task is to develop criteria or rules that can be used to distinguish functions from non-functions. You may also want to explain that they have worked with several different representations of functions and that they will see those representations in the Investigate.

Give students a few minutes to read the instructions and orient themselves to the task. You may need to assist students in getting started. The following questions may help them:

- Why are the relations organized into two different groups?
- Does it help you to first look only at the relations that are represented in the way you are most comfortable with?
- Choose one of the functions. Can you formulate a description of it that does not apply to its corresponding relation in the other group?
- Once you have a description or rule that fits one function-relation pair, does it apply to the other pairs? If not, how can you revise your rule?
- Would changing one or more of the representations given make it easier for you to complete this task?
- Does the size of the domain and/or range have anything to do with whether a relation is a function?
- Can a function be linear?
- Can a function be non-linear?

As you circulate and monitor students' work, pay attention to misunderstandings and ask questions like those above to redirect students whose definitions of functions are not correct or complete. Students may be distracted by the representation of the relation, its domain, or its linearity, and miss the relevant criterion that each member of the domain must be related to exactly one member of the range.

You might want to ask the following questions:

- Why does the vertical line test work?
- Would a slanted line work as well?

As they complete the Reflect and Respond, you may want to encourage students to consider the expanations developed by other groups. Ask them if each expanation works for each of the eight pairs given in the student resource. After some discussion, provide time for students to revise their explanations.

Meeting Student Needs

- Open this section by displaying a variety of four-sided objects (quadrilaterals) to students, ensuring that you include several rectangles of various sizes. Ask students the following questions:
  - What do all of the objects have in common?
  - Are there objects that are unique?
  - What makes them unique?

  Lead students to the conclusion that rectangles are a special type of quadrilateral. Then, inform students that they have studied various relations but that in this section they will develop an understanding of a special type of relation called a *function*.
- *Input* and *output* are useful ways of explaining the relationship between dependent and independent variables. Use these terms interchangeably to help students understand this relationship.
- The section opener gives examples of two quantities depending on each other in a certain way. You might wish to discuss other examples that are relevant to students' experiences. For example, students in the North might discuss how the pressure on the throttle determines the speed of a snowmobile, or how the speed at which ice melts in a drinking-water tank depends on the thermostat setting in the house.
- For more assistance with the investigation, refer to notes found on the Internet about concept attainment. See the related Web Link that follows in this Teacher's Resource.

ELL

- Some students may need help with terms such as *battery*, *recharged*, *Bella Coola*, *altitude*, and *storage space*. Use visuals, descriptions, and examples to facilitate their understanding.
- Show students pictures to assist them in understanding what is meant by *above ground oval swimming pools* and *hot-air balloon*.

WWW Web Link

For more information about concept attainment, go to www.mhrmath10.ca and follow the links

Answers

Investigate Functions

1. Example: The functions all move horizontally along the page, while the non-functions have some x-values with more than one y-value.

2. a) Example: In a function, the x-value can have only one y-value, but the y-value may have one or more x-values.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Invite each pair of students to share their answers to #1, since not each pair will see the connection between function and non-function immediately. • Ensure that the explanation students write for #2 clearly identifies an understanding of the difference between function and relation. You may wish to have a class discussion and provide several function and non-function graphs and sets of ordered pairs. Suggest to students that they may wish to come back to their explanation after completing the examples.

Link the Ideas

You may wish to ask students if they have a definition of *function* that they prefer to the two alternatives given. While the wording may be different, you can ask them if the two definitions given convey the same meaning. Students may want to record their definition of a function in their Foldable or math journal.

Function notation is mathematical shorthand for a concept students have already explored. You might ask them the following questions:

- How does this compare to the process of substituting a value into an expression?

- Why would we use this notation instead of the terminology for substituting?

Example 1

This example allows students to test their definition of a function. You may wish to give students a few minutes to decide, independently, which relations are functions. Then, you could quickly poll the class. For example, you could ask students to give a thumbs-up if the first relation is a function, and a thumbs-down if it is not. After discussing the results and achieving consensus, you could repeat the process for the second pair and third pair of relations.

You may wish to have students complete the Your Turn independently and then compare results with a partner. If desired, you could have one partner report to the class. You may wish to ask the following questions:

- Why does the vertical line test not serve as a definition for a function?
- Is it possible to rephrase the test so that it may?

Example 2

The Think-Pair-Share strategy may be an effective way to approach this example. Ask each student to answer the questions and then find a partner and come to a consensus on the correct answers. Choose a number of students to report the answers on which they agreed or to present their work to the class. Discuss the work presented to ensure that all students understand. You may wish to use the same strategy for the Your Turn.

Example 3

To help students understand the translation of the equation to function notation, you may wish to focus on the variables and refer to the Link the Ideas. Ask the following questions:

- Which variable is the independent variable? How do you know?
- Which variable is the dependent variable? How do you know?

You may also wish to point out the sidebar question and discuss that the points on the graph are not connected because the function is not discrete.

You might ask students to explain why they prefer either Method 1 or Method 2 for answering part c) or have them propose a different method that they prefer.

For the Your Turn, you may want to have students work in pairs or groups of three. Ask them to identify the domain of the function to highlight the fact that the domain is understood to be the real numbers if no restrictions are given or implied.

Key Ideas

Have students rewrite the Key Ideas in terms that are most convenient and memorable for them. You may wish to simulate a "function machine" using a calculator. There are also online function machines that may prove useful. See the related Web Link that follows in this Teacher's Resource.

Meeting Student Needs

- The new notation, $f(x)$, can be a very abstract concept for students. Be sure to take time to explain how to read the notation ("f of x" or "f at x"). To help students understand, explain that it is another way of representing y, but that it gives us more information: $f(3) = 4$ instead of $y = 4$ shows that the value of y is 4 when $x = 3$.
- You may wish to develop the idea of function notation alongside an example of evaluating an expression.

x	$3x + 1$	$f(x) = 3x + 1$
-2	$3(-2) + 1 = -5$	$f(-2) = 3(-2) + 1$ $= -5$
-1	$3(-1) + 1 = -2$	$f(-1) = 3(-1) + 1$ $= -2$
0	$3(0) + 1 = 1$	$f(0) = 3(0) + 1$ $= 1$
1	$3(1) + 1 = 4$	$f(1) = 3(1) + 1$ $= 4$

- Ensure students note that Example 1 confirms what was developed in the first part of the Link the Ideas section.
- Example 2 develops a link to a practical application of function notation. Ask students if there are other examples that they can contribute.

ELL

- Encourage students to include the term *function notation* in their vocabulary dictionary, Foldable, or other organizer that they may be using.

- Use diagrams and words to clarify the meaning of *input* and *output*.
- Students may need assistance with such terms as *corresponding*, *associated*, and *cell phone plan*.

Common Errors

- Students may want to use the vertical line test as a definition for function.

Rx Ensure students understand that this is only acceptable if the function is represented graphically. It may be possible to use the vertical line test as a definition if it is carefully phrased to indicate that the relation is first represented graphically.

Web Link

To try an online function machine, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) Not a function; the relation does not pass the vertical line test.

b) Function; each x-value has only one y-value.

c) Function; each x-value has only one y-value.

Example 2: Your Turn

a) $F(86) = 186.8$; this means that 86 °C is the same as 186.8 °F.

b) $F(37) = 98.6$; this means that 98.6 °F is the same as 37 °C.

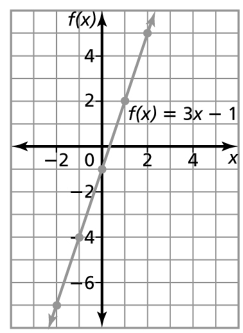
c) $K(80) = 353.15$; this means that 80 °C is the same as 353.15 K.

Example 3: Your Turn

a) $f(x) = 3x - 1$

b) Example:

x	f(x)
−2	−7
−1	−4
0	−1
1	2
2	5



c) $x = 18$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Initially, some students may wish to plot the points and then use the vertical line test. Coach students to identify the characteristics of a function or non-function from looking at ordered pairs and tables of values. Have them write the characteristics into their Foldable for future reference.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from reviewing how function notation is different from an equation in terms of *y*. • Ensure students are not confused by the meaning of *f*(3). Some students will want to multiply 3 and *f*. Provide several examples before students move forward.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from reviewing how functional notation is different from an equation in terms of *y*. • Ensure students are not confused by the meaning of *f*(3). Some students will want to multiply 3 and *f*. Provide several examples before students move forward. • You may wish to set up two tables of values initially, one with function notation and the other in terms of *y*. Help students to see the parallels between the two. • Have students verbalize the function-writing process and identify the ordered pairs that are generated. Ask students to identify how they determine the “order” of the ordered pairs. This skill is important for graphing the given equations manually or using technology.

Check Your Understanding

Practise

For #1, ask students whether their explanations make reference to concepts such as domain, range, and independent and/or dependent variables.

For #2, ask students to explain their choice of variables and/or to identify the dependent and independent variables.

Conceptually, #4 and 5 are equivalent; however, the fraction in #5 may provide more challenge to students.

To help students with #6, you might ask the following questions:

- On which axis does the independent variable appear?
- Would it be helpful to express the question in another way to help you understand what is being asked?

In #7, you may wish to draw students' attention to the suggestion that they make a table of values in order to draw the graphs. You may want to ask students how they know whether the points on the graph should be connected.

Apply

In #8, students may be tempted not to write the explanations for part b). Encourage them to complete this part carefully, as this should help with their skills in creating functions to model problems.

In part d) of #9, student's answers for an appropriate domain may vary. The emphasis is on their ability to justify their choice and to draw a graph that reflects their stated domain.

While completing #9 is not required, doing so will assist students in understanding the unit project. If students are going to complete the unit project in groups, you may want them to work with those groups to complete this question. In particular, students may find the final task of measuring the length of their radius bone and their height to be more easily completed with help from a peer.

Most students will likely be able to begin question #11, but the discussion of Mach numbers presents an enrichment opportunity and may be more challenging for some students.

Extend

Students will likely need to experiment to complete #12. This question gives a glimpse of systems of equations, which is the topic of Chapters 8 and 9.

In #13, a unit project question, function notation is used to refer to points on the graph. If students completed #6, refer them to that question as a starting point.

You may wish to discuss the importance of bison to Blackfoot culture, including the Siksika, Kainai, and Piikani. Much of tribal life revolved around preparing for the bison hunt and utilizing the results of that hunt. Nothing was wasted. Bison meat was roasted for immediate use or made into pemmican and stored. The hides provided winter clothing, blankets, ceremonial costumes, and tipi covers. Bison sinew was used as thread, bowstrings, and snowshoe webbing. The bones made useful garden tools, arrowheads, scrapers, and pipes. Bison horns provided cups, spoons, bowls, toys, and headdresses. The tails were used as fly swatters and whips. Bison dung fueled the campfires. Their teeth were made into necklaces. Bladders became medicine bags, water containers, and pouches. Bison skulls were used for ceremonial purposes. For additional information on Head-Smashed-In Buffalo Jump, see the Web Link at the end of this section.

For #14, you might guide student's thinking by asking one of the following questions:

- What is the difference between a point that is closed and a point that is open on a graph?
- In what context have we previously seen open and closed dots? What did they mean then, and do you think they mean something similar now?

Students may think that #15 appears intimidating, but ask students to determine a pattern to help their understanding of the concept. For example, you could ask a short series of questions:

- What do you do to determine $h(1)$?
- What do you do to determine $h(-7)$?
- What do you do to determine $h(142)$?
- What do you do to determine $h(t)$?

These questions should help students to recognize that the underlying idea is that of substituting a value into an expression.

Create Connections

Invite students to discuss their answer to #16 with a partner or in a small group.

In #17, students need to understand the role of the independent and dependent variables in function notation. If students have difficulty with the abstraction of the unspecified function $f(x)$, you may want to invite them to choose a specific function to use in its place.

In #18, students encounter a situation in which their intuition often leads them astray. To help students see Jean-Marie's error, ask them to compare the function $f(x + 2)$ with the polynomial $y(x + 2)$ to highlight how the two are different, though they appear to be the same. This concept will prove to be important in the study of functions and relations in the future.

Unit Project

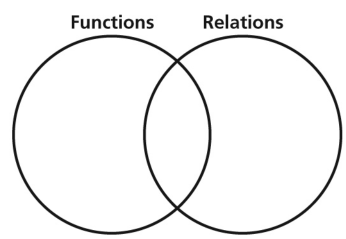
The Unit 3 project focuses on forensics and how mathematics might be useful in forensic science. You might take the opportunity to discuss the Unit 3 project described in the Unit 3 opener if you have not done so already. Students should enjoy this project, as it ties into many of the current television shows and scientific studies that they have likely been exposed to.

In #9, students have an opportunity to apply their knowledge of solving for an unknown variable or evaluating a function to solve for the missing height of a man. The question has a direct connection to their project, so students would benefit from completing this question for the final assignment. A good understanding of functions is necessary for this question. Further, students will apply their knowledge of domain and range to a similar function representing the length of female bones.

Students may find #13 more difficult. This question requires a good understanding of functions from a graphing perspective. In addition, students must be able to demonstrate an ability to interpret information from a graph and apply it to a context to solve a problem.

Meeting Student Needs

- Provide **BLM 6–8 Section 6.4 Extra Practice** to students who would benefit from more practice.
- Post examples and non-examples of functions in each of the forms: ordered pairs, mapping diagram, table of values, and graph.
- For #7, students could graph the functions on their graphing calculator and then transfer the graph onto grid paper. Emphasize that the graphs cannot have arrows.
- Encourage all students to complete #9. Some students will be able to complete the question with paper and pencil. Allow others to use a graphing calculator to determine the values for parts a), b), and c). They may require some instruction as to how to change the table of values or to determine the information from the graph.
- In relation to #13, students may be interested to learn the following: The Métis relationship with the buffalo was central to the cultural way of life of the early Métis. The buffalo hunt was an event that happened twice a year (spring and autumn) and dictated most other Métis community events. The organization of the buffalo hunt was rigorous and regimented, creating a chain of command that was respected outside of the hunt as well. St. Albert, AB, was established by Métis, and the first laws that governed the community were based upon the rules of the buffalo hunt.
- You may need to display a blank Venn diagram suitable for answering #16.



ELL

- You may need to explain to students what is meant by *saving pattern*, *force of gravity*, *elevation*, and *initial*.
- For #9, 11, and 13, have students work with a partner who can explain the language and can assist them through the steps of the questions.
- Note that if students experience difficulty with #12, their difficulty may be caused by the language in the question rather than lack of mathematical understanding.

Enrichment

- Challenge students to draw and explain a function that shows the temperature of water in a bath from when a person first draws the bath to when the tub is completely empty at the end of the bath. Ensure that they consider the effect of the person entering the tub and any other factors they think of that may affect the water temperature, such as bubbles. (Example: The graph may show the temperature of the water as initially being hot, then decreasing as the coolness of the tub affects it, then rising again as the water volume increases. A person entering the tub cools the water as long as the person's temperature is less than the water temperature. Bubbles insulate the warmth of the water.)

Gifted

- Challenge students to examine the following situation: Suppose an office worker buys a coffee from the building's cafeteria. She wants her drink to be at the maximum temperature possible when she returns to her desk. Should she put the cream in the coffee before she leaves the cafeteria or when she arrives back at her desk? Have them determine their answer by graphing time versus temperature. Ensure that they explain their answer. (Cooling is more rapid the greater the temperature difference. Adding the cream decreases the temperature difference between the coffee and the air. Therefore, the office worker should add the cream before leaving the cafeteria. The negative slope of this graph is greater initially, but has a lesser negative slope than the graph for the coffee that has the cream added later.)

Web Link

For additional information on buffalo jumps and the importance of bison to Blackfoot culture, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4 and 6–8. Students who have no problems with these questions can go on to the remaining questions.	• For #1, you may wish to review the rules for determining a function. Encourage students to refer to their Foldable or Example 1 if further scaffolding is needed. • For #2 to 6, students need a good understanding of both function notation and evaluating functions. For #4c), ask students to verbalize what 42 represents. A reminder of solving basic equations may be helpful. • For #7, ask students to demonstrate another way the domain could be used in function notation. Coach them through the first question and have them complete the rest independently. • In #8, students are given the formula. They should be confident in answering #8b). Ensure that they are not simply learning to substitute values in for a function without understanding what the variables represent.
Unit 3 Project If students complete #9 and 13, which are related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 6-4 Chapter 6 Unit 3 Project**, and have them finalize their answers. • Since #9 is an entry-level project question, all students should be able to complete it. It links directly to #8 in that they must understand what the variable represents. • You may consider having students work in pairs to complete #9. • Assign #13 only to students who are confident and strong in their ability to interpret graphs.
Assessment *as* Learning	
Create Connections Have all students complete #16 and 17.	• You may wish to use #16 as an Assessment *as* Learning or Assessment *of* Learning. It provides an opportunity for students to explain their understanding of functions. Similarly, #17 provides students an opportunity to determine the difference between evaluating a function at a value and the final value of the function. • Assign #18 to students who require more challenge and have a good understanding of functions.

6.5 Slope

Mathematics 10, pages 315–329

Suggested Timing

180–240 min

Materials

- grid paper
- plastic transparent ruler
- toothpick
- tape
- ruler
- one or more small boxes

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–9 Section 6.5 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 3a)–d), 4, 5, 7, 8, 17, 19
Typical	#1–5, two of 8–12, 17–19
Extension/Enrichment	#9, 13–19

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

You may wish to connect this section to section 6.1, in which rates of change were discussed extensively. For a line or line segment, the rate of change is usually referred to as the slope. Students may be familiar with the concept and calculation of slope from science classes.

Investigate Slope

Students are likely best to work in pairs or small groups for this Investigate. If you have students that are familiar with slope from other classes, you may want to arrange the groups so that each group contains one such student.

After #1, you may wish to poll the class to ascertain that they agree on the relative steepness of the lines. One possible method is to have students write the lines, in order of increasing steepness, on the back of a sticky note. If you have them place the sticky notes on a transparency, you can quickly read the responses by turning over the transparency and placing it on an overhead.

You may wish to poll the class again after they have completed the measurement of the slopes in #3. It may be most efficient to simply ask if the measurements confirmed their initial decisions. A brief whole-group discussion may or may not be necessary, depending on the results of the groups.

In #6, if students are familiar with the concept of slope from science, you may ask if anyone recognizes the ratio or knows the usual terms for AB (rise) and AC (run).

You may want to take a few minutes for discussion after students complete each of #6 and 7. Each group of students might compare results and diagrams with one or two other groups to ensure that each of them is correctly following the directions. Students may also benefit from hearing the ways other groups express their discoveries.

The Reflect and Respond for this Investigate may be most beneficial to students if they include diagrams in their explanations and descriptions. When each group has answered one part of the Reflect and Respond, you could have them exchange with another group and come to a consensus on the answer. It would be best if students exchanged with a different group for each question.

Meeting Student Needs

- Develop a list of several places where slope can be evident to students: the incline on a treadmill, the road around a speed curve, the path of a hockey puck as it is shot toward the top corner of the net, the steepness of a ski hill or hiking trail, the edges of a framed picture or doorframe, the lines in the letter Z, etc.
- This investigation is very important for students who learn more effectively by manipulating objects. You might wish to include lines with negative slope. For these, students need to flip the ruler over horizontally to the "opposite" side. The ruler needs to be flipped because the run is now –1 not +1, as it is when the line rises from left to right. The slope, then, will be the additive inverse of the value shown on the ruler.
- Make sure grid paper is available to students.
- You may wish to have the investigation completed as an individual assignment so students have the opportunity to explore on their own.

ELL

- Have students work with a partner on the Investigate so that they have assistance with the language and in completing the steps.

Answers

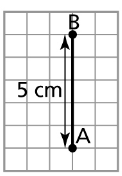
Investigate Slope

1. OC, OD, OA, and OB. Example: I compared the lines to a hill and considered which would be easiest and hardest to climb.

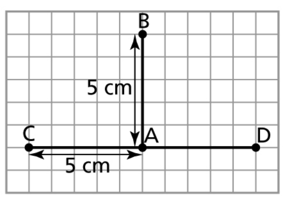
3. a) OA: 1.6 cm; OB: 2.6 cm; OC: 0.6 cm; OD: 1.2 cm

b) Example: The slope represents the number of units that the line rises for each horizontal unit.

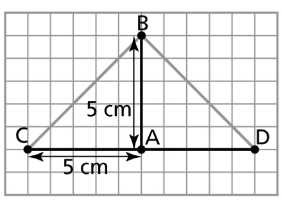
4.



5.



6. a)



$\frac{AB}{AC} = 1$

b) The ratio and the slope are equal.

7. a) Check that on one model mountain AB = 15 cm and AC = 5 cm, and on the other AB = 5 cm and AC = 10 cm.

b) The first mountain is steeper and the second mountain is less steep than the original mountain.

c) Example: The slope value for the first mountain should be greater than the slope of the original mountain, and the slope value for the second mountain should be less than the slope of the original mountain.

d) First mountain: $\frac{AB}{AC} = 3$ and the slope is 3.

Second mountain: $\frac{AB}{AC} = 0.5$ and the slope is 0.5.

8. a) Example: The lengths of AB and AC must be equal, say 2 cm. The slope is 1. Check that students drawings match their description.

b) 4 cm

c) Example: Yes. If the line intersects the slope ruler at 2.5 mm, the slope is $\frac{1}{4}$. If the slope ruler is constructed so that the length of the part of the toothpick that is exposed measures 4 cm and the line intersects the slope ruler at 1 cm, the slope is $\frac{1}{4}$.

d) Example: Yes. If a line intersects a slope ruler below the toothpick, the slope has a negative value.

e) $\frac{8}{2} = \frac{4}{1}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students might require coaching to understand that a larger scale model of the mountain drawn in #6 would have the same slope on all its sides. • Coach students in a quick reminder of scale models. You might ask how equivalent fractions relate to them. • Use the responses regarding equivalent fractions to assist them in solving #8c).

Link the Ideas

There is quite a bit of content in this Link the Ideas. You may want to have students express the main ideas in their own words, either in discussion with their group or in a whole-class discussion. The first idea is that of positive and negative slopes. While many students may find this intuitive, you may wish to have them look carefully at the four possibilities for the sign of the rise and run and ensure that they agree that this exhausts all possibilities.

You may want to allow some time for students to record a strategy of their choice. To allow for personal strategy, the student resource does not give an explicit algebraic formula for slope. Some students may want to develop one for their use, some may prefer to continue to work in terms of rise and run, and other students may have other methods. It may be productive to have students look back and make sure that their chosen strategy obtains the correct slopes for the line segments they worked with in the Investigate.

You may also want to invite students to discuss why the slope ruler they constructed works. If this is not obvious to them, you might prompt with guiding questions:

- What is the run represented by on your slope ruler when you use it?
- What is the rise represented by on your slope ruler when you use it?
- How do you get the slope from the rise and run?
- How is the slope related to the rise on your ruler? Explain why this is the case.

You may want students to think of rates of change when they consider slopes of horizontal and vertical lines. You might ask the following questions:

- Given a horizontal line, what rate of change does it show? How do you know?
- What does this rate of change mean about the rise?
- Given a vertical line, what rate of change does it show? How do you know?
- What does this rate of change mean about the run?

Students may be interested to know that in North America, m is the most commonly used variable to represent slope. Although there is speculation about the origins of its use, math historians have yet to determine why m became the popular choice. Also note that slope is not expressed with units.

Example 1

The class may complete this example quite quickly; for example, students might write *positive*, *negative*, *zero*, and *undefined* on four index cards or small pieces of paper. Then, as you call out each segment, in any arbitrary order, students hold up the card that shows the slope of that segment. Alternatively, you might use another method, such as an electronic student-response system.

Example 2

Encourage students to compare their personal strategy to the one shown in the solution. Ask students why the pitch is given as a positive value despite the fact that half of the truss has a negative slope. You may also want them to think about the following questions:

- Would it be possible to construct a truss of pitch zero? Explain.
- Would it be possible to construct a truss with undefined pitch? Explain.

Example 3

Students should be accustomed to the idea that many different strategies may be used in mathematics. Students may wish to debate the merits of each method, initially with a partner or small group and then perhaps as a class. Some of the questions they might consider in the debate are the following:

- Is each method valid? Explain.
- Is it possible to apply each method to different problems or situations? Explain.
- Will one method be more (or less) appealing to some types of learners? Explain.
- Is one method more appealing to you? Explain.
- Is each method equally efficient? Explain.
- Is each method easy to explain to another student, such as someone who missed math class?
- Is there another method that you prefer? If so, how would you answer the previously asked questions for that method?

The Your Turn is intended to help students decide on a personal strategy. If students have an alternative method that they prefer, invite them to use that strategy on the Your Turn questions to check that the strategy is correct and to confirm that it is preferable to the methods shown.

Example 4

Before beginning the example, you may want to ask students whether a point and a slope are enough information to graph a line. Allow a number of students to give their answers and rationale. Then, extend the discussion by asking if there is only one point that could be used to determine the line.

Next, make sure students have grid paper so that they can draw the line given in the example. Students can then check their work against the given solution. Then, have them complete the Your Turn. You may want to ask students to consider how they would draw a line with a slope that is an integer, such as 2 or –3. If they cannot answer readily, remind them with prompts:

- What are the numerator and denominator of an integer?
- How does this help you draw a line with that slope?

Example 5

Before they look at the graph, ask students to predict characteristics of the line based on the written information:

- In which quadrant would you expect to find the graph? Why?
- Can you predict the sign of the slope? Explain.
- What would a slope of zero look like? What would that mean in the context of a race?

To emphasize that the graph is a model, or approximation, of the boat's travel, ask students the following questions:

- What does the graph tell us about the way the boat travels? Is this realistic?
- Why would it be a good idea to use a simplified model?

Students should be able to quickly complete the Your Turn individually. You may want to have them compare responses with a peer. Alternatively, take a quick poll of the class to check understanding.

Key Ideas

Suggest that students put the Key Ideas in terms that are most convenient and memorable for them. They may wish to record the Key Ideas in their Foldable or math journal.

Meeting Student Needs

- When explaining rise and run, it is important students realize that the direction they move in has either a positive or negative value. For the rise, a movement up will have a positive value and a movement down will have a negative value. For the run, a movement right will have a positive value and a movement left will have a negative value. Emphasize that they should begin at point A and describe the rise and the run until they get to point B. A common mistake is to start at A, describe the rise, then start at B and describe the run. This mistake results in the slope having the wrong sign.
- Point out to students that the slope of the line is the same between any two points on the line. Take the time to plot a line with eight or more points and have pairs of students determine the slope between various points on the line. Record the results for all students to see.
- Once students have seen the formula for slope, $m = \frac{\text{rise}}{\text{run}}$, have them return to the investigation and determine the slope of each line.
- When the two-point slope formula is introduced, discuss the use of subscripts such as x_1 and x_2, and y_1 and y_2. You may wish to illustrate that the naming of the two points as 1 and 2 will make no difference to the outcome: both will produce the same slope. Also, ensure that students leave the signs of the formula intact and correctly substitute both positive and negative values into the formula. You may wish to show a colour-coded example of substitution.
- Example 3 develops the concept that horizontal lines have a slope of zero and vertical lines have an undefined slope. Have students create separate graphs for all three parts, to visualize the lines. Encourage them to connect the graphs to something in real life.
- Example 5 develops a concept that will also be used in physics. You may wish to provide one or two more examples for students.
- Some students may benefit from exploring slope online. See the related Web Link that follows in this Teacher's Resource.

ELL

- Suggest that students add *slope*, *rise*, and *run* to their vocabulary dictionary, Foldable, or other organizer.
- You may need to have English language learners work alongside another student for Examples 1 and 5, since there are a number of terms in these questions that they may not be familiar with.
- For Example 2, ensure students understand the connection between *width* and *span*, and between *slope* and *pitch*.
- You may need to help students with the terms *roof truss* and *undefined*.

Common Errors

- Some students may confuse slopes of zero with those that are undefined.

Rx You might help them by asking the following questions:
  - For which type of line is the run equal to zero?
  - For which type of line is the rise equal to zero?
  - Which type of line shows no change in the quantity?

- In Example 3, when using the slope formula, students may substitute the x-coordinates and y-coordinates in the wrong order:

$m = \frac{6-2}{5-(-3)}$.

Rx One way to assist students with this potential error is to guide them with prompts:
  - Did you make a small sketch of the points and line segment?
  - Does your answer seem appropriate given your sketch?

WWW Web Link

For online interactive activities related to slope, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

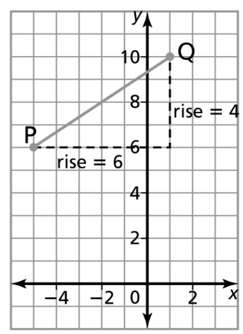
AB: positive; CD: neither; EF: negative; GH: negative; IJ: positive; KL: neither

Example 2: Your Turn

$\frac{1}{4}$. This means that the roof rises 1 unit for every 4 units of horizontal distance.

Example 3: Your Turn

a)



$m = \frac{2}{3}$

b) $m = -1$

Example 4: Your Turn

Example: (–3, 2), (0, 3), and (3, 4). Check that (–6, 1) and students' three other points are plotted on the graph.

Example 5: Your Turn

$\frac{500}{92}$, or approximately 5.43 m/s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • An alternative approach for students having difficulty could be to have them focus on the top of each line (the largest point) and ask whether the line would point to the right of the y-axis (positive slope) or to the left of the y-axis (negative slope). Vertical lines do not point left or right and therefore are undefined; horizontal lines, if extended, go left and right at the same height and therefore have a slope of zero.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure students understand that slope is $\frac{\Delta y}{\Delta x}$ and not $\frac{\Delta x}{\Delta y}$. • Remind students to reduce the fractional values of slopes where possible. • Ensure students know how to write an integer value for slope as a fraction to represent rise and run.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • When substituting into the slope formula, it is common for students to confuse the values of *x* and *y*. Having them label each ordered pair as (x_1, y_1) and (x_2, y_2) will assist them. • Caution students to watch for when they need to subtract negatives. • Coach students to realize that the placement of a single negative can appear in the numerator or denominator without changing the slope of the line. You may wish to model this case for students.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that slope is $\frac{\text{rise}}{\text{run}}$. Review with them the counting method that provides another approach to graphing a line. Count units downward for the negative rise value, and count upward for the positive rise value. Count units to the right for the positive run value, and count to the left for the negative run value. A slope of $\frac{-2}{5}$ would be down 2 and right 5. Ask students what would happen if it was $\frac{2}{-5}$.
Example 5 Have students do the Your Turn related to Example 5.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Review with students that rate of change can be thought of as a move from one ordered pair or location to another. • Coach them to see how the slope formula links to the calculation of rate. • Having them label each ordered pair as (x_1, y_1) and (x_2, y_2) will assist them.

Check Your Understanding

Practise

For #1, students may wish to refer to the Key Ideas, and for #2, they may wish to refer to Example 1.

Point out to students that #4 is related to Example 4. You may wish to refer students to the Key Ideas if they need assistance. Since the question includes lines with slopes that are integers, you may want to ask how students can rewrite a number like –4 as a ratio between the rise and the run.

For #5, to help students interpret the physical meaning of the slope in the situation, ask the following questions:
- What does the slope mean in words?
- Who would be concerned about this slope? Explain.

Apply

The task in #7 is similar to Example 4, so encourage students to look back at this example if necessary. Have students compare their explanation with at least one peer. To help them understand the meaning of the points on the graph, you could ask the following questions:
- What does each of the coordinates represent?
- What are the independent and dependent variables in the situation?
- The rise describes a change in which quantity?
- The run describes a change in which quantity?

For #8, guide students' thinking by asking the following questions:
- If you think of the slope as a ratio, what have we previously done in math to compare ratios?
- Why does $\frac{1}{16}$ represent a gentler slope?
- How can you use a diagram to show that $\frac{1}{16}$ is a gentler slope?

For #9, prompt students to recall previous learning by asking the following questions:
- What does it mean when a number is expressed as a percent?
- How can you write a percent in a different way so that it looks more like what we expect of a slope?

To help students with #10, you might ask the following questions:
- Which is the independent variable in this question?
- Which of rise or run refers to the independent variable?
- Which is the dependent variable in this question?
- Does *rise* or *run* refer to the dependent variable?

- How could a graph help you solve this problem?
- What units would you use to express the answer to this problem?
- How does that help you determine the correct slope?

In #11, students will likely need to sketch a graph representing the situation. You might ask the following questions:
- Which quantity should be graphed on the x-axis? Why?
- Which quantity should be graphed on the y-axis? Why?

When students have obtained answers, have them look at the signs of their slopes and ask them how they know whether the slopes are appropriate.

Students need to recognize that the numbers in #12 are written in two different forms. As they sketch a graph for the situation, you might ask the following questions:
- How would you label the values on the axes?
- What is another way to write a number like 2.8 million?

A real-life application of slope is presented in #13. Assist students' understanding by asking the following questions:
- What other representation or representations of this function can you use to make determining slope easier?
- How can you tell which quantity is changing in the run?
- How can you tell which quantity is changing in the rise?

In part c), students solve a proportion. Ask them what strategy they have used previously when they know two ratios are the same but do not know one part of one ratio.

Extend

For #14, have a number of boxes available to help students visualize the situation. Students will need to use the Pythagorean Theorem to determine the run along the diagonal of the base of the box. Guide them by asking the following questions:
- What type of triangle is formed by the three vertices indicated in the diagram?
- Are other triangles of this type part of this situation?
- What strategy or strategies do we have for working with this type of triangle?

For #15, to emphasize that volume, not length, is the quantity under consideration, you may wish to ask students what they would place on the y-axis to sketch a graph.

At first, #16 may look very different to students. Ask them to think of the rule they use to determine the slope and apply it without regard to the appearance of the coordinates. For example, to show a pattern, ask the following questions:
- How do you determine the slope if your coordinates are (1, 3) and (4, 15)?
- How do you determine the slope if your coordinates are (–2, 1) and (5, –13)?
- How do you determine the slope if your coordinates are (0, 0) and (–3, –12)?

Then, ask students to apply the pattern to the coordinates given.

Create Connections

You may wish to make #17 more accessible to students by prompting their thinking:
- What does the word *constant* mean in English?
- What does the word *constant* mean in math?
- For any line, does it matter which two points you use to determine the slope?

For #18, you may need to assist students by using guiding questions:
- What type of triangles are you able to analyse using trigonometry?
- How can you form that sort of triangle if you know a slope?

For the Mini Lab, #19, have students work in small groups. After groups have completed Step 1, you may want them to compare answers to ensure that they understand how to read a topographic map. As you monitor students' work, watch for correct units in Step 3. When groups have finished the question, you may want to have a whole-class discussion for students to compare answers and hear alternative explanations and rationales.

Meeting Student Needs

- Provide **BLM 6–9 Section 6.5 Extra Practice** to students who would benefit from more practice.
- For #2, remind students to use the formula $m = \frac{\text{rise}}{\text{run}}$.

- When students graph lines given a point and a negative slope, encourage them to put the negative sign on the "rise" and always "run" to the right. Some students may be confused unless directly informed of how to deal with a negative value.
- When first viewed, the graph in #5 may be confusing to some students. Ask them to describe why this graph appears in quadrant IV.
- For #7, students interested in pursuing carpentry might be encouraged to determine the regulations for ramps in their own province.

ELL

- You may wish to have students work with a partner on #8, 10, 11, and 19 due to the challenging language.
- Students may need assistance in understanding the terms *mountain pine beetle*, *infested*, *lightning*, and *thunderclap*. Use descriptions and visuals to clarify.

Enrichment

- Have students list potential problems when graphing a relation to show real-life information. For example, they might mention the zero-denominator problem, the vertical-line issue, or the difference between discrete points and lines. (Example: When showing time versus the population of an endangered species, for small numbers of the species, it is incorrect to use a line rather than discrete points.)

Gifted

- Have students create a list of ordered pairs for a linear graph that includes fractions, square roots, and negative numbers. Have them draw the graph of the ordered pairs. (Check that students create a list of values that can be graphed, yet meet the criteria. One strategy is to use such numbers as $\sqrt{4}$ for the square root and $\frac{4}{1}$ for the fraction.)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2, 3a)–d), 4, 5, 7, 8, 17, and 19. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 and 2 may wish to review the Link the Ideas, Key Ideas, and Example 1. • For #2 and 3, review the meaning of *slope* and the terms *rise* and *run*. Have students link rise and run to the parts of the slope formula related to each. A common mistake for students is to confuse the rise and run in the calculation. Coach them through one of the parts in each question. Some students may find "counting" the vertical changes and then the horizontal changes an easier way to determine the slope. • For #4, clarify that students know how to interpret the negative sign. Show them that it will not alter the final answer whether the negative is applied to the numerator or to the denominator. • Students who prefer to count the slope rather than use the formula must remember that a downward count is interpreted as a negative when solving #5. Have them identify the rise units and the run units so that they use the correct terminology. • For #7, ask students to provide an equivalent value for –3 in fraction form. This will assist those looking for the run value in the slope. • For #8, encourage students to draw a diagram and coach them through the labelling of it. You may wish to go over a few equivalent fraction questions before asking students to solve the shortest length of the ramp.
Assessment *as* Learning	
Create Connections Have all students complete #17 and 19.	• For #17, ask students to explain their thinking behind the meaning of *constant* (a number that does not change). Have them verbalize why the meaning is important when they draw a line. Have them draw a line with a slope of their own choice and ask them to select any two points on their line. Ask what the rise and run are. Encourage them to generalize their thinking to answer the question. • All students should work through #19. Put students in pairs or small groups. The information and processes used in this Mini Lab will assist them with their project. Coach students through the contour line map if they have never worked with one before. Select one of the mountains and review the indicated elevations, how they were determined, and what they mean. Some students may have a difficult time visualizing the map.

6 Chapter 6 Review

***Mathematics 10*, pages 330–334**

Suggested Timing

60–90 min

Materials

- grid paper
- ruler
- graphing calculator

Blackline Masters

BLM 6–5 Section 6.1 Extra Practice
BLM 6–6 Section 6.2 Extra Practice
BLM 6–7 Section 6.3 Extra Practice
BLM 6–8 Section 6.4 Extra Practice
BLM 6–9 Section 6.5 Extra Practice

Planning Notes

Have each student complete the review for the chapter individually, perhaps with reference to their notes and their Foldable or math journal, as well as the student resource. Suggest that they note the questions that they are unable to complete. Have students work in groups of two to four to solve these problems collaboratively. Suggest that they use previously unassigned questions in the student resource to practise areas that require remediation.

Have stuggling students discuss strategies with you or a classmate. Encourage them to refer to their notes, worked examples, and previously completed questions in the student resource.

Have students make a list of questions that they needed no help with, a little help with, and a lot of help with. They can use this list to help them prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to BLM **6–5 Section 6.1 Extra Practice**, **BLM 6–6 Section 6.2 Extra Practice**, **BLM 6–7 Section 6.3 Extra Practice**, **BLM 6–8 Section 6.4 Extra Practice**, and **BLM 6–9 Section 6.5 Extra Practice**.
- Have students refer to all information posted in the classroom that is related to the chapter: student learning outcomes, diagrams, and definitions. Encourage students to analyse their own understanding and to create a list of topics that require further investigation for understanding. This exercise will assist students in preparing for the summative assessment.
- Have students choose the specific areas in which they require the most review. Group students with others who need to review the same material. Once they have completed these review questions, have them proceed to other review questions.
- For #3, students may benefit from using volume manipulatives, "measuring" the depth, and then recording the data in a table. This data could be graphed to allow students to make the connections.
- Allow students to have access to grid paper and graphing calculators.

ELL

- Encourage students to refer to their vocabulary dictionary, Foldable, and any other organizer they have been using as they work on the questions.

Gifted

- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 6 Review The Chapter 6 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test. • Consider assigning students #1, 3–10, 13, 14, and 16 as the minimum questions that will cover the chapter.

Chapter 6 Practice Test

6

***Mathematics 10*, pages 335–337**

Suggested Timing

50–60 min

Materials

- grid paper
- ruler
- graphing calculator

Blackline Masters

BLM 6–10 Chapter 6 Test

Planning Notes

Unlike the questions in the chapter review, the questions in the practice test are not grouped according to the sections in the student resource. Use the Study Guide below to refer students to the appropriate place in the student resource if they have difficulty with a question in the practice test. Have students complete the practice test individually.

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–8, and 10.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	6.2	Example 1	✓ determine if a relation is linear
#2	6.5	Example 2	✓ determine the slope of a line
#3	6.3	Examples 1, 3	✓ express domain and range in a variety of ways
#4	6.1	Example 1	✓ sketch a graph for a given situation
#5	6.4	Example 1	✓ sort relations into functions and non-functions
#6	6.5	Examples 3, 4	✓ determine the slope of a line ✓ use slope to draw lines
#7	6.5	Example 3	✓ understand slope as a rate of change ✓ solve problems involving slope
#8	6.4	Example 2	✓ use notation specifically designed for functions
#9	6.3	Example 3	✓ express domain and range in a variety of ways
#10	6.3	Example 1	✓ express domain and range in a variety of ways
#11	6.1	Example 3	✓ sketch a graph for a given situation

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 6 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their chapter Foldable.	• Have students use their responses on the practice test and work they have completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 6 Test After students complete the practice test, you may wish to use **BLM 6-10 Chapter 6 Test** as a summative assessment.	• Consider allowing students to use their Foldable. • Allow students to have access to grid paper and graphing calculators.

Linear Equations and Graphs

7

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF5 Determine the characteristics of the graphs of linear relations, including the:
- intercepts
- slope
- domain
- range.

RF6 Relate linear relations expressed in:
- slope–intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope–point form ($y - y_1 = m(x - x_1)$)

to their graphs.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
7.1	✓ identify the slope and *y*-intercept of a straight-line graph
	✓ determine a linear equation using slope and *y*-intercept
	✓ rewrite a linear relation in slope-intercept form
	✓ graph equations in slope-intercept form
	✓ solve problems using equations in slope-intercept form
7.2	✓ convert a linear equation to general form
	✓ use intercepts to graph a line
	✓ relate the intercepts of a graph to the situation
	✓ solve problems using equations in general form
7.3	✓ write the equation of a line from its slope and a point on the line
	✓ convert equations among the various forms
	✓ write the equation of a line from two points on the line
	✓ solve problems involving equations in slope-point form
7.4	✓ identify whether two lines are parallel, perpendicular, or neither
	✓ write the equation of a line using the coordinates of a point on the line and the equation of a parallel or perpendicular line
	✓ solve problems involving parallel and perpendicular lines

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 7–1 Chapter 7 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 338 in *Mathematics 10* to activate students' prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about linear relations. You might provide the following prompts: • Where have you encountered a graph of a linear relation? • What were the variables involved in the graph? • Did you know the equation of the graph? How could you determine it? • When might you have seen two lines graphed together? What story could you make up about the graph?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 7–2 Chapter 7 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 7 Foldable As students work on each section in Chapter 7, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter.
Assessment *for* Learning	
BLM 7–3 Chapter 7 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mathematics calculations.

Chapter 7 Planning Chart

					Assessment		
Section/ Suggested Timing	**Prerequisite Skills**	**Materials/Technology**	**Teacher's Resource Blackline Masters**	**Exercise Guide**	**Assessment *as* Learning**	**Assessment *for* Learning**	**Assessment *of* Learning**
Chapter Opener • 30–40 min (TR page 257)			BLM 7–1 Chapter 7 Self-Assessment BLM 7–2 Chapter 7 Prerequisite Skills BLM 7–4 Chapter 7 Unit 3 Project BLM U3–2 Unit 3 Project Checklist		TR page 256 Chapter 7 Foldable, TR page 256	TR page 256	
7.1 Slope-Intercept Form • 100–140 min (TR page 259)	Students should be familiar with • plotting ordered pairs • independent and dependent variables • drawing a line graph • slope and *y*-intercept • algebraic modelling • isolating a variable • integer operations • evaluating expressions	• two metre sticks • elastic band • foam cup • paper clips, string, or tape • toothpick or straightened paper clip • six identical marbles or other items of equal mass • ruler • grid paper or graphing technology	BLM 7–3 Chapter 7 Warm-Up BLM 7–4 Chapter 7 Unit 3 Project BLM 7–5 NATO Emblem on a Grid BLM 7–6 Section 7.1 Extra Practice TM 7–1 How to Do Page 343 Example 1 Using TI-Nspire™ TM 7–2 How to Do Page 343 Example 1 Using Microsoft® Excel	**Essential:** #1, 3–6, 8–10, 13, 14, 18, 24–26 **Typical:** #1, 2, 4–7, 9, 10, 12–14, 15 or 16, 18, 24–26 **Extension/Enrichment:** #17, 19–24, 26	TR pages 262, 267 Chapter 7 Foldable, TR page 256	TR pages 264, 265, 267	
7.2 General Form • 80–120 min (TR page 268)	Students should be familiar with • domain and range • drawing a line graph • slope-intercept form of the equation of a line • least common denominator • evaluating expressions	• bottle of water • stopwatch • grid paper and ruler, or graphing technology	BLM 7–3 Chapter 7 Warm-Up BLM 7–7 Section 7.2 Extra Practice	**Essential:** #2, 3, 5, 6, 8, 10, 18, 19, 21 **Typical:** #1–7, 10 or 11, 12 or 13, 15, 18, 19, 21 **Extension/Enrichment:** #1, 4, 14–17, 20, 21	TR pages 270, 275 Chapter 7 Foldable, TR page 256	TR pages 272, 273, 274	
7.3 Slope-Point Form • 80–120 min (TR page 276)	Students should be familiar with • slope-intercept form of the equation of a line • general form of the equation of a line • isolating a variable • distributive property • evaluating expressions	• grid paper • ruler • SI measuring tape	BLM 7–3 Chapter 7 Warm-Up BLM 7–4 Chapter 7 Unit 3 Project BLM 7–8 7.3 Investigate: Figure 1 BLM 7–9 Section 7.3 Extra Practice	**Essential:** #1–3, 5a), b), 6a), c), d), 7–9, 11, 21–24 **Typical:** #1–5, 7, 8, 10, 11, 14, 24 **Extension/Enrichment:** #12, 13, 15–20, 22, 24	TR pages 278, 281 Chapter 7 Foldable, TR page 256	TR pages 280, 281	
7.4 Parallel and Perpendicular Lines • 80–120 min (TR page 282)	Students should be familiar with • meaning of parallel and perpendicular • slope-intercept form of the equation of a line • general form of the equation of a line • slope-point form of the equation of a line • isolating a variable • distributive property • evaluating expressions	• grid paper • scissors • ruler	BLM 7–3 Chapter 7 Warm-Up BLM 7–10 Section 7.4 Extra Practice	**Essential:** #1–5, 6a), c), e), 7a), c), e), 9–11, 25, 26 **Typical:** #1–5, 6a), c), e), 7a), c), e), 8, 11, 13, one of 14–16, 25, 26 **Extension/Enrichment:** #12, 14–19, 24–26	TR pages 283, 287 Chapter 7 Foldable, TR page 256	TR pages 285, 287	
Chapter 7 Review • 60–90 min (TR page 288)		• grid paper • ruler	BLM 7–6 Section 7.1 Extra Practice BLM 7–7 Section 7.2 Extra Practice BLM 7–9 Section 7.3 Extra Practice BLM 7–10 Section 7.4 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 7 Foldable, TR page 256	TR page 289	
Chapter 7 Practice Test • 45–60 min (TR page 290)		• grid paper • ruler	BLM 7–11 Chapter 7 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1–9, 12	TR page 291		TR page 291 BLM 7–11 Chapter 7 Test
Unit 3 Project • 80–100 min (TR page 292)		• ruler	Master 1 Project Rubric BLM 6–4 Chapter 6 Unit 3 Project BLM 7–4 Chapter 7 Unit 3 Project BLM U3–3 Unit 3 Project Final Report				TR page 294 Master 1 Project Rubric
Unit 3 Review and Test • 60–90 min (TR page 296)		• grid paper • ruler	BLM 7–12 Chapter 7 BLM Answers	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapters 6 and 7 Foldables	TR page 296	TR page 296

Linear Equations and Graphs

Opener

***Mathematics 10*, pages 338–339**

Suggested Timing

30–40 min

Blackline Masters

- BLM 7–2 Chapter 7 Prerequisite Skills
- BLM 7–4 Chapter 7 Unit 3 Project
- BLM U3–2 Unit 3 Project Checklist

Key Terms

y-intercept	*x*-intercept
slope-intercept form	slope-point form
parameter	parallel lines
general form (of a linear equation)	perpendicular lines

What's Ahead

This chapter introduces students to three different forms of writing linear equations: slope-intercept form ($y = mx + b$), general form ($Ax + By + C = 0$), and slope-point form ($y - y_1 = m(x - x_1)$). Students explore the information that can be easily obtained from each form, such as the y-intercept, slope, or x-intercept, and they use this information to graph linear equations. Students create their graphs using paper and pencil and with technology.

Students convert linear equations among the three forms presented and they use the slope-point form to determine the equation of a line given a graph. Students determine the equation of a line that is parallel or perpendicular to a given line using a specified point that lies on the line.

Planning Notes

Begin Chapter 7 by having students discuss what aspects of their daily life might involve a linear equation. Invite students to suggest an example of what one equation would be. Then, have students compare the forms that they wrote the equations in. You may wish to write two or three sample equations that are in different forms on the board and then ask students the following questions:

- Do these equations represent the same relationship?
- What variables are involved in this relation?
- In what ways are these equations similar? In what ways do the equations differ?
- How could you get one equation to resemble the other equation?

Unit Project

You may wish to discuss the Unit 3 project described in the Unit 3 opener. Unit project questions are integrated throughout the chapter. The questions are not mandatory, but are recommended, because they provide some of the work needed for the Unit 3 project assignment. You will find questions related to the project in the Check Your Understanding of sections 7.1 and 7.3. You may wish to introduce the project by discussing the popularity of mystery novels, television shows, and movies. As background information, you may wish to share the history of the Klondike Gold Rush.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 339 and how it might be used to summarize Chapter 7. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. You may wish to suggest the four-door fold as an alternative design. Allowing personal choice in this way will increase student ownership of their work.

Give students time to develop the summary method. You may wish to suggest that the concept map provided in the text be incorporated into the Foldable as a summative tool. Ask students to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

Ensure that students understand that linear relations can be determined, written, and graphed in a variety of ways. The advantage to the Foldable design used in this chapter is that each section has its own blank two-page spread for notes, key definitions, reflections on the students' own learning, and a grid page that provides an opportunity to model the graph associated with an equation. Encourage students to write their own examples, using those in the textbook as a model, and have a partner check their equations and graphs.

The final tab provides a location for students to complete or keep a record of the questions that have been completed relating to the unit project. An additional benefit of this design is that pages can be easily added to the Foldable.

As students progress through the chapter, provide time for them to keep track of what they need to work on. The back of the Foldable could be used for students to record samples of project work. The checklist for the unit project provided on **BLM U3–2 Unit 3 Project Checklist** could be stapled to the back of the Foldable and students could then keep track of the project-related questions and concepts that they have completed.

Meeting Student Needs

- Consider having students complete the questions on **BLM 7–2 Chapter 7 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Hand out to students **BLM U3–2 Unit 3 Project Checklist**, which provides a list of all of the requirements for the Unit 3 project.
- Some students may benefit from completing all unit project questions.
- **BLM 7–4 Chapter 7 Unit 3 Project** includes all of the unit project questions for this chapter. These questions provide a beginning for the Unit 3 project.
- Have students discuss where they may have seen equations used in their life. Encourage students to discuss equations with one or both of their parents to determine how equations affect their life. Have students report their findings to the class or fill in a short questionnaire next class.
- To help students relate to the chapter opener, you may wish to discuss the kinds of archaeological material Inuit have left behind. Brainstorm a list of information that might be obtained from tent rings, kayak stands, and discarded tools. You may wish to discuss the kinds of things our present cultures will leave behind and what this says about us.

Enrichment

- When real-life data is used to create an expression, outliers are often not included in the expression. This is because outliers are often thought to be incorrect or errors. Challenge students to describe a real-life situation where a table of values contains an outlier that should not be ignored. For example, outliers that are appropriate include black holes in mass data or diamonds in hardness testing.

Gifted

- The equation $d = vt$ is used to find distance travelled by an object moving at a certain velocity for a certain period of time. Einstein found that as objects move at velocities approaching the velocity of light, time slows down. Challenge students to speculate what this theory means to the graph of distance versus time for objects travelling close to the velocity of light. (Example: As velocity approaches the velocity of light, the time axis becomes extended between values as the value of time itself changes.)

Career Connection

Several careers that use linear equations are mentioned in the chapter opener. These include archaeologists, paleontologists, police detectives, forensic analysts, developers, and civil engineers. Have students who are interested in any of these careers explore newspapers, the Internet, and local employment offices to find out what career opportunities exist in their local area and how that particular career uses mathematics.

WWW Web Link

For information about some of the careers mentioned in the chapter opener, go to www.mhrmath10.ca and follow the links.

Slope-Intercept Form

7.1

***Mathematics 10*, pages 340–356**

Suggested Timing

100–140 min

Materials

- two metre sticks
- elastic band
- foam cup
- paper clips, string, or tape
- toothpick or straightened paper clip
- six identical marbles or other items of equal mass
- ruler
- grid paper or graphing technology

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–4 Chapter 7 Unit 3 Project
BLM 7–5 NATO Emblem on a Grid
BLM 7–6 Section 7.1 Extra Practice
TM 7–1 How to Do Page 343 Example 1 Using TI-Nspire™
TM 7–2 How to Do Page 343 Example 1 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF5 Determine the characteristics of the graphs of linear relations, including the:

- intercepts
- slope
- domain
- range.

RF6 Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y – y_1 = m(x – x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3–6, 8–10, 13, 14, 18, 24–26
Typical	#1, 2, 4–7, 9, 10, 12–14, 15 or 16, 18, 24–26
Extension/Enrichment	#17, 19–24, 26

Unit Project Note that #18 is a Unit 3 project question.

Planning Notes

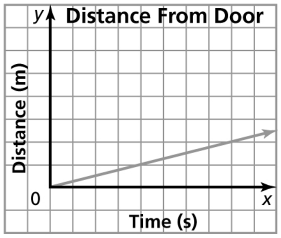
Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections. It is important that students be able to determine the slope of a graph. Have students discuss situations in their own life that may be modelled by the graph of a straight line. One example could be the relationship between costs on a prepaid cell phone and the calling time. You may wish to begin the discussion by asking the following questions:

- If you own a cell phone, under what conditions can a graph of a straight line model the amount you are charged?
- Besides cell phones, what other examples in your life can be modelled by the graph of a straight line?

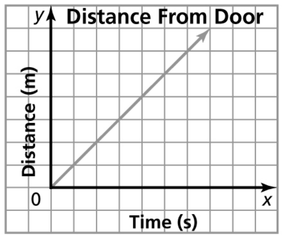
Make a list of students' examples. As you work through the chapter, add new examples to the list as they come up.

As an introduction to the chapter, you may wish to show students the following graphs. Each graph represents the distance, in metres, a student is from the door of the classroom after t seconds.

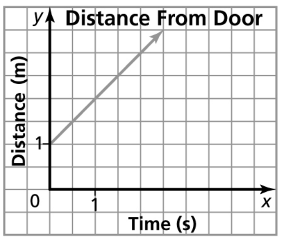
Graph A



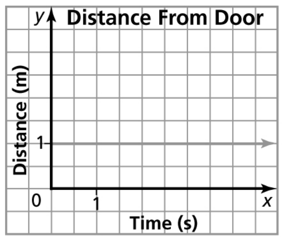
Graph B



Graph C



Graph D

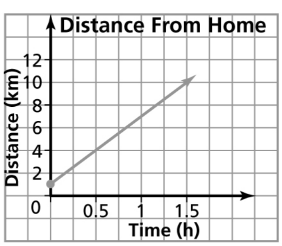


Ask students to compare and contrast the graphs. You may wish to use the following questions:

- How are the graphs similar? How do they differ?
- What are the independent and dependent variables?
- What does the slope of each line represent?
- The y-intercept in Graphs C and D is 1. What is a y-intercept? In this situation, what does the y-intercept represent?

Have students act out each graph.

Alternatively, you may wish to present students with the following scenario: In the morning, Adam joins his father for a walk. The graph shows the distance, in kilometres, Adam and his father are from their house, after t hours.



Have students determine the equation of the line. Then, ask them if that is the only way to express the equation. Many students may not be able to determine an equation. In this chapter, students will learn different strategies to determine the equation of a straight-line graph. The first method is called slope-intercept form. You may wish to return to the graph as a summary activity after completion of the Investigate and Link the Ideas sections.

Investigate the Graph of a Linear Equation

This Investigate could be done as a class demonstration if necessary. You could also have students set up different stations in which the elastic band, masses, and /or foam cup size vary. Alternatively, you may wish to have different-sized materials available for students to conduct their own investigation with. When gathering the materials, long, thin elastic bands work best. Make sure the elastic band is not stretched beyond its elastic limit. It should always return to its original length when the masses are removed. The items used as masses must have enough mass to result in a measurable extension of the elastic band. Instead of marbles, bolts or washers may be used. Various types of candies, such as miniature chocolate bars, may also work well.

You may wish to discuss with the class what a scatter plot is and how it shows the relationship between two variables. Have students explain why it is or is not appropriate to connect the points on this graph. You may need to remind students of the terms *discrete* and *continuous*. Ask students to describe discrete and continuous data. Technology could be used to find the equation of the line or to check the accuracy of the equations found from students' hand-drawn lines.

This investigation is based on Hooke's law, named after the English scientist Robert Hooke (1635–1703). Students may be studying this relationship further in science. The relationship between the expansion or compression of any spring and the force acting on the spring is linear, within the elastic limits of the spring. This relationship makes springs very useful. Ask students to provide examples of how springs are used in everyday life. Possible applications include vehicle suspension springs, garage door springs, anti-vibration springs, bathroom scales, grocery produce scales, and road-side truck scales.

Meeting Student Needs

- Students may need to review how to identify independent and dependent variables and how to place them on the graph. Students may also benefit from reviewing the definition of a linear function/relation.
- Students may need to extend their graphs. You may wish to take the opportunity to discuss interpolation (estimating coordinates of points lying *between* plotted points) and extrapolation (estimating points lying *beyond* those that are plotted).

- When discussing the section opener, you may wish to have students discuss whether food for a dog team could be modelled by a linear equation.

Common Errors

- Some students determine slope as the ratio of Δx to Δy, or run over rise.

Rx Remind students of the meaning of slope: the steepness of a line. You may wish to provide students with two graphs: Graph A has a line with slope 3, and Graph B has a line with slope $\frac{1}{3}$. Students can clearly see that Graph A is steeper and therefore should have the greater slope. Graph B is less steep and therefore should have the smaller slope. Have students determine the slope both ways, i.e., rise over run and run over rise. This will help students see which formula matches what they know to be intuitively true.

Graph A

$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{\text{run}}{\text{rise}}$

$m = \frac{3}{1}$ $m = \frac{1}{3}$

Graph A should have the greater slope.

$m = \frac{\text{rise}}{\text{run}}$ gives the expected slope.

Graph B

$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{\text{run}}{\text{rise}}$

$m = \frac{1}{3}$ $m = \frac{3}{1}$

Graph B should have the smaller slope.

$m = \frac{\text{rise}}{\text{run}}$ gives the expected slope.

- Students mix up positive and negative slopes.

Rx Reinforce the concept of positive and negative slope by teaching students that graphs "run to the right." To determine the slope of a line, have students pick any point on a graph and run to the right. To reach another point on the line, they would have to go up (+ slope) or down (– slope).

WWW Web Link

To view a video showing how the extension of a spring changes when different forces are applied, go to www.mhrmath10.ca and follow the links.

Answers

Investigate The Graph of a Linear Equation

3. The independent variable is the number of marbles. The dependent variable is the length of the apparatus.

4. Example: The slope of the line is 0.5. The units are centimetres per gram. The slope of the line represents an increase of 0.5 cm in the length of the apparatus per gram.

5. a) Example: The line intersects the y-axis at (0, 14). This point represents the length of the apparatus before any marble is placed in the cup.

b) Example: Multiply the slope by the number of marbles and add 14.

6. Example: 19 cm; Yes.

7. a) Example: $y = 0.5x + 14$

b) Example: 21.5 cm

8. a) Example: All measurements will be 4 cm more than the initial measurements.

b) Example: The graph will be vertically translated 4 units up.

c) Example: $y = 0.5x + 18$

9. a) Example: Measurements in #2 will be twice the apparatus length compared to the initial measurements.

b) The slope will be steeper, 2(0.5) or 1.

c) Example: $y = x + 14$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Some students may benefit from having extra time to try the suggested change in #8. Without completely redoing the Investigate, suggest that they physically hold the larger cup in the place of the original. This may prompt their thinking about the effects on the *y*-intercept. • For #9, provide sufficient time for students to try one or two objects of different masses. It may benefit some students to physically try to visualize the effects of the change before predicting how the graph will change. • You may wish to divide the class into two groups and have one group complete and respond to #8 while the other group completes #9. If students completed trials, have representatives present their results of #8 and #9 to the rest of the class.

Link the Ideas

To help students remember the equation $y = mx + b$, they can think of b as the beginning y-value when x is zero, and m as the rate or slope needed to move to another point on the line. Discussing the information in the table will reinforce these concepts. You may wish to have students extend the table. You could fill in the first column with a table of values and have students complete the other four columns. You may wish to have a class discussion about the following questions:

- When calculating slope, does it matter which coordinates are chosen? Why?
- For a particular example, why is Δy positive or negative? Why is Δx positive or negative?

This will help students understand the meaning of slope rather than merely applying a formula.

Example 1

Students should be able to do this without difficulty. They could also find the slope of the line by picking two “nice” points and counting the rise and run between the two points. Encourage students to determine the sign of the slope by the direction of the line in the graph. This will help them confirm that their answer has the correct sign. Remind students to reduce any fractions for slope to lowest terms. You may wish to draw students’ attention to coefficients of 1 and 0. Remind students that $x = 1x$, $-x = -1x$, and $0x = 0$. Students may benefit from making these distinctions.

You may wish to draw students’ attention to the order of the solution for part a). The question asks for the slope and y-intercept of the line. The first answer provided is the y-intercept, not the slope, because the y-intercept is easily read off the graph without performing any calculations. Encourage students to always try to answer the parts of a question that they are most confident with and can answer quickly first.

For part c), students need to use graphing technology to check their equation. You may wish to have students use **TM 7–1 How to Do Page 343 Example 1 Using TI-Nspire™** or **TM 7–2 How to Do Page 343 Example 1 Using Microsoft® Excel** to work through this solution.

Example 2

You may wish to discuss the positive and negative values of y within the context of this question (making money, losing money) and what a value of y of 0 means. Students may have difficulty grasping the concept of breaking even. You may want to discuss other situations that could involve a break-even point.

Emphasize to students that there are several methods for isolating a variable in an equation. You may wish to have a discussion about some strategies for reducing the number of steps and thereby reducing the possibility of introducing an error. One such strategy is shown in the Mental Math and Estimation box beside the solution.

Example 3

Work closely with students when they are defining their variables. Ensure that students choose variables meaningfully and with the proper units. Ask students to indentify the independent and dependent variables and to justify their choices.

In part c), students are asked to identify and interpret the meaning of the y-intercept. You may wish to have sudents do the same for the x-intercept. Students have likely worked with parameters before, but they may not be familiar with the term. You may wish to have students provide some examples of parameters they have used, perhaps from science class.

Although it is virtually unknown, Canada has a distinguished heritage of submarine service that dates back to 1914. As part of the British fleet during World War II, Canadian submariners were involved in every type of naval operation. Since 1961, the Canadian Navy has had at least one submarine in its fleet. You may wish to discuss the following questions as a class or have students interested in this topic research it further and report back to the class.

- Why is it important for Canada to have operational submarines?
- What are the advantages to having non-nuclear submarines?

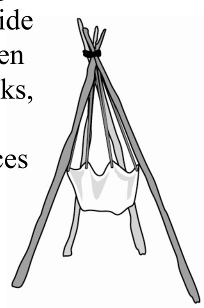
Example 4

This example uses a real-life situation to reinforce the concepts in this section. Make sure students have a solid understanding of the previous examples before working through this one. Emphasize to students the importance of using brackets to show a number being substituted. Some students may need to be reminded of the order of operations.

You may wish to write the slope-intercept form of a linear equation on the board with (x, y) beside it. Then, write the equation for boiling water with (t, W) below it to help students understand the link between the equations. Colour-coding the similar variables might also be helpful.

Draw students' attention to the Did You Know? box beside the solution. The property described can be referred to as the point-on property (POP). Make sure students understand this concept, because it will be useful when they work through many problems involving equations.

In addition to the method described in the student resource, First Nations people used another method of boiling water. They used to hang a sac made from a bison stomach or piece of hide on a wooden stand. The sac was then filled with water. Using forked sticks, red-hot rocks were scooped from a campfire and added to the sac. Pieces of meat and vegetables were then added to make a soup or stew.



Key Ideas

Remind students to always include the following when graphing:

- labels for the axes, equation of the line, and scale on the vertical and horizontal axes
- labels for the coordinates of the x-intercept and y-intercept

Meeting Student Needs

- Allow students to use manipulatives, such as algebra tiles, when isolating y in an equation.
- You may wish to create an overhead displaying the table from the Link the Ideas and then use markers to highlight the slope and y-intercept in each graph and corresponding equation.
- Colour-coding may be useful when working through Example 1.
- Allow students to work through Example 4 in pairs or small groups. Encourage discussion and a written explanation of each group's findings.

Enrichment

- Challenge students to compare the lines $x = 0$ and $y = 0$ in terms of slope-intercept form. ($y = 0x + 0$ is appropriate, but $x = 0$ is a vertical line.)

Gifted

- Ask students to create a poster that shows three applications of slope-intercept form that meet the following criteria: they are real-life applications, they show a step-by-step approach to using the slope-intercept form, and both slope and y-intercept are shown.
  (Examples might include slope of a distance-time graph representing speed and the y-intercept representing initial starting point.)

Common Errors

- Many students have difficulty converting equations to slope-intercept form.

R_x Remind students that there are many strategies they can use to isolate y in an equation. Encourage students to use the methods they are comfortable with. You may wish to have different students show their methods and discuss some of the strategies they use and why they prefer a certain method.

- Students may have difficulty interpreting the meaning of slope and y-intercept in a context.

Rx To help students interpret the meaning of slope in a particular situation, direct them to the units being used. To interpret the meaning of the y-intercept, ask students what happens when the independent variable is zero.

WWW Web Link

To learn how to make a dunk tank, go to www.mhrmath10.ca and follow the links.

For a graphical representation of loss, break-even, and profit, go to www.mhrmath10.ca and follow the links.

For information on how submarines work, go to www.mhrmath10.ca and follow the links.

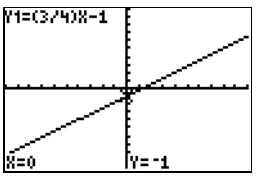
Answers

Example 1: Your Turn

a) The slope is $\frac{3}{4}$ and the y-intercept is –1.

b) $y = \frac{3}{4}x - 1$

c)



Example 2: Your Turn

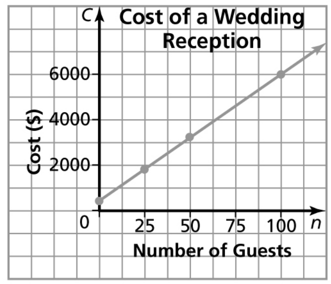
a) The slope of the line is 12. It represents income of \$12 per ticket.

b) The y-intercept is –840. It represents the cost of \$840 to rent the hall.

c) 70 tickets

Example 3: Your Turn

a)



b) The slope of the line is 55. This means that the price increases by \$55 per guest. The y-intercept is 425, and it represents the cost of renting the ballroom.

c) Let C represent the cost, in dollars. Let n represent the number of guest. $C = 55n + 425$

d) \$8125

e) 265 guests

Example 4: Your Turn

a) \$150 **b)** 11 h

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may need coaching and a review of the terms *slope* and *y-intercept*. • Remind students to select two ordered pairs that are “nice” or fall exactly on grid values that make them easier to read when calculating slope. • Some students may benefit from using the counting method to determine the slope from the y-intercept. Have students start at the y-intercept and locate the closest ordered pair that is on the line and made up of integer values. Then, have students count from the y-intercept the number of units up (+) or down (–) and then left (–) or right (+) to the ordered pair. The up or down count represents the rise, and the left or right count represents the run. • Once students have completed the count, ensure they can correctly place the slope and y-intercept into the form $y = mx + b$.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students will have difficulty with the interpretation of the slope and/or *y*-intercept. They may become proficient at identifying the values and the process to graph them, but are unable to understand their meaning. Coach students through rewriting the general form into slope-intercept form. Have them verbally identify the slope and the *y*-intercept. Ask students to identify the dependent and independent variables from the question. Then, ask students how the slope and *y*-intercept are related to the problem. • Remind students they are not asked to graph this question, but if they choose to do so, they may need assistance in choosing appropriate scales for the axis. • Help students to understand that at the break-even point, revenue equals expenses. If necessary, provide an example with smaller numbers. For example, you buy ten pencils at 15¢ each. How many pencils do you have to sell at 25¢ each to break even?
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that graphing from a table means to write ordered pairs from the data. Ask them to identify what the ordered pair represents. • Remind students that changes in *x* and *y* can be determined from the table: $\left(\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 425}{25 - 0}\right)$ • Have students verbally explain what is happening in the graph before they answer part c).
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • When students write their own equations, remind them to select variables that are easy to remember and make sense in the context of the problem. For this problem, ask students to identify the meaning of the variables. • You may wish to review some simple equations and solving for a variable; then, use the same equations and use substitution to solve for a missing value.

Check Your Understanding

Practise

For #1c), reinforce that the slope is not 0. The number 1 is implied in front of *x*, so the slope is 1. In part d), you may wish to explain that the *x*-intercept is 0. Since the equation has no constant, the constant is 0. This equation could be rewritten as $y = -4x + 0$. Make sure students understand that there is a *y*-intercept and that it is 0. They might mistakenly assume this equation has no *y*-intercept.

For #3, students may need to be reminded to place brackets around fractions if they are entering these equations into a graphing calculator.

For #8, you may wish to show students a quick way to determine slope given a graph. On the graph, identify the *y*-intercept and one other nice point (a point with whole number coordinates). Start at the point on the left and count up or down and then to the right to the second point. Place the number of times you count up or down with the sign (+ for up, – for down) in the numerator and the number of times you count right in the denominator. Reduce the fraction to lowest terms.

For #9 and 10, students may need help seeing the connection between the *x* and *y* in the equation and the *x* and *y* in the coordinate pair. You may wish to emphasize the point-on property mentioned earlier.

Apply

Question #14 will take some time. Allow students to complete this question either with a partner or in a group. Encourage students to look for patterns to make the work easier.

For #15, students need to realize that springs can expand or compress. In either case, the relationship between the length of the spring and the force applied is linear.

Question #18 is similar to Example 3. It is an important question because it ties together many of the concepts in the section.

Extend

For #20, encourage students to use more than one approach to solve this problem. Students can start by letting $x = 0$ or by isolating the variable y in both equations.

Create Connections

Question #24 could be done after Example 1.

Have students complete #25 in conjunction with #3 and 6.

For #26, you may wish to create a wall of family portraits. Students can post the families of lines they created, showing the equations of the lines, their corresponding graphs, and a description of how the lines are related. Posted families could be grouped along common themes. Encourage students to use patterning as they work through this Mini Lab.

Unit Project

The Unit 3 project questions give students an opportunity to solve problems that will assist them in solving an archaeological mystery in the final project assignment.

Question #18 is a Unit 3 project question. It is an application of determining whether data in a table of values represent a straight line, the calculation of the slope and what it represents, and writing an equation and interpreting the results. The topic of how weather affects humans is appropriate practice for the final project, in which students will set out to solve the mystery of discovered human bones and the disappearance of three Klondike gold miners.

Meeting Student Needs

- Allow students to use a graphing calculator if available.
- Keep the equation $y = mx + b$ on display in the classroom with the slope and y-intercept identified.
- Provide **BLM 7–6 Section 7.1 Extra Practice** to students who would benefit from more practice.
- Question #9 can be made more visual by using a color-coded example illustrating the substitution of x and y into the equation to find b. Question #10 can also be illustrated this way.
- For #14, provide students with **BLM 7–5 NATO Emblem on a Grid** to allow students to extend the lines and determine each equation.
- For #15, some students may need to graph the data shown to determine the slope and y-intercept. If students must graph first, encourage them to write a response demonstrating knowledge that the y-intercept corresponded to where x had a value of zero, and that the slope is determined by studying the y-values.

Common Errors

- Students often confuse no intercept with an intercept of 0.

R_x Have students sketch a line that has no y-intercept and have them sketch a line that has a y-intercept of 0. Explain to students that a line with no y-intercept does not intersect the y-axis. It is a vertical line. A line with a y-intercept of 0 intersects the y-axis at the origin, or point (0, 0). Encourage students to determine the equation of each line they drew, and to justify their equations.

WWW Web Link

For information about NATO, go to www.mhrmath10.ca and follow the links.

To learn more about how scientists use bones to find the age of dinosaurs, go to www.mhrmath10.ca and follow the links.

For information about extinct animals, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3–6, 8–10, 13, and 14. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to partner students of like ability to work together. • Review with students how to identify the slope and y-intercept. Specifically, ensure that they can identify that given $y = x + 2$, the slope is 1, and given $y = 2$, the slope is 0. Also review that the y-intercept is zero when the equation does not contain a constant term. For example, $y = 3x$ could be written as $y = 3x + 0$. This will assist students who might have difficulty with #1–3. • Questions #4 and 5 require students to rewrite equations in slope-intercept form. Students who are having difficulty should be coached through solving for y in simpler forms. Encourage students to verbalize the steps they perform and why they are doing so. Coach and remind students that they are working on opposite operations. Try examples such as solving for y in the following: $y + 1 = 2x$ $y – 3 = 5x$ $y + 2x = 4$ $2y = 4x + 6$ $3y = x + 9$ $–2y = 4x – 5$ • For #8, encourage students to use their own personal strategy to find a slope. Some will count, some will find the difference, and some may wish to sketch a triangle created by the line. • For #9 and 10, students may require coaching on evaluating and solving for a missing variable. Review the meaning of ordered pair (x, y) and ensure students are clear on which value to substitute for x and which for y. • Students having difficulty with #13 should review Example #3 and their response to the Your Turn. • Some students may find the diagram overwhelming in #14. Assure them that the diagram shows multiple straight lines. Have students use a ruler to help them focus on one line at a time. Remind them that it is easier to determine the slope when they select ordered pairs that do not fall between two integer values. Choose "nice" points.
Unit 3 Project If students complete #18, which is related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• Question #18 is modelled after Example 4. Students could refer to the example and pattern their work after it. In part b), encourage and listen for the approaches students use. Encourage them to describe the one that works best for them. • Part c) can be approached in several ways. Encourage students to explain or describe their strategy and why. • Part f) will require students to interpret a graph. Before they respond, have them verbally estimate from the values in the table; for example, what altitude is likely to have a temperature of –20 °C? Visual learners should verify their estimates from their graph. Challenge students to answer this using more than one strategy. • You may wish to provide students with **BLM 7–4 Chapter 7 Unit 3 Project**, and have them finalize their answers.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Questions #24 and 25 will provide important feedback for both you and students in determining their understanding of solving for slope and y-intercepts using more than one method. These questions would be an excellent summary to include in the student Foldable. • For #26, partner students of like ability. Encourage students to write out all the similarities and differences they see in each group of equations before they complete their graphs. Have students do the same after they graph a family. Ask how the answers and graphs compare. For students who need more prompting, ask them to create a chart for each family and identify the slope and y-intercept for a family before they graph it. Ask how they can use these patterns to explain the graphs.

7.2 General Form

***Mathematics 10*, pages 357–369**

Suggested Timing

80–120 min

Materials

- bottle of water
- stopwatch
- grid paper and ruler, or graphing technology

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–7 Section 7.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF5 Determine the characteristics of the graphs of linear relations, including the:
- intercepts
- slope
- domain
- range.

RF6 Relate linear relations expressed in:
- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y – y_1 = m(x – x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#2, 3, 5, 6, 8, 10, 18, 19, 21
Typical	#1–7, 10 or 11, 12 or 13, 15, 18, 19, 21
Extension/Enrichment	#1, 4, 14–17, 20, 21

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce prerequisite skills needed for this section.

Make sure students understand that the slope-intercept form of an equation refers to the slope and the *y*-intercept. If you get students to focus on the "*y* =" part of the slope-intercept equation, then they will make the connection that the only equation that cannot be written in this form is an "*x* =" equation. This will help students answer the question in the opener and understand why vertical lines cannot be written in slope-intercept form. You may wish to specifically discuss the equations of the lines shown on the Canadian flag. Use the following questions to encourage discussion:

- Does every line have a *y*-intercept?
- Does every line have a slope? If not, what type of line has a slope that cannot be expressed as a number? Why?
- What type of line is not a function? Why?

Investigate Intercepts and General Form

Do not allow students to consume more than 1 L of water. This needs to be emphasized to students for their own health and safety, along with the fact that the activity is not a drinking race. The goal is to drink at a constant rate. Ensure that students are allowed adequate bathroom breaks following this investigation. This investigation provides a great opportunity to talk about independent and dependent variables. To help students identify the independent and dependent variables, you may wish to ask the following questions:

- Does the volume of water in the glass depend on the time that passes?
- Does the time that passes depend on volume of water in the glass?

For #1, allow students to express the domain and range in any form they choose, i.e., words, number lines, interval notation, lists, or set notation. To help students work with domain and range, you could ask the following questions:

- Are the data a set of individual points? (The domain will be the x-coordinates of those points, and the range will be the y-coordinates of those points.)
- Is there any restriction on the variable; that is, are there values the variable cannot equal? (The domain or range will be anything but those values, i.e., $x \neq \#$ or $y \neq \#$.)
- Is there a limitation on the variable; that is, must the variable be between two values? (The domain or range will be an interval, for example, $\# < x < \#$ or $\# < y < \#$.)
- Is there no limitation or restriction on the variable; that is, can the variable can be any value? (The domain or range will be all real numbers, i.e., $x \in$ R or $y \in$ R.)

In #5, reinforce to students that x-intercepts and y-intercepts are the values of the x-coordinate or y-coordinate and that they can be represented as coordinate pairs: $(x, 0)$ and $(0, y)$.

For #6, each student can drink a bottle of water or you can ask for volunteers. Use a stopwatch to measure how long it takes to drink the bottle of water at a constant rate. Have students sketch the graph using the capacity of the bottle as the vertical intercept and the time to drink the water as the horizontal intercept. Use students' graphs for a classroom discussion.

To help students find the x-intercepts and y-intercepts in #7, ask them to focus on what they are trying to find. At the point where a line intersects an axis, the coordinate of the other axis is 0. For example, to find the x-intercept for the equation $6x - 9y + 18 = 0$, they need to find the value of x at the x-axis. At the x-axis, the value of y is 0, so they can rewrite the equation as $6x + 18 = 0$ and solve for x.

Similarly, to find the y-intercept for the equation $6x - 9y + 18 = 0$, they need to find the value of y at the y-axis. At the y-axis, $x = 0$, so the equation can be rewritten as $-9y + 18 = 0$ and solved for y.

Meeting Student Needs

- Discuss the student learning outcomes related to this section.
- Allow students enough time to carry out the investigation and to write their strategies for finding the intercepts of a line. Alternatively, you may wish to have students work with a partner or in small groups.

ELL

- Students may need to be reminded that the *whole numbers* are 0, 1, 2, 3, ….

Common Errors

- Students may not follow the conventions for writing equations in general form.

R$_x$ Reinforce to students that the x-term is written on the left, the y-term is next, and then the constant comes last. The right side of the equation is 0. You may wish to discuss the benefits of having the x-term be a whole number.

- Students sometimes graph the dependent variable on the horizontal axis and the independent variable on the vertical axis.

R$_x$ Work closely with students, helping them properly identify and graph each variable.

Answers

Investigate Intercepts and General Form

1. Domain: $\{x \mid 10 \leq x \leq 12, x \in \mathrm{R}\}$,
Range: $\{y \mid 10 \leq y \leq 600, y \in \mathrm{R}\}$

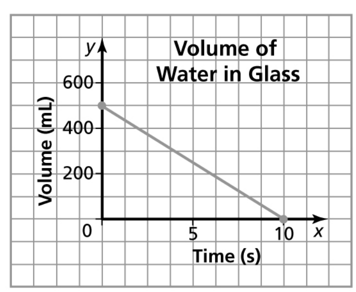
2. Slope = –50. The negative slope means that the volume of water decreases over time. The volume of water decreases by 50 mL every.

3. y-intercept: (0, 600). It represents how much water was in the glass before Leora started drinking.

4. a) $y = -50x + 600$ **b)** $50x + y - 600 = 0$

5. a) x-intercept: (12, 0).
b) x-intercept: (12, 0); y-intercept: (0, 600).

6. a) Example:



b) Example: Leora drank more. Leora's graph starts at 600 and mine starts at 500. I finished first in 10 s. We drank at the same rate of 50 mL/s.

c) Slope-intercept form: $y = -50x + 500$;
General form: $50x + y - 500 = 0$.

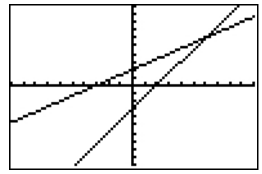
Answers

7. a) yes

b) 0, 0. Example: I can substitute 0 for one intercept and then solve the formula to calculate the value of the other intercept.

c) Example: To determine the x-intercept, set $y = 0$ and solve for x. To determine the y-intercept, set $x = 0$ and solve for y.

d) Example: $3x - 2y - 6 = 0$, x-intercept: $(2, 0)$, y-intercept: $(0, -3)$ and $2x - 3y + 6 = 0$, x-intercept: $(-2, 0)$, y-intercept: $(0, -2)$



8. General form. Example: No, y cannot be isolated.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Some students will choose to use the slope-intercept form to identify the *y*-intercept, as they are familiar with the format. If this is the case, reinforce the process by asking the following questions: – What is the value of *x* at the *y*-intercept? – What is the value of *y* when the line intersects the *x*-axis? – How would you use the slope-intercept form to find both *A* and the *y*-intercept? • As students improve at identifying the *x*-intercept and *y*-intercept from the slope-intercept form, point out that they could save some time by using the general form. Complete an example with them where one equation is in general form and beside it is the same equation in slope-intercept form. Solve for the intercepts in both equations and have students compare the steps required, watching for similarities. It is important that students find a strategy that is clear for them but equally important that they understand other approaches.

Link the Ideas

As you read through the Link the Ideas, you may wish to ask students the following questions based on the four equations.

Equation 1: $0.5x + y - 3 = 0$
Equation 2: $2y = -x + 6$
Equation 3: $x + 2y - 6 = 0$
Equation 4: $-x - 2y + 6 = 0$

- Which equations are equivalent to $y = -0.5x + 3$?
- Which equations are expressed in general form?
- Which general form equation is recommended, by convention?

You may wish to mention that *standard form* is a slight variation of general form. The *standard form* equation of a line is $Ax + By = C$, where A, B, and C are real numbers, and A and B cannot both be zero. By convention, A is a whole number. Sometimes students will be given an equation in standard form.

Example 1

In this example, rather than using the distributive property and multiplying each side by 3, encourage students to multiply each term by three. Students may forget to multiply the constant by 6 if they use the distributive property.

As a warm-up activity, you may want to start with simpler equations like the ones below.

Express each equation in general form.

$y = 3x$
$y = 2x - 5$
$y = -6x + 7$

Then, you could challenge academically capable students with the following question:

Express the equation $y = \frac{3}{2}x - \frac{4}{5}$ in general form.

Example 2

You may wish to encourage students to use mental mathematics by modelling the following strategy while working through parts a) and b):

- To find the x-intercept, put your finger over the y-term, and with what remains visible, solve for x. This works because $y = 0$ at the x-axis.
- To find the y-intercept, put your finger over the x-term. Solve for x with what remains visible. This works because $x = 0$ at the y-axis.

Throughout this chapter, students solve many equations. You may wish to have students recall some strategies for isolating a variable and avoiding complicated steps, such as multiplying by a constant to eliminate fractions. In part b), the y-term was moved to the right side to eliminate a negative coefficient for y. Students could have chosen to divide each term by –3 in the third line of the solution.

Consider showing students how to use a calculator to graphically find the intercepts. Remind students to label the x-intercepts and y-intercepts on their graph.

Part c) suggests that students consider another method of graphing the equation. Discuss ways that students could check their work, perhaps by converting the equation to slope-intercept form and comparing the direction of the slope and the y-intercept with their graph. Students could even count the rise and run of their graph if they wish to check the slope, which is easily read from an equation in slope-intercept form.

Example 3

This example explores the special case of the general form involving vertical and horizontal lines. You may wish to review domain and range, including the various ways of expressing domain and range, before working through this example.

Present students with the following six equations and have students sort the equations into vertical lines, horizontal lines, and oblique lines, that is, neither vertical nor horizontal.

Equation 1: $y = x + 6$ Equation 2: $x = 5$
Equation 3: $y = 0$ Equation 4: $x - y = 0$
Equation 5: $x + 6 = 0$ Equation 6: $y - 5 = 0$

Ask students to explain how they categorized the lines, giving as many different reasons as possible. For each relation, have students sketch the graph, identify the intercepts, and state the domain and range.

Example 4

This example allows students to use the skills they have learned in this section in a real-life application. Make sure students understand how to develop the equation in part a) and how it relates to Spencer's disk space on his laptop. You may wish to point out to students that they could divide each term in the original equation $1.1T + 4.4M = 66$ by 1.1, which would provide an alternative way to reduce it to lowest terms with a whole-number coefficient for T.

Give students time to process the terminology of the T-intercepts and M-intercepts and what the variables represent in the context.

Key Ideas

Reinforce to students that both the general form and the slope-intercept form allow them to graph a linear relation using different methods. Make sure students feel comfortable using both methods of graphing and encourage them to check their work using each method.

The table shows examples where an equation can have two intercepts or one intercept (x or y). Ask students whether other options are possible and why. For example, can a linear equation have no intercepts? an infinite number of intercepts?

Meeting Student Needs

- Post the general form of a linear equation on the board. Emphasize that A, B, and C are real numbers, A is generally a whole number, and A and B are not both zero.
- For Example 2, allow students to use a small table of values to determine the x-intercept and y-intercept, similar to the following:

x	y	
	0	(#, 0)
0		(0, #)

- Create an exit slip for the end of this section that has students identify three to five key ideas learned. Vocabulary may also be included. Create a spot where students can assess their own learning by circling a thumbs up/thumbs down or smiley face/sad face.

ELL

- Students may not be familiar with the unit GB (gigabyte). Explain to them that it is roughly equal to one billion bytes, and that the byte is a unit that measures the storage capacity of a computer system.
- You may wish to discuss the term *infinite*. Some students have trouble grasping this concept. It will be revisited in section 8.3.

Enrichment

- Ask students to predict the following outcomes:
  – Suppose the value of B in the general form of an equation is doubled. How are the slope and y-intercept of the line affected? (They are both halved.)

– If the value of A in the general form is doubled, what happens to the slope? (The slope is doubled.)
– If the value of C is doubled, what happens to the slope? (no change to the slope)

Gifted

- Challenge students to assess the circumstances in the enrichment question above, and write a generalized statement about the effect of changing the general forms affect on slope of a line.

Common Errors

- Students may confuse domain and range.

R$_x$ You may wish to remind students to think alphabetically: d (domain) comes before r (range) and x comes before y. So, domain contains x-values and range contains y-values.

Web Link

For information about computer memory as referred to in Example 4, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

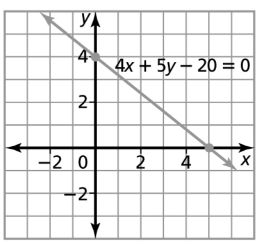
$3x - 4y - 8 = 0$

Example 2: Your Turn

a) x-intercept: (5, 0)

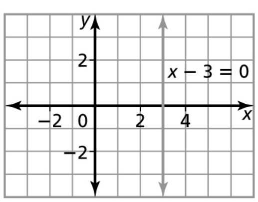
b) y-intercept: (0, 4)

c)

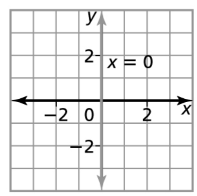


Example 3: Your Turn

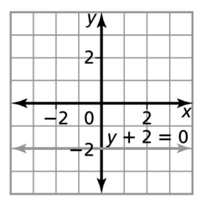
a) x-intercept: (3, 0). There is no y-intercept. Domain: $\{3\}$, Range: $\{y \in R\}$



b) x-intercept: (0, 0). There are an infinite numbers of y-intercepts. Domain: $\{0\}$, Range: $\{y \in R\}$



c) There is no x-intercept. y-intercept: (0, –2) Domain: $\{x \in R\}$, Range: $\{-2\}$



Example 4: Your Turn

a) $12S + 16T = 336$

b) S-intercept: (28, 0). If Brooke does not work as a tutor, she needs to work 28 h as a snowboard instructor.

c) T-intercept: (0, 21). Brooke needs to work 21 h as a tutor if she does not work as a snowboard instructor.

d) 15 h

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students have difficulty with the distributive property and multiply only the terms containing a variable, forgetting the constant term. It may benefit students to visually see each step and multiply each term. For example, $y = \frac{3}{4}x - 2$ $\frac{y}{1} = \frac{3x}{4} - \frac{2}{1}$ The common denominator is still 4. $4\left(\frac{y}{1}\right) = 4\left(\frac{3x}{4}\right) - 4\left(\frac{2}{1}\right)$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Success with Example 2 is directly related to students' understanding of the process in Example 1. If students are still struggling, an alternative approach would be to have students enter the equation into their calculator and teach them how to locate the intercepts from a table of values generated by the technology. Students must, however, understand and be able to relate what the meaning of the intercepts is. • Most students find substituting in the value of zero for *x* and *y* relatively easy. They do, however, confuse what value is evaluated at zero. Many students will want to substitute the value of zero into the variable they are trying to solve for. For example, to find the *x*-intercept, they will substitute zero in for x. Remind students about working with opposites. Show them that if they substitute 0 for x when finding the *x*-intercept, they will end up with an equation and value in the form of *y* = #, not *x* = #.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students have difficulty with vertical and horizontal lines. Remind them to locate the point on the axis in the equation. For example, if *x* = 4, locate 4 on the *x*-axis. Where else do ordered pairs have an *x*-value of 4? Plot some of the points. This process will assist in providing a visual that helps students link the *x* = value to a vertical line. The same holds true for horizontal lines.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • It may benefit students to discuss what variables might be appropriate for the equation. Discuss which axis each will go on. • If students are having difficulty determining the intercepts because the variables are no longer *x* and *y*, point out what the axis is labelled where it is typically "*x*". Do the same for the vertical axis. Ask what the ordered pairs look like (*S*, *T*). • Some students may have difficulty with part d), not recognizing what the operation is. Prompt students by asking what the variable is for snowboarding and ask what variable they wish to solve for.

Check Your Understanding

Practise

You may wish to have students complete #6 with a partner or in a group because this question involves several tasks.

Apply

For #7, some students may need assistance working through the question. You may wish to help students with the equations of vertical and horizontal lines. Vertical lines run up and down and pass through the x-axis. Therefore, the equations are in the form $x = \ldots$. Horizontal lines run left and right and pass through the y-axis. Therefore, they are in the form $y = \ldots$.

Question #10 is similar to the Investigate activity. If students need help with this question, you may refer them to their answers to the Investigate questions.

In #12, students work with density, mass, and volume. Ensure that students have set up the equation correctly before they complete parts b) and c). Encourage students to inspect the units in their equations as a way to check their equation. If the equation is correct, the units on the right and left sides of the equal sign will be the same.

Create Connections

These questions could be done after the Practise questions.

Question #21 is a Mini Lab and provides an opportunity for students to discover relationships among the parameters in an equation and the corresponding graph. Students can use technology to graph the lines or sketch the lines by hand using intercepts. Students should be prepared to share their conclusions.

Meeting Student Needs

- Provide **BLM 7–7 Section 7.2 Extra Practice** to students who would benefit from more practice.
- Post both forms of linear equations studied so far: slope-intercept form and general form. You may wish to identify equations of horizontal lines ($y = \#$) and vertical lines ($x = \#$).
- Allow students to work in pairs or small groups to complete #7–14.
- Have each student complete #18; then, discuss as an entire class.
- Have students complete #21 in small groups and compare answers.
- Allow students to determine the percent in #12c) using any method they wish.
- Some students may benefit from using money manipulatives for #13, as well as manipulatives representing the tickets.
- Encourage students to use coloured pencils when drawing the triangles in #17 and to clearly label their graphs.
- In #19b), students may come up with one method of determining the y-intercept, but they may not be able to think of another method. Encourage students to look back on their work in this chapter and to review what they have learned about finding y-intercepts.

ELL

- You may wish to explain the two swimming styles mentioned in the table in #11, *backstroke* and *butterfly*.
- In #12, some students may not be familiar with the terms *traction*, *density*, *volume*, and *mass*. Explain these terms. Students will likely encounter the three measurements in science class, and they may even discuss *traction*, but it will be called *friction*.
- For #20, students may need to be reminded that *oblique* means slanting.

Common Errors

- For #3g), students may recognize that the line $y = 0$ intercepts the y-axis at 0, but they may neglect to consider where this line intercepts the x-axis.

R_x Encourage students to think of each axis separately when determining the intercepts.

- For #5 and 8, students may confuse whether a linear equation has an intercept of zero or no intercept with a particular axis.

R_x Review and clarify these two situations with students. Have them come up with an example of each type of line. Ask them to graph the line and determine an equation of the line.

Web Link

For information on different types of exercise and the calories burned as discussed in #11, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #2, 3, 5, 6, 8, and 10. Students who have no problems with these questions can go on to the remaining questions.	• Encourage students to work with a partner. • Remind students that writing the general form in #2 requires them to work with opposite operations. You may need to review the characteristics of general form and how to remove fractions from an equation. Suggest that students complete one of each section on each question and have a partner check their work before completing all the assigned problems. • Some students may be confused by the many lines in #6. Have them make a list in their books of the equation numbers 1 to 8 and identify the intercepts for each equation. Suggest that below the list, they determine the x-intercepts and y-intercepts of the given equations. Use information from both to match the graphs. • Students having difficulty with #8 should be prompted to describe a line that does not intersect the y-axis. Ask what type of line it is. Encourage students to sketch a graph with the point (3, 6) on it. Ask students how they could sketch the line so it did not touch the y-axis. • For #10, encourage students to write the intercepts as ordered pairs, as this could then be used to find the slope in part c). Prompt students to identify what their preferred form is for writing the equation of a line. Ask them to identify what information they have that will be used (slope and intercepts).

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #18, 19, and 21.</td><td>• Students capable of finishing the recommended questions with few supports should attempt #20.
• Question #18 provides an excellent Assessment as Learning question that could be placed into their Foldable.
• For #19, prompt students to think of opposites. If we are looking for the x-intercept, what must be zero? Some students may prefer to graph the equation and read off values from the graph. Encourage whatever strategy is easier for them to remember.
• Question #21 is an extension of the idea of families of lines that was started in the previous section. Encourage students to label information for each grouping before they graph the lines. A suggested list of questions to refer to might include the following:
– When the equations are written in slope-intercept form, what is the same? What is different?
– When the equations are written in general form, what is the same? Different?
– When the equations are graphed, what is the same and what is different?</td></tr>
</table>

7.3 Slope-Point Form

Mathematics 10, pages 370–382

Suggested Timing

80–120 min

Materials

- grid paper
- ruler
- SI measuring tape

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–4 Chapter 7 Unit 3 Project
BLM 7–8 7.3 Investigate: Figure 1
BLM 7–9 Section 7.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF6 Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1 = m(x - x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:

- a graph • a point and the slope • two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5a), b), 6a), c), d), 7–9, 11, 21–24
Typical	#1–5, 7, 8, 10, 11, 14, 24
Extension/Enrichment	#12, 13, 15–20, 22, 24

Unit Project Note that #24 is a Unit 3 project Mini Lab.

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

In this section of the chapter, students are introduced to a third form for writing an equation of a line. They will discover that this new form can only be used for non-vertical lines.

Present students with the following three equations: $y - 3 = 2(x - 4)$, $2x - y - 5 = 0$, and $y = 2x - 5$. Ask students to identify the equation that is written in slope-intercept form. Next, have them identify the equation that is in general form. The third equation is in slope-point form. Ask students to show that all three equations are equivalent.

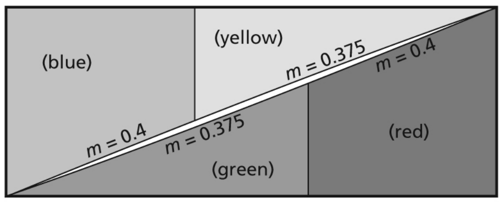
Investigate Equations in Slope-Point Form

In this Investigate, students discover that any two points on a line can be used to write the equation of the line. This fact is then generalized and written as the slope-point form of a linear equation.

In #3, students may need assistance with determining the discrepancy between the areas. As a hint, suggest that students determine if the path from point E to G in Figure 2 is a straight line. They should justify their answer.

It may be useful for students to use **BLM 7–8 7.3 Investigate: Figure 1** to explore what is happening here. This BLM provides an enlarged version of Figure 1 on page 370 in the textbook. If students cut out this figure and carefully put it together, they may see that there is an empty area between the two sets of polygons.

The figure below demonstrates why this occurs. The yellow and green shapes from Figure 1 have a rise of 3 and a run of 8. This gives a slope of $\frac{3}{8}$ or 0.375. The triangles on the blue and red shapes have a rise of 2 and a run of 5. This gives a slope of $\frac{2}{5}$ or 0.4. Since these two slopes are very close, the difference will not be evident on a small version of the visual. On a larger version, however, the difference will be clear. The larger you make the visual, the clearer the difference will be.



In #4, students should start with points that produce a line with an integral y-intercept and slope. For academically capable students, ask them how they could find the equation of the line when the y-intercept is not integral. For example, find the equation of a line with a slope of $\frac{1}{2}$ .passing through (3, 5).

Students may need assistance with #5. Using their graphs as a guide, ask students how they could numerically find the y-intercept of a line passing through a given point with a given slope. For example, state the equation of a line with a slope of 2 passing through (3, 10). Then, have a discussion similar to the following:

- To move from point (3, 10) to the y-intercept, the run would be –3. Using the slope relationship $2 = \frac{\text{rise}}{-3}$, the corresponding rise would be 2(–3).

Hence, the y-intercept is 10 + 2(–3), or 4. This leads to the generalization that a line with slope m passing through point (x_1, y_1) has a y-intercept of $y_1 + m(-x_1)$. Hence, the equation of the line in slope-intercept form is $y = mx + (y_1 + m(-x_1))$. This simplifies into slope-point form:

$$y = mx + (y_1 + m(-x_1))$$
$$y = mx - mx_1 + y_1$$
$$y - y_1 = m(x - x_1)$$

For #7, have students develop this explanation in their own words. You may wish to have partners explain this to each other.

In #8, you could ask students to explain why the equation is for a non-vertical line. Why is it not possible to represent a vertical line in this way?

Meeting Student Needs

- Discuss the student learning outcomes for this section. Post them in the classroom.
- Post the three forms that can be used to represent a linear equation. Explain that students will focus on the third form, slope-point form. It is to be used when given the slope of the line and a point on the line.
- Give students an enlarged photocopy of Figure 1. Have them cut the square into the four polygons and assemble the parts into a rectangle by placing them on grid paper. What do they notice?
- Allow students to work in pairs on the Investigate.

Answers

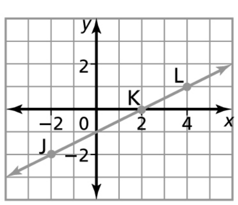
Investigate Equations in Slope-Point Form

1. 64 square units

2. 65 square units

3. Example: The second figure is not a rectangle. “Diagonal” EG is not a straight line.

4. Example:



slope = $\frac{1}{2}$

a) $y = \frac{1}{2}x - 1$

b) $y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x - 1$; The equations are the same.

c) Example: If you know the slope and the coordinates of one point, you can use those values in $y = mx + b$ to find the value of b and the equation of the line.

5. $y - y_1 = m(x - x_1)$

7. Encourage students to develop and share their own strategies. Discuss these strategies as a class before proceeding to #8.

8. Example: Substitute x for x_2 and y for y_2 and multiply both sides by $(x - x_1)$.

9. Vertical line; the slope is undefined. $x_2 - x_1 = 0$

10. Example: Always true. The slope of a line can be determined by any two points on the line.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• If students are having difficulty getting started with #7, write the slope-intercept form on the board and ask how the look of it might be changed using the slope formula. • For #8, suggest students sketch a vertical and a horizontal line anywhere on a coordinate grid. Ask them to determine the equation for each of their lines. This should reactivate previous learning and provide the support to answer the question. • You may wish to complete #9 as a class. Perhaps sketch a parabola on the board and label the points (0, 0), (1, 1), and (2, 4). Ask students to use these to respond to the question.

Link the Ideas

Slope-point form is best expressed as $y - y_1 = m(x - x_1)$ and not as $\frac{y - y_1}{x - x_1} = m$. That way all points on the line, including the point (x_1, y_1), satisfy the equation of the line, $y - y_1 = m(x - x_1)$.

Students need to realize that x_1 and y_1 represent the coordinates of a specific point on the line, whereas x and y represent the coordinates of any point on the line. You may wish to ask the following questions:

- In the formula $y - y_1 = m(x - x_1)$, what do the coordinates (x_1, y_1) represent?
- What do the coordinates (x, y) represent?

Example 1

In this example, students will be asked for the equation in slope-point form and slope-intercept form. Then, students use technology to graph their equation. Encourage students to consider looking for ways to verify that their answer is correct. Refer students to the Mental Math and Estimation box beside the solution to part b) for one way to check part of an answer.

After completing the example, have students work through the Your Turn.

Example 2

Once the slope is determined, the coordinates of any point can be used to find the equation of a line in slope-point form. Calculations are sometimes easier for one point than the other. In part a), you may wish to ask the following questions:

- Do you prefer using one point over the other?
- How could you check your equation?

Have students use the coordinates of the point they did not use to check their equation. Students could also use technology to check their equation.

When going over the procedure to determine the equation in general form for part c), you could show students that multiplying each side by 2 is equivalent to multiplying each term by 2. This might eliminate some errors.

Example 3

This example allows students to use the skills they have learned in a real-life application. Ask students who have done mountain climbing to share their experience.

Note that the sample solution uses the y-intercept as the point to substitute. This often results in the most efficient solutions.

In part a), emphasize that d is the distance from the *top* of the first pitch. The slope is negative since the distance to the *top* of the pitch is decreasing. A decrease suggests a negative value. Similarly, an increase would suggest a positive value.

To help students visualize the situation and show that they understand the problem, you may wish to have them sketch and label a diagram showing the heights of the pitches and Brad's height at each of the times given in the problem.

In part a) of the solution, you may wish to discuss how the slope formula is related to the given formula, $m = \frac{d_2 - d_1}{t_2 - t_1}$. Encourage students to discuss how they need to consider how far the climber has moved from the second distance to the first distance. Some students may find this confusing because Brad is moving up the slope; however, the numbers are given from the top of the slope so are getting smaller. So, when Brad moves from 60 m down to 40 m down, he has actually moved 20+ m. That move took him ten minutes, which is a negative number because it refers to time elapsed.

Have students complete the Your Turn question.

Key Ideas

Have students summarize what they have learned. Ensure that students understand why a vertical line cannot be written in slope-point form. Emphasize that any point on a line can be used to determine the equation of a line in slope-point form if you know the slope. A second point on the line can be used to check the equation. Reinforce to students that they only need two points to graph a linear equation. They can use the points to find the slope and then use the slope-point form with either of the points given.

Meeting Student Needs

- Establish which form you want students to use when writing an equation, if it is not stated in the question. Students should also be given opportunities to choose the form they write the equation in.
- Create two posters on chart paper illustrating Examples 1 and 2. Ask students to compare and contrast the two examples.
- To help students understand the benefit of writing an equation in slope-point form, explain that this form requires less work and fewer calculations. That is because students will only need to determine the slope of the line and any point on the line, not a specific point, such as the y-intercept, as required for the slope-intercept form.

Enrichment

- Ask students to write in slope-intercept form four lines whose internal area is 100. For example,
$y = \frac{4}{3}x + 0$, $y = \frac{4}{3}x + 8$, $y = -\frac{4}{3}x + 8$,
$y = -\frac{4}{3}x + 33$

Gifted

- The enrichment question above cannot be done for a number of real shapes created by four lines and having an internal area of 100. Challenge students to investigate what the common characteristic of these shapes is. (Any shape with a vertical line on the x-y coordinate grid cannot be written in slope-intercept form because the slope of a vertical line is undefined.)

Common Errors

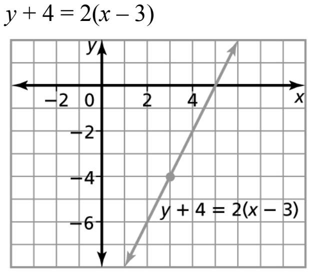
- Students may not apply the distributive property correctly. For example, to expand $y + 4 = \frac{3}{2}x - 3$, some students will write
$2(y + 4) = 2\left(\frac{3}{2}x - 3\right)$
$2y + 4 \neq 3x - 3$

R_x Show students that what you do to one *term* you do to all the others.

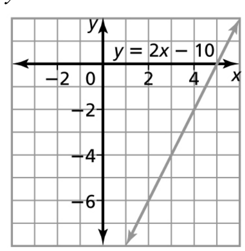
Answers

Example 1: Your Turn

a) $y + 4 = 2(x - 3)$



b) $y = 2x - 10$



c) The graphs are identical.

Example 2: Your Turn

Example: First, find the slope, m. Then, substitute the value of m and the coordinates of one point into the slope-point form of the equation. $x + 3y - 1 = 0$

Example 3: Your Turn

a) $d = -90t + 540$ **b)** 3 p.m.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Students may benefit from starting a list of formulas in their Foldable or notes with the title beside them. Have students verbalize which value in the ordered pair represents x and which represents y, and where they will substitute each in. • Remind students of the distributive property that requires that they multiply each term in the brackets. • You may wish to have students graph their line by hand first and then verify it using technology.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to write out the ordered pairs and label them below each coordinate as x_1, y_1 and x_2, y_2. It may help organize their thinking when substituting into the slope formula. • Some students might be challenged by suggesting that there is a two-point formula, and it appears as $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. Ask students to determine whether it is correct and whether it will work each time they encounter two points.
Example 3 Have students do the Your Turn related to Example 3.	• Students who have difficulty determining the equation for the context may need prompting to clarify the meaning of certain values. You may wish to review Example 3 and ask what the initial distance was (60 m) at time 0 min. Ask what the initial distance is in the Your Turn (540 km). Go back to the example and ask what the finishing distance was (40 km in the example and 315 km in the Your Turn). Help students to identify where these fit into the slope formula. Repeat the same process for time and then allow students to find the slope.

Check Your Understanding

Practise

To save time, you may wish to ask students to express equations in slope-intercept form or general form, but not necessarily both. You can also let students choose the form of the equation.

Completing #4 will help students work through #8.

Question #5 prepares students for #6. In #5, students express the answer in slope-point form only. There are two possible answers for each graph. In #6, students express the equation in slope-intercept form and general form.

Question #7 gives students the opportunity to communicate and apply their understanding of linear equations. You may wish to discuss part a) as a class.

Apply

The strategies from #9 for determining the y-intercept of a line can be applied in #10.

In #11 and 12, students have the opportunity to apply skills from sections 7.1 and 7.2.

For #17 and 18, students must determine the equation of a line and then determine a particular characteristic of that line. Question #18 has students read information from a label on a bag of potatoes. You may wish to have a class discussion about the information provided on the label.

Create Connections

These questions could be done after the Practise questions.

For #22, students summarize their understanding of what information is needed to determine the equation of a line.

In #23, students use their creativity to illustrate their thought process for deciding on a form of a linear equation.

Unit Project

The Unit 3 project question, #24, provides an opportunity for students to apply their ability to determine the equation of a line to real data. Discuss with students the conditions under which the equation of their line would be useful in predicting a person's height from their bone remains. The equation would be useful if the person was of the same gender and similar ethnic background. It would not be useful for the remains of a young child nor of an elderly person. You may wish to discuss and compare the results for males and females.

Meeting Student Needs

- Provide **BLM 7–9 Section 7.3 Extra Practice** to students who would benefit from more practice.
- Before students begin, illustrate the three forms of a linear equation and emphasize the key components for each equation: m, b, A, B, C, y, y_1, x, and x_1.
- Students are given the opportunity to practise converting from one form to another. They should be aware of whether fractions or decimals are acceptable for certain situations, and when a value can be negative (e.g., in general form, A can never be negative by convention).
- Encourage students to write two equations for each graph in #5. Can they prove why each equation is correct?
- Allow students to use graphing calculators for all questions in this section.
- Questions #13, 14, 17, and 18 may be difficult for some students. Ensure they have a partner who will be able to assist them. Encourage a discussion of the question prior to writing the response.

WWW Web Link

To learn more about paleontology in Canada, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5a), b), 6a), c), d), 7–9, and 11. Students who have no problems with these questions can go on to the remaining questions.	• Remind students about multiplying through brackets when using the distributive property. This is important for #1 and for subsequent questions. To help organize and compare, you may wish to suggest that students include columns on their page. • Coach students to verbalize how the labelled ordered pairs in #2 can be used and identified in the formula. Encourage them to use the dotted triangle to determine the vertical change first and then the horizontal. For #5, suggest students could either use the slope formula or dot in a triangle of their own to mark the moves from one point to another. • For #6, suggest that students label the points so they are not confused when substituting the values into the slope-point form. Visual students may prefer to graph the points to determine the slope and *y*-intercept first. • Students who are having difficulty with the slope-point form might benefit from completing the first two questions by writing the equations first in slope-intercept form. Coach students to see the link between the process in #2 and 8. • Once students have verified their answer for #9, encourage them to include it in their Foldable, as it identifies two personal strategies that students could use.
Unit 3 Project If students complete #24, which is related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 7–4 Chapter 7 Unit 3 Project** and have them finalize their answers. • Students will enjoy the activity in the Mini Lab. Remind them that the information here can be useful for their final project. You may wish to make a large chart on the board and have student pairs indicate their results as they work through the Mini Lab.
Assessment *as* Learning	
Create Connections Have all students complete #21 and 22.	• Some students may benefit from working with a partner of like ability in completing #21 and 22. Have them discuss their responses. Both questions are excellent Assessment *as* Learning pieces and would benefit the students to include in their notes or Foldable.

7.4 Parallel and Perpendicular Lines

***Mathematics 10*, pages 383–395**

Suggested Timing

80–120 min

Materials

- grid paper
- scissors
- ruler

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–10 Section 7.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–5, 6a), c), e), 7a), c), e), 9–11, 25, 26
Typical	#1–5, 6a), c), e), 7a), c), e), 8, 11, 13, one of 14–16, 25, 26
Extension/Enrichment	#12, 14–19, 24–26

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter concentrates on parallel and perpendicular lines. Students identify, write equations, and solve problems involving parallel and perpendicular lines. When discussing the section opener, you may wish to have students visualize that the gymnast's arms are perpendicular to the top of the balance beam. Have students identify objects in the classroom or on their person that are parallel or perpendicular to each other.

Investigate Slopes of Parallel and Perpendicular Lines

This Investigate uses the sides of squares and the diagonals of a rectangle to explore the slopes of parallel and perpendicular lines.

In #4, students will each have different rectangles. You may wish to circulate and check the appropriateness of their answers.

Step #5 enables students to see why the slopes of perpendicular lines are negative reciprocals. When the rectangle is rotated 90° about a vertex, the rise of the diagonal becomes the run, the run of the diagonal becomes the rise, and the orientation of the diagonal changes.

Meeting Student Needs

- You could lead a discussion on parallel and perpendicular lines. Have students point out locations in the room where they see parallel lines and then perpendicular lines.
- Based on previous knowledge, you may wish to determine what students know about the slope of vertical lines and the slope of horizontal lines. Discuss whether all vertical lines are parallel and whether all horizontal lines are parallel. Compare the slopes of horizontal lines and vertical lines.
- The Investigate lends itself to students who require hands-on activities. Through the investigation, students will come to a better understanding of parallel and perpendicular lines.

ELL

- Some students may need to be reminded of the meaning of *counterclockwise*.

Answers

Investigate Slopes of Parallel and Perpendicular Lines

2. C(1, 7) and D(–3, 4); $m_{AB} = \frac{3}{4}$, $m_{BC} = -\frac{4}{3}$, $m_{CD} = \frac{3}{4}$, $m_{AD} = -\frac{4}{3}$; Example: The slopes of opposite sides are equal, and the slopes of adjacent sides are negative reciprocals of each other.

3. C(7, 17) and D(–5,12); $m_{AB} = \frac{5}{12}$, $m_{BC} = -\frac{12}{5}$, $m_{CD} = \frac{5}{12}$, $m_{AD} = -\frac{12}{5}$; Example: The slopes of opposite sides are equal, and the slopes of adjacent sides are negative reciprocals of each other.

4. Example: A(3, 1), B(9, 1), C(9, 4) and D(3, 4); $m_{AB} = 0$, m_{BC} is undefined, $m_{CD} = 0$, m_{AD} is undefined; $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$

5. Example: $m_1 = \frac{1}{2}$ and $m_2 = -2$; the two slopes are negative reciprocals of each other.

6. The slopes of parallel sides are equal. Example: $m_{AB} = m_{CD} = \frac{3}{4}$ in #2.

7. The slopes of perpendicular sides are negative reciprocals. Example: $m_{AB} \times m_{AD} = -1$.

8. **a)** The slopes are undefined.
b) The slope of a vertical line is undefined. The slope of a horizontal line is 0. The lines are perpendicular to each other.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Before beginning #6, as a class discuss the coordinates of the vertices of the various shapes. You may wish to label them on the board. • Some students may benefit from having the slope formula placed on the board for the investigation. • Most students will determine that the slopes of parallel lines are the same; however, they may not realize that the slopes of perpendicular lines are negative reciprocals of each other. Many students will determine that they are reciprocals but miss the opposite sign. Point out to students that perpendicular slopes always have a product of –1. • Some students may need interpretation of the slopes of horizontal and vertical lines because they may have difficulty remembering which fractional value is zero and which is undefined. You may wish to use the analogy of an animal walking. A zero slope means there is no tilt, and that occurs with a horizontal line, as the *y*-values are identical. In a vertical line, it is impossible to tell whether the animal is going straight up or straight down. It can not be distinguished, therefore the slope is undefined.

Link the Ideas

The Link the Ideas section provides formal definitions for parallel lines and perpendicular lines. Reinforce that parallel lines do not intersect because they travel in the same direction (hence the equal slopes). The slopes of two non-vertical parallel lines, m_1 and m_2, are equal; that is, $m_1 = m_2$. Vertical lines are a special case. The slopes of two non-vertical perpendicular lines, m_1 and m_2, are negative reciprocals; that is, $m_1 = \frac{-1}{m_2}$ or $m_1 \times m_2 = -1$.

Vertical and horizontal lines are a special case.

Example 1

Example 1 demonstrates how to determine whether two lines are parallel, perpendicular, or neither. Prompt students' thinking with the following questions:

- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines whether the lines are parallel?
- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines whether the lines are perpendicular?
- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines if the lines are neither parallel nor perpendicular?

Example 2

In this example, students must determine the slope of a linear equation given in general form and then determine the equation of a line that is parallel and passes through a given point.

Make sure students understand why they are converting the original equation from general form to slope-intercept form. Once the equation is in slope-intercept form, students can find the slope of the line, thereby finding the slope of a line parallel to it.

In part a), two methods are shown for arriving at the equation in slope-intercept form. In Method 1, the slope-point form is converted to the slope-intercept form. In Method 2, the slope-intercept form is used directly. Academically strong students could be shown the following third method, which uses the general form and the point-on property.

The slope of the line $Ax + By + C = 0$ is represented by $-\frac{A}{B}$. The only values that affect the slope are A and B. Since the slopes of parallel lines are equal, an equation of a parallel line in general form can be obtained by using the same values for A and B as in the original line. The equation $2x - y + C = 0$ represents a line parallel to the original line, $2x - y + 4 = 0$.

To find C in $2x - y + C = 0$, substitute the coordinates of the point $(1, -6)$ for (x, y).

$$\begin{aligned} 2x - y + C &= 0 \\ 2(1) - 1(-6) + C &= 0 \\ 8 + C &= 0 \\ C &= -8 \end{aligned}$$

Therefore, the equation of the line in general form is $2x - y - 8 = 0$.

For part b), students could also use the slope-point form to determine the equation of the line in general form. The solution would be as follows:

$$\begin{aligned} y + 6 &= 2(x - 1) \\ y + 6 &= 2x - 2 \\ 0 &= 2x - 2 - y - 6 \\ 0 &= 2x - y - 8 \end{aligned}$$

The equation of the line in general form is $2x - y - 8 = 0$.

Part c) illustrates how to use technology to verify that the lines are parallel. Have students experiment with the graphing calculator they are using and make notes about how to perform this check.

Students can use the Try This to confirm that they understand how to use their graphing calculator to verify that lines are parallel.

Example 3

In this example, students must determine the slope of a linear equation in general form and then determine the equation of a line that is perpendicular and passes through a given point.

Students can confirm that the slopes are of perpendicular lines by multiplying the slopes to get -1.

You may wish to show students the following third method, which uses the general form and the point-on property.

The slope of the line $Ax + By + C = 0$ is represented by by $-\frac{A}{B}$. The only values that affect the slope are A and B. Since the slopes of perpendicular lines are negative reciprocals, an equation of a perpendicular line in general form can be obtained by switching the A and B coefficients and changing the sign of one of them. Hence, the equation $2x - 3y + C = 0$ represents a line perpendicular to the original line, $3x + 2y - 6 = 0$.

To find C in $2x - 3y + C = 0$, substitute the coordinates of the point $(9, 0)$ for (x, y).

$$\begin{aligned} 2x - 3y + C &= 0 \\ 2(9) - 3(0) + C &= 0 \\ 18 + C &= 0 \\ C &= -18 \end{aligned}$$

Therefore, the equation of the line in general form is $2x - 3y - 18 = 0$.

The equation of the line in slope-intercept form is $y = \frac{2}{3}x - 6$.

To answer the blue type near the bottom of the page, students could use the slope-point form of the equation and convert that equation to general form. The advantage to working with equations developed earlier in the solution is that there is less chance of incorporating an error than when working with a solution that has been manipulated many times.

Key Ideas

Have students summarize what they have learned. Ensure that students understand the properties of parallel and perpendicular lines.

Meeting Student Needs

- You may wish to post the information from the beginning of this section for students to use as a reference for the section. This information can be placed on a bookmark with other key information from the chapter, or it may be enlarged on a photocopier and posted at the front of the room for continual reference.
- Be sure students understand the concept of negative reciprocals (reciprocals that are opposite in sign; they have a product of –1).
- Allow students to use a graphing calculator for Example 1.
- You may wish to demonstrate that a line parallel to $2x + y + 1 = 0$ will be of the form $2x + y + C = 0$ (based on the fact that the slope = $-\frac{A}{B}$, so the values of A and B do not change). An extension for perpendicular lines can show that the slope of a line perpendicular to $Ax + By + C = 0$ will be $\frac{B}{A}$. Hence, the equation of a perpendicular line will be of the form $Bx - Ay + C = 0$.

Enrichment

- An old-fashioned analog clock has three hands, for hours, minutes, and seconds. How often in a 24-h day will the second hand be parallel to either of the other hands?

Gifted

- Challenge students to explain mathematically why parallel railway tracks appear to touch in images of such tracks as on the prairies. (Example: Suppose a big triangle represents the relationship between the actual image and the image that appears on the retina. One side is the distance to the tracks and a right angle between the two tracks. The farther the tracks are away, the lesser the angle from the eyes' perspective. The change in angle lessens the distance between far-away objects, such as the two sides of the tracks in the distance. Since the change in angle is linear, the lines still appear straight, but they converge.)

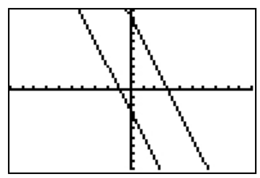
Answers

Example 1: Your Turn

a) neither **b)** parallel **c)** perpendicular

Example 2: Your Turn

$y = -3x + 9$, $3x + y - 9 = 0$



Example: The graphs are parallel lines with different x- and y-intercepts.

Example 3: Your Turn

$y = \frac{1}{4}x - 8$ or $x - 4y - 32 = 0$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work in pairs. • Encourage students who are having difficulty to develop a chart that allows them to fill in the characteristics of parallel and perpendicular lines. Specifically, the column for slope and y-intercept would serve as a quick reference. • Remind students that the negative reciprocals' product is –1. • Ensure that students understand that the comparisons can only be made when the forms of the equations are identical.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Review with students the characteristics of parallel lines. Remind them that knowing the slope is the key as the tilts of the lines must be the same. • Encourage students to use their favourite personal strategy for determining the equation of a parallel line. Where possible, encourage them to try more than one method of explaining how a second approach could be completed.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Review with students the characteristics of perpendicular lines. Remind them that knowing the slope of the original allows them to determine the negative reciprocal of the value and find a perpendicular line. • Encourage students to use their favourite personal strategy for determining the equation of a perpendicular line. Where possible, encourage students to try more than one method of explaining how a second approach could be completed.

Check Your Understanding

Practise

For #1g), you may wish to suggest that students begin by writing 0 as a fraction, for example, $\frac{0}{1}$. So, the reciprocal becomes $\frac{1}{0}$. Since division by 0 is not allowed, the slope must be undefined. A similar approach can be used for #1h).

For #2, you may wish to show students how to find the slope of an equation in general form. Please refer to the notes for Example 2.

Before students complete #4, you may want to present a simpler example, such as the following: The slopes of two lines are $\frac{2}{5}$ and $-\frac{6}{n}$. Find the value of n if the lines are parallel. Find the value of n if the lines are perpendicular.

Apply

In #8, students should learn that drawing conclusions from a sketch may be misleading. Slope is the best way to prove two lines are parallel. You may wish to reinforce this concept by asking students to prove two lines are parallel using the distance formula.

For #10, students may need to be reminded that $y = 0$ is the equation for the x-axis.

For #11, suggest that students equate expressions for the slopes of the lines and then solve for n.

For #12, students can prove ΔABC is a right triangle either by using slopes or by using the Pythagorean relationship.

Question #13 requires students to use skills from sections 7.1 and 7.2.

For #16, direct students to the definition of a tangent to a circle shown beside #16. A tangent to a circle was introduced in grade 9.

For #18, using the three end points given, (20, 14), (23, 14) and (23, 12), will give equal slopes only if decimals are used for calculating the slope. Students working with fractions will have slopes that are very close, but not equal.

Extend

If you have shown students how to find the slope of an equation in general form, questions #19, 20, 23, and 24 should be straightforward.

For #21, students must realize that the shortest distance between two lines is a perpendicular distance. This question has many steps to it. Students should pick a point on one line and find the equation of the line passing through that point and perpendicular to the second line. Students then need to find the intersection point of the perpendicular line and the second line. Finally, the perpendicular distance between the lines is the distance from the original point and the intersection point.

For #23, you may wish to inform students that there are two values for n that will make the lines parallel.

Create Connections

Encourage students to experiment with perpendicular lines in varying orientations as they work on #25. They need to determine the slopes of vertical, horizontal, and oblique perpendicular lines in order to fully answer the question.

You may wish to have students complete #26 after the Link the Ideas section.

Meeting Student Needs

- Provide **BLM 7–10 Section 7.4 Extra Practice** to students who would benefit from more practice.
- Questions #1 to 5 should be completed to fully develop the concept of slope as related to parallel and perpendicular lines.
- Remind students of the three forms of expressing linear equations. Students may need to use different forms to assist them to complete part of the assignment. For example, #6 requires the slope-point form.
- For the Apply questions, allow students to work in either pairs or small groups. These questions are abstract and students will need assistance. You may wish to have students complete the questions in a station approach, where the questions are written on chart paper and each group adds at least one key comment/response on the chart paper.
- For #12 and 22, students could also plot the points on grid paper to visualize which lines need to be perpendicular in order for the triangle to be a right triangle.

Enrichment

- Related to #17, ask students to play with the formula to answer the following questions:
  – Will the lines get closer together or farther apart as you age?
  – When will the lines intersect? What would this mean? (Answer: You would be 220 years old and your heart rate would be 0.)
  – Is the above answer valid?

Common Errors

- Some students may not determine a negative reciprocal correctly.

R_x Reiterate that the slopes of perpendicular lines are related in two ways: they have opposite signs and are reciprocals. Having opposite signs means if m_1 is positive, then m_2 is negative, and vice versa. Being reciprocals means their numerators and denominators are interchanged.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–5, 6a), c), e), 7a), c), e), and 9–11. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to have students work in pairs to discuss and compare responses. • You may wish to review how an integer can be written as a fraction when finding the perpendicular slopes in #1, 2, and 7. • Encourage students to refer to their Foldable or any summary page they have created for solving problems relating to formulas. • Some students may benefit from reactivating what an equivalent fraction is. Provide them with several questions that they can practise with before starting #4. • For #9, struggling learners may benefit from recalling the definition of a parallelogram. • If students have difficulty with #10, have them sketch a horizontal line and perpendicular line of their own choosing. Have them determine the slope. Ask them which is parallel to the *x*-axis and which to the *y*-axis.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Both questions are excellent Assessment *as* Learning questions and should be considered for the students' Foldable. Encourage students to include specific examples in their explanation.

7 Chapter 7 Review

Mathematics 10, pages 396–398

Suggested Timing

60–90 min

Materials

- grid paper
- ruler

Blackline Masters

BLM 7–6 Section 7.1 Extra Practice
BLM 7–7 Section 7.2 Extra Practice
BLM 7–9 Section 7.3 Extra Practice
BLM 7–10 Section 7.4 Extra Practice

Planning Notes

Have students work individually on #1 and 2. Ensure that students can identify the slope and y-intercept from the equation of a line and that they can determine the equation if given both of these parameters.

Have students who are not confident with a specific form of linear equations discuss strategies with another classmate. Encourage students to refer to their chapter Foldable, summary notes, classroom-developed posters, worked examples, and previously completed questions in the related student resource.

Encourage students to make a list of questions that they found difficult and to use this list to help prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 7–6 Section 7.1 Extra Practice**, **BLM 7–7 Section 7.2 Extra Practice**, **BLM 7–9 Section 7.3 Extra Practice**, and **BLM 7–10 Section 7.4 Extra Practice**.
- You may wish to provide students with a copy of the student outcomes. Have students rate their level of understanding for each outcome. Students can then use their responses to direct them as to which topics to begin their review with.
- You may wish to review the Key Ideas for each section and summarize the points on the board or overhead.
- Allow students to create flash cards to help them study the chapter's key terms and formulas.
- Keep all posters and chart papers displayed in the classroom while students complete the review questions.

Enrichment

- Builders often need to ensure that structures are "square," meaning perpendicular. They also need to check that structures that need to be parallel are parallel. Challenge students to explain how using triangle facts can be used to check for accuracy, and why the bigger the triangle, the greater the accuracy. (Example: Builders often use 3-4-5 triangles to ensure right angles. Builders use a tape measure to find a 3-ft distance on one wall and a 4-ft distance on another wall. If the distance between those two points is 5 ft, then the angle between the walls is 90°. If the builder uses a 6-8-10 triangle instead, the accuracy improves because the percent of error in the measurement is diminished by the greater distance.)

Gifted

- Challenge students to explore the following question: "If parallel lines never meet and are the same distance apart, are concentric circles parallel?" Have students explain their reasoning in terms of the similarities and differences between parallel and concentric lines. (Example: In plane geometry a line is straight but in solid geometry it may be curved. The main thought on this seems to be that there are parallel curves and parallel lines.)
- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Common Errors

- Some students isolate the y-term and then read the y-intercept and slope from the equation. For example, in #1b), students rewrite the equation as $6y = 5x + 12$ and state the slope as 5 and the y-intercept as 12.

R_x Have students practise rearranging equations to isolate a specified variable. Encourage students to check their work, for example, by verifying that the coordinates of the y-intercept satisfy the equation.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 7 Review The Chapter 7 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test.

7 Chapter 7 Practice Test

***Mathematics 10*, pages 399–401**

Suggested Timing

45–60 min

Materials

- grid paper
- ruler

Blackline Masters

BLM 7–11 Chapter 7 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Students can then indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions. Tell them that this test is for practice, and identifying which questions are most challenging shows them which concepts they need to revisit.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–9 and 12.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	7.1	Example 1	✓ identify the slope and y-intercept of a straight-line graph
#2	7.2	Example 2	✓ identify the x-intercept and y-intercept of a line
#3	7.1	Example 2	✓ rewrite a linear relation in slope-intercept form
#4	7.2	Example 2	✓ convert a linear equation to general form
#5	7.3	Example 1	✓ determine the equation of a line using its slope and a point on the line
#6	7.4	Example 1	✓ identify whether two lines are parallel, perpendicular, or neither
#7	7.4	Investigate Link the Ideas	✓ identify whether two lines are parallel, perpendicular, or neither
#8	7.3	Example 2	✓ determine the equation of a line from two points on the line ✓ convert a linear equation to general form
#9	7.2	Example 4	✓ solve problems using the equation of a linear relation
#10	7.3	Examples 1, 2	✓ identify equivalent linear relations from a set of linear relations
#11	7.1 7.2 7.4	Examples 2, 4 Link the Ideas	✓ solve problems using the equation of a linear relation ✓ relate the intercepts of a graph to the situation
#12	7.4	Example 2	✓ determine the equation of a line using the coordinates of a point on the line and the equation of a parallel or perpendicular line
#13	7.3	Examples 1, 2	✓ explain strategies for graphing a linear relation in slope-point form
#14	7.1 7.2	Example 4 Investigate	✓ solve problems using equations in slope-intercept form ✓ relate the intercepts of a graph to the situation
#15	7.3	Example 3	✓ determine the equation of a line using two points on the line ✓ convert equations among the various forms ✓ explain a method of graphing and sketch a graph of an equation

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 7 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Have students use their responses on the practice test and work they completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 7 Test After students complete the practice test, you may wish to use **BLM 7–11 Chapter 7 Test** as a summative assessment.	• Consider allowing students to use their Foldable.

UNIT 3

Unit 3 Project

***Mathematics 10*, pages 402–405**

Suggested Timing

80–100 min

Materials

- ruler

Blackline Masters

Master 1 Project Rubric
BLM 6–4 Chapter 6 Unit 3 Project
BLM 7–4 Chapter 7 Unit 3 Project
BLM U3–3 Unit 3 Project Final Report

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF5 Determine the characteristics of the graphs of linear relations, including the:
- intercepts
- slope
- domain
- range.

RF6 Relate linear relations expressed in:
- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1 = m(x - x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Planning Notes

Discuss what type of evidence a forensic archaeologist might look for and how the evidence can give information related to time or distance travelled. Discuss how such bones are treated after study. For example, modern archaeologists return bones like these to the families or descendants for proper burial.

Emphasize to students that they need a system for putting all of the information together once they complete the questions in Part B.

Unit project questions occur throughout Chapters 6 and 7. These questions are optional, but some of them do provide additional information that students may need to complete the unit project. For example, students may need to know what carbon-14 dating is or how the length of a particular bone can enable one to predict the height of the person. Information about these processes is included in the relevant unit project questions. You may wish to have students record their finalized answers for the Unit 3 project in a student booklet that you create from **BLM 6–4 Chapter 6 Unit 3 Project** and **BLM 7–4 Chapter 7 Unit 3 Project**.

As you read through the project, highlight the information in Part A. Students can begin to hypothesize about want happened to the trio. In Part B, students will use slope to verify that the points are collinear. When determining the time it would take to travel from point A to point B, students will need a multi-step approach. Remind them to pay attention to the map scale. Encourage students to be creative. Their explanation, however, must be consistent with the answers to Part B.

Give students time to review the contents of their project portfolio and ensure that they have completed all required components for their final report or presentation. Students should have completed all of the questions related to Part B on pages 404 and 405 in the student resource, and they should include a description of a new clue as part of their explanation of what happened to the three gold seekers.

Meeting Student Needs

- The Unit 3 project is a great way to showcase student learning. Post Part A outside the classroom when it is assigned. Create excitement by posting leading questions or hints, either just one a day, or possibly three or four one day to encourage students to come back to your classroom to gain assistance to finish the project. You may wish to offer a "Pot of Gold" for the best presentation that was finished quickly. (The "Pot of Gold" could be chocolate or gum coins wrapped in gold foil.)
- Students may need to recall how to read a contour map. Tell them that each line represents the same elevation above sea level. You may wish to discuss the meaning of the contour interval given in the legend.
- To find the distance to the top of Mount Tempest, students will need to approximate. Encourage them to try a variety of methods, such as the distance formula and Pythagorean relationship.
- Some students may benefit from redrawing the path that the gold seekers took using grid paper. Encourage students to determine where the origin on their coordinate grid should be located.
- You may need to remind students of the directions: north, south, east, and west.

ELL

- Students may not be familiar with the term *terrain*. Explain the meaning of this word and emphasize to students that in the context of #4b), students are asked to describe the slope of the mountain.
- The term *blueprints* may be new to students. You may wish to bring in some blueprints to show what they are and discuss how they are used.

Enrichment

- Have students determine the length of time it would take to travel from point B to the top of Mount Tempest and down to point C.

Answers

Part B

1. 6.5 h **2.** Yes; $m_{AB} = m_{BC} = -\frac{1}{8}$

3. Yes; no contour lines are crossed.

4. a) 3000 m

b) Example: The contour lines around Mount Tempest are much closer than around other rock formations, indicating a faster altitude change and steeper slope. In addition, the slope of the ascent is steeper than that of the descent.

5. a) 6.7 h **b)** 1.3 h

6. 4 °C at the base of Mount Tempest and –15.2 °C at the top
Example: This low temperature would cause the travellers to expend extra energy in keeping themselves warm and require the use of heating elements since water would freeze.

7. Example: Taller male: 2.0 m, Shorter male: 1.6 m, Female: 1.5 m

8. a) 120.66 years

b) The first femur belongs to the woman, since $H(40.9) \approx 155$. The second femur belongs to the taller male, since $M(56.1) \approx 195$.

9. a) $y = -\frac{1}{8}x + \frac{83}{8}$ **b)** $8x - y - 274 = 0$ **c)** $n = 38$

Part C

Example: The three explorers started their journey by canoe. The canoe capsized due to rapids and most of the supplies were lost. Sam, the leader of the group and the tallest, suggested that they continue their trip for gold and ration their supplies more strictly. The ascent up Mount Tempest was harsh due to snow (which required the party to wear snowshoes) and cold weather. After this climb, the snowshoes were left behind to both serve as a checkpoint and to lighten the load of the traveller's cargo. Upon arrival at Honey Lake, tension had arisen between Sam and Kittie, both of whom wanted to take all of the gold for themselves. The argument only increased in frequency and in intensity after the gold was retrieved. Halfway through the ascent back up Mount Tempest they both collapsed due to exhaustion. This was probably caused by the extra energy used in their arguing as well as the limited quantity of food and supplies. This left Billy with more than enough food, supplies, and treasure to return to civilization. After finding his snowshoes, he began the final descent down Mount Tempest. Once he reached the closest village, which happened to be located near the banks of Moon River, he purchased a car with a small portion of the gold he retrieved. The seller of the car told authorities that he thought it strange to be paid in gold coins. This final clue helped the investigator explain what happened to Sam, Kittie, Billie, and their treasure.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 3 Project This unit project gives students an opportunity to demonstrate the concepts, skills, and processes learned in Unit 3. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on the Unit 3 project.	• You may wish to have students use **BLM U3–3 Unit 3 Project Final Report**, which provides a checklist for students to identify where in their project they demonstrate the skills, concepts, and processes explored in Unit 3. • Reviewing **Master 1 Project Rubric** with students will help clarify the expectations and the scoring. It is recommended to review the scoring rubric at the beginning of the unit, as well as intermittently throughout the unit to remind students about the project assessment.

The Specific Level Notes below provide suggestions for using **Master 1 Project Rubric** to assess student work on the Unit 3 project.

Score/Level	Specific Level Notes
5 (Standard of Excellence)	• provides a complete and correct response with clear and concise communication; may include a minor error that does not affect the understanding of the overall project; may include weak communication in no more than one calculation
4 (Above Acceptable)	Provides *one* of the following: • a complete response to all parts of the project, with possibly weak or missing justification in no more than two calculations; includes good communication that addresses the relationships among slope, intercepts, and linear equations and how they can be used to make predictions • a complete response with one error that is carried through correctly (i.e., uses an incorrect point in determining an equation but then graphs and interprets the results correctly based on an incorrect equation); includes good communication that addresses how the concepts relate to solving the puzzle of the gold seekers • a response which addresses all parts of the project but is difficult to follow and lacks organization; does not provide obvious support and connection for each explanation of the disappearance of the gold seekers; includes good communication
3 (Meets Acceptable)	Demonstrates *one* of the following: • makes initial correct start to all sections of the project • correctly substitutes a given length into a linear equation to determine height and to solve problems; determines the span of values correctly; tests calculations against additional data with some errors present; is able to identify a function and a non-function; is able to sketch a straight line from a given equation with accuracy; can determine the slope and y-intercept and explain their representations with some consistency; models a basic understanding of slope and equations and their applications; communication contains some connections to the overall project • provides answers to all parts without supporting work or justification
2 (Below Acceptable)	• makes initial start to various sections of the project; provides some correct links • determines an appropriatee range for two functions but is unable to explain why it is appropriate • sketches a straight-line graph of given data with and without technology, with some success • classifies a relation as a function or non-function and makes some connections to the problem • determines the slope of a line and explain what it represents with some success • uses the graph of a linear function to solve problems with limited success • identifies the y-intercept but can not explain what it represents • attempts to determine the equation of a line with limited success • uses a given equation to solve problems • includes some communication
1 (Beginning)	• makes initial start to various sections of the project but is unable to carry through or link concepts together • substitutes a given value into a linear equation to determine height • sketches a straight-line graph of given data using technology only • classifies a relation as a function or non-function but makes no connection to the problem • determines the slope of a line but is unable to explain what it represents • attempts to predict the altitude of a given temperature from a straight-line graph with limited success • attempts to determine the equation of a line with little or no success • includes little or no communication

UNIT 3

Review and Test

Mathematics 10, pages 406–409

Suggested Timing

60–90 min

Materials

- grid paper
- ruler

Planning Notes

Have students work independently to complete the review and then compare their solutions with a classmate. Alternatively, assign the Unit 3 Review to reinforce concepts, skills, and processes learned so far. If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage students to refer to their notes in each chapter Foldable and then to the specific section in the student resource and/or their notebook. Once students have found a suitable strategy, have them add it to the appropriate section of their chapter Foldable. Consider having students make a list of the questions that they found difficult. Students can then use the list to help them prepare for the unit test.

Meeting Student Needs

- To determine whether a set of coordinate pairs represents a function or not, some students may need to graph the data, whereas other students may wish to predict the answer by comparing the x-values and y-values.
- Before they create the graph, allow students to convert equations to the form they feel most comfortable using.
- Encourage students to use their chapter Foldables and to add new notes if they wish.

Common Errors

- For #1b) on the unit review, some students will sketch a horizontal line because the cost of peanuts is constant.

R_x Have students create a table of values showing the number of bags of peanuts and the cost. Ask students to explain the information in the table and ask them if the cost for purchasing each number of bags of peanuts seems appropriate. Encourage students to identify the values plotted on each axis and to explain what the slope of the line should show. Have students verify whether this is represented in the graph they drew.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 3 Review The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.	• Have students review their notes from each Foldable and the tests from each chapter to identify items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter. • Have students revisit any chapter section they are having difficulty with.
Assessment *of* Learning	
Unit 3 Test After students complete the unit review, you may wish to use the unit test on pages 408 and 409 as a summative assessment.	• Consider allowing students to use their Foldable.

UNIT 4

Systems of Equations

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Mathematics 10, pages 410–413

Suggested Timing

30–40 min

Blackline Masters

BLM U4–1 Unit 4 Project
BLM U4–2 Unit 4 Project Checklist

What's Ahead

In Unit 4, students extend their knowledge of linear relations to investigate systems of linear equations. They learn how to identify the solution to a system of linear equations shown on a graph and how to use algebraic manipulation to solve a linear system. Students use both the substitution and elimination methods. Throughout the unit, students model real-life situations using systems of linear equations. Students interpret the information shown on the graphs and in the equations and discuss how the information relates to the particular situation. In this unit, students analyse various types of linear systems where the system of equations may have no solution, one solution, or an infinite number of solutions. Students learn several strategies for predicting the number of solutions.

Planning Notes

Introduce Unit 4 by pointing out the systems of equations organizer on page 410 of the student resource. This organizer shows the topics in this unit—solving systems of linear equations graphically and algebraically. It encourages students to record different ways to solve linear equations. The organizer is repeated at the beginning of each chapter and is shaded to show which topics are covered in that particular chapter.

In the opening paragraphs, you may wish to discuss specific decisions that business owners need to make that may be represented using a system of linear equations. Some examples are employee salaries, selling prices of products or services, or use of business supplies.

The Looking Ahead box at the bottom of page 411 identifies the types of problems students will solve throughout the unit. You may wish to reactivate students' knowledge of intersection points.

Unit 4 Project

The Unit 4 project focuses on the real-world application of water conservation and how linear systems can be applied to analyse information related to water use, wildlife that depends on water sources, and water conservation through retrofitting.

Introduce the Unit 4 project by reading and discussing the introductory notes on page 412 of the student resource as a class. Consider distributing **BLM U4–1 Unit 4 Project** to inform students about how the project develops throughout the unit. This master provides an overview of the project.

The Unit 4 project questions are identified throughout Chapters 8 and 9 with a project logo. These questions are not mandatory but are recommended because they provide some of the background needed to complete the Unit 4 project. The questions are also available on masters, one for each chapter. You may decide to use these masters to create a student booklet and have students record their finalized answers in the booklet. They can do this after they have completed their in-class work, during assigned project work time, or in conjunction with chapter assignments. Alternatively, you may wish to provide students with **BLM U4–2 Unit 4 Project Checklist**, which lists *all* of the related questions for each chapter. Students can use the checklist to monitor their progress and prepare their presentation. Have students collect all their work for the Unit 4 project in a portfolio.

Career Connection

Use the collage of photographs to direct a discussion about decisions made in daily life and in the careers of people who may need to make similar decisions. For example, in a fast-food sandwich shop, the manager may need to determine how many scoops of tuna can go on one sandwich, or a construction crew may need to monitor the fuel efficiency of its fleet of vans. Students will likely be able to name many careers that involve mathematics. Ask them what decisions they think need to be made in these careers and discuss whether the relationships involved are linear.

Web Link

For information about careers related to mathematics, go to www.mhrmath10.ca and follow the links.

Solving Systems of Linear Equations Graphically

8

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:
- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
8.1	✓ explain the meaning of the point of intersection of a system of linear equations
	✓ solve a system of linear equations graphically, with and without technology
8.2	✓ model a situation using a system of linear equations
	✓ interpret information from the graph of a system of linear equations
	✓ solve problems involving systems of linear equations
8.3	✓ explain why a system of linear equations may have no solution, one solution, or an infinite number of solutions
	✓ predict and verify the number of solutions a system of linear equations has

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 8–1 Chapter 8 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 414 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about linear relations. You might provide the following prompts: • Where have you encountered a graph of a linear relation? • What were the variables involved in the graph? • Did you know the equation of the graph? How could you determine it? • When might you have seen two lines graphed together? What story could you make up about the graph?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 8–2 Chapter 8 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 8 Foldable As students work on each section in Chapter 8, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter.
Assessment *for* Learning	
BLM 8–3 Chapter 8 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing mathematics calculations.

Chapter 8 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters
Chapter Opener • 30–40 min (TR page 303)			BLM 8–1 Chapter 8 Self-Assessment BLM 8–2 Chapter 8 Prerequisite Skills BLM 8–4 Chapter 8 Unit 4 Project BLM U4–2 Unit 4 Project Checklist
8.1 Systems of Linear Equations and Graphs • 100–120 min (TR page 306)	Students should be familiar with • ordered pairs and tables of values • evaluating expressions • drawing a line graph • slope-intercept form of the equation of a line • isolating a variable • evaluating expressions	• 0.5-cm grid paper • ruler • coloured pencils • computer with graphing or spreadsheet software	BLM 8–3 Chapter 8 Warm-Up BLM 8–4 Chapter 8 Unit 4 Project BLM 8–5 Section 8.1 Extra Practice TM 8–1 How to Do Page 420 Example 2 Using TI-Nspire™ TM 8–2 How to Do Page 420 Example 2 Using Microsoft® Excel
8.2 Modelling and Solving Linear Systems • 100–120 min (TR page 316)	Students should be familiar with • drawing a line graph • point of intersection • algebraic modelling • evaluating expressions • rate of change	• map of Canada • ruler • grid paper or computer with graphing software	BLM 8–3 Chapter 8 Warm-Up BLM 8–4 Chapter 8 Unit 4 Project BLM 8–6 Section 8.2 Extra Practice
8.3 Number of Solutions for Systems of Linear Equations • 100–120 min (TR page 324)	Students should be familiar with • drawing a line graph • parallel lines • algebraic modelling • reducing equations to lowest terms • isolating a variable	• stopwatch or other timer or clock showing seconds • measuring tape • grid paper and coloured pencils • computer with graphing software • graphing calculator	BLM 8–3 Chapter 8 Warm-Up BLM 8–7 Section 8.3 Extra Practice
Chapter 8 Review • 60–90 min (TR page 331)		• grid paper • ruler • graphing calculator or computer with graphing software	BLM 8–5 Section 8.1 Extra Practice BLM 8–6 Section 8.2 Extra Practice BLM 8–7 Section 8.3 Extra Practice
Chapter 8 Practice Test • 60–90 min (TR page 333)		• grid paper • ruler	BLM 8–8 Chapter 8 Test BLM 8–9 Chapter 8 BLM Answers

Section/ Suggested Timing	Exercise Guide	Assessment: Assessment *as* Learning	Assessment: Assessment *for* Learning	Assessment: Assessment *of* Learning
Chapter Opener		TR page 302 Chapter 8 Foldable, TR page 302	TR page 302	
8.1 Systems of Linear Equations and Graphs	**Essential:** #1, 3, 5, 6, 7b), d), 8a), 10, 12, 16, 22 **Typical:** #1–5, 7–11, 13–16, 21, 22 **Extension/Enrichment:** #14, 16, 17–20, 23	TR pages 307, 315 Chapter 8 Foldable, TR page 302	TR pages 311, 312, 314, 315	
8.2 Modelling and Solving Linear Systems	**Essential:** #1, 3–5, 8, 9, 12, 23 **Typical:** #1 or 2, 4, 6, 7, 9, 10 or 11, 12, 13, 15 or 16, 22, 23 **Extension/Enrichment:** #9, 11, 12, 14, 17 or 18, 19–22, 24	TR pages 317, 323 Chapter 8 Foldable, TR page 302	TR pages 320, 323	
8.3 Number of Solutions for Systems of Linear Equations	**Essential:** #1, 2, 4, 5, 7, 9, 12, 19, 20 **Typical:** #1, 3, 4, 6, 9, 10, 12, 13, 19–22 **Extension/Enrichment:** #6, 8, 11, 12, 14–17, 18, 21, 22	TR pages 326, 330 Chapter 8 Foldable, TR page 302	TR pages 328, 330	
Chapter 8 Review	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 8 Foldable, TR page 302	TR page 332	
Chapter 8 Practice Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1–9	TR page 334		TR page 334 BLM 8–8 Chapter 8 Test

Solving Systems of Linear Equations Graphically

Opener

Mathematics 10, pages 414–415

Suggested Timing

30–40 min

Blackline Masters

BLM 8–2 Chapter 8 Prerequisite Skills
BLM 8–4 Chapter 8 Unit 4 Project
BLM U4–2 Unit 4 Project Checklist

Key Terms

point of intersection
system of linear equations
solution (to a system of linear equations)
coincident lines

What's Ahead

In this chapter, students graph and solve systems of linear equations in two variables. Students graph systems by hand, as well as using technology. They work with systems expressed algebraically, as well as in words. In sections 8.2 and 8.3, students interpret situations in order to develop a system of linear equations, then interpret the solution to the system within the context of the situation. The chapter concludes with a thorough examination of the possible types of systems of equations and the possible numbers of solutions, including applications to real-world situations.

Planning Notes

The chapter opener begins by discussing cell phones, transportation, and business. You could have students briefly participate in a placemat activity to start the chapter. Students sit in small groups of four to six students, with at least four groups within a class. Place a large sheet of paper in the centre of each group. Have students read the chapter opener. Students then write, in the space in front of them, everything that they know about the topics mentioned in the chapter opener. Students are not familiar with systems of equations, but they have lots of experience with linear equations. Allow a short time, no more than 3 min, for this activity.

Then, each group moves to the placemat of a different group. Looking at that group's work, they add anything that they can to that placemat. Allow up to 2 min for this rotation. In the third rotation, each group moves to a new placemat and repeats the task one last time. Finally, each group returns to its own placemat. Allow a few minutes for each group to read its placemat and see the contributions of other students.

You can choose one or two of the placemats and then begin a class discussion to highlight the Key Terms and Big Ideas. For example, look for student work that will lead to the idea of a solution, or point of intersection. Have students recall and discuss instances of using a graph to model a problem. Ask them to remember how they used graphs in this way in the past. You may want to have them recall some of the Key Terms from their past experience, for example, slope, intercepts, and tables of values. This discussion could lead into the Investigate in section 8.1.

Unit Project

The Unit 4 project focuses on water and its use by wildlife and humans. The project was introduced prior to the chapter opener. Embedded in each section of the student resource are questions that will help students as they work on the project. In particular, #16 in section 8.1, and #9 and 12 in section 8.2 are related to the unit project. While students may not need to complete these questions to successfully complete the project, they are intended to be helpful and it is recommended that students complete them whenever possible.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 415 and how it might be used to summarize Chapter 8. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

Ensure students understand how to do the following:
- isolate a variable in an equation
- identify a slope and interpret its meaning
- determine and locate the y-intercept
- use the form $y = mx + b$ for graphing.

Students may benefit from reactivating some of these skills and modelling their understanding through some opening questions. A tab in the Foldable has been set aside for such a review.

Section 8.1 Systems of Linear Equations and Graphs introduces students to finding the point of intersection of a system of linear equations in a variety of ways and then asks them to identify multiple approaches to verifying whether the solution is correct. This section has been divided into two tabs to provide students sufficient space to model multiple approaches to both solutions and verification. The use of the 0.5-cm grid paper will provide a place that is unlined for work to be shown and an attached grid area for ease of graphing. A similar approach is taken for section 8.3 Number of Solutions for Systems of Linear Equations, where both graphs and equations are used to determine whether there are zero, one, or an infinite number of solutions to a system.

The first tab of the Foldable will provide a space for recording information regarding any concepts students find difficult or would need more practice in. As students progress through the chapter, provide time for them to keep track of what they need to work on. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 8–2 Chapter 8 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Consider having students staple or tape **BLM U4–2 Unit 4 Project Checklist** to the back of their Foldable. This checklist will help students keep track of the project-related questions and concepts that they have completed.
- Some students may benefit from completing all unit project questions.
- **BLM 8–4 Chapter 8 Unit 4 Project** includes all of the unit project questions for this chapter. These questions provide a beginning for the Unit 4 project.
- Demonstrate what a system of linear equations can represent. Create a set of posters that illustrates solving linear systems graphically as well as algebraically, by substitution and by elimination. You could use these posters to introduce the student learning outcomes for the chapter and unit.
- Post the student learning outcomes for the entire chapter. You may also wish to post the Key Terms for students to view prior to starting.
- You may wish to discuss the decisions involved in buying a snowmobile or ATV. Hybrid cars are not feasible in the Arctic, given the price of electricity; and leasing cars is likely not an option, due to the condition of most roads.

ELL

- Some students may not understand what is meant by a *system of linear equations* or the term *two-variable*. You may wish to discuss these concepts before beginning the lesson.

Enrichment

- Ask students to consider the following questions:
  - On a graph of speed versus time, what does the point of intersection of the lines that represent two race cars mean? (the time at which they are travelling at the same speed)
  - On a graph of distance versus time, what is the point of intersection if one line represents the travel of a person riding a bike and the other line represents a person who left before the cyclist walking along the same path as the cyclist? (the point where the cyclist meets the walker)

Gifted

- Have students solve the following problem: A wealthy financier invests $12 000 in two investments. One pays 5% when sold after one year and the other pays 10%. If the total interest earned is $900, write and graph two equations that represent the situation. What part of the graph shows the amount actually invested in each investment? (The two equations are $x + y = 12\ 000$ and $0.05x + 0.10y = 900$. $6000 is invested in each investment.)

Career Connection

The photo in the chapter opener shows Dr. Ian Stirling removing a satellite collar from a polar bear in the Arctic. Dr. Stirling is doing work for Environment Canada in the Wildlife Research Division. Many students are familiar with the science of biology, but may not understand the importance of mathematics within the field. When students have a few minutes to research, you could direct them to try to discover how mathematics is used to perform the following tasks:

- estimate the size of a particular population
- analyse DNA
- determine the number of organisms an ecosystem can support
- evaluate the effect of climate change on coral reefs
- design footwear for specific uses

WWW Web Link

For information about biologists, go www.mhrmath10.ca and follow the links.

8.1 Systems of Linear Equations and Graphs

Mathematics 10, pages 416–431

Suggested Timing

100–120 min

Materials

- 0.5-cm grid paper
- ruler
- coloured pencils
- computer with graphing or spreadsheet software

Blackline Masters

BLM 8–3 Chapter 8 Warm-Up
BLM 8–4 Chapter 8 Unit 4 Project
BLM 8–5 Section 8.1 Extra Practice
TM 8–1 How to Do Page 420 Example 2 Using TI-Nspire™
TM 8–2 How to Do Page 420 Example 2 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3, 5, 6, 7b), d), 8a), 10, 12, 16, 22
Typical	#1–5, 7–11, 13–16, 21, 22
Extension/Enrichment	#14, 16, 17–20, 23

Unit Project Note that #16 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce prerequisite skills needed for this section. Browse the Internet to find examples of graphs that represent systems of linear equations. You may wish to display the graphs using a projector. Encourage students to discuss and interpret what the point of intersection represents in each case.

Investigate Ways to Represent Linear Systems

You may wish to have students predict the outcome of the investigation before they begin. To open a discussion, ask students, "Will one plan always be better than the other?" Alternatively, you may wish to have students use technology to graph the lines. For example, students could input the tables of values into a spreadsheet and use the spreadsheet to generate the graph.

In discussing responses to #3, you may wish to lead students to the idea of slope. In this situation, the slope of each line is the cost per minute of the corresponding plan. You could ask students to discuss the lines in light of their work in previous chapters. Ask students the following questions:

- How are the graphs of the lines similar? How are they different?
- Do both lines have the same y-intercept?
- What does the y-intercept represent in this situation?
- Do both lines have the same slope? What does the slope represent in this situation?

As an extension to #5, you may wish to ask students to decide under which conditions each plan would be preferable.

Meeting Student Needs

- To make the section opener more relevant to Northern students, you may wish to discuss the gaps in cell phone coverage in the North and whether there is any possibility of this changing. You may also wish to discuss how cell phone plans work and how to choose an appropriate one.

- Invite students to share the details of their cell phone plans with the class, including base monthly cost and cost per minute for local or long distance calls. Pair students such that the students in each pair are on different plans, if possible. Then, have students create a table of values and sketch and analyse the graph. Encourage students to include a description of the plan they prefer and why.
- Some students may benefit from a discussion about how a system of two linear equations involves two variables and that there could be systems of three or more linear equations.

Common Errors

- Students may not pay careful attention to units, in this case dollars and cents.

R_x Be sure that students have used these units properly in their tables and graphs. Encourage students to consider whether their answers make sense in the context of the problem.

- Students may not follow the conventions for making graphs.

R_x As you watch them work, remind students to label axes, show correct units, and indicate which graph relates to each cell phone plan.

Answers

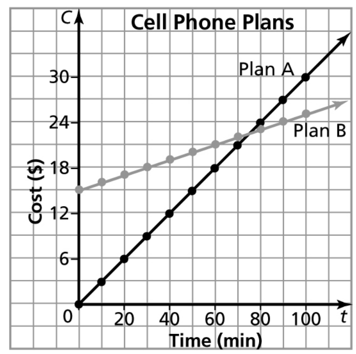
Investigate Ways to Represent Linear Systems

1.

Plan A	
Time (min)	**Cost ($)**
0	0
10	3
20	6
30	9
40	12
50	15
60	18
70	21
80	24
90	27
100	30

Plan B	
Time (min)	**Cost ($)**
0	15
10	16
20	17
30	18
40	19
50	20
60	21
70	22
80	23
90	24
100	25

2.



3. The graph of Plan A starts at zero and goes up more quickly than the graph of Plan B. The graph of Plan B starts higher but goes up more slowly. The graphs cross between 70 min and 80 min.

4. The point of intersection is the number of minutes where both plans would cost the same. The point of intersection is at 75 min and a cost of $22.50 for each plan. The table hints at these numbers being between 70 min and 80 min and between $22 and $23.

5. Example: I prefer Plan A since I use less than 75 min per month and this plan would save money for my use.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Encourage students to write the coordinates of the intersection point on the graph to make a visual link to the table of values. • Some students will be able to complete the table of values using patterning without really understanding what they are determining. Draw their attention to the connection between the numbers in the table and the coordinates on the graph. Use the labels on the axes to prompt students in their responses to #3 and 4. • The response to #4 should be discussed as a class. • To assist students in answering #5, have them choose a value for minutes that exceeds their point of intersection and ask them to describe what each plan would cost at that time. Use this response to prompt their thinking about how the point of intersection could assist students in determining which plan is the better option. • Provide students with an opportunity to include the final summation in their Foldable or graphic organizer.

Link the Ideas

After discussing the different representations of systems of linear equations and how to identify the solution, you may wish to have students consider the advantages and disadvantages of each method. For instance, you could ask the following questions:

- Will the solution to a linear system always appear in the table of values?
- Is it always convenient to create a graph?
- When might it be difficult to accurately read values from a graph?

You may wish to emphasize that the solution to a system of linear equations is not a single value, but rather a coordinate pair. It is not sufficient, for example, to obtain only a value for x when solving a system of linear equations.

Example 1

Draw students' attention to the type of relation involved in this example by asking the following questions:

- What does it mean when points on a graph are connected by a line?
- What term describes this type of data? (continuous data)
- What does it mean when the points are not connected? What term describes this type of data? (discrete data)
- Should the data points for this example be connected? Why?

As three different solution methods are shown, you may wish to have students work in small groups. You could assign groups to each of the three methods, with the understanding that each group will explain the solution method to the whole class. If an interactive whiteboard is available, you may wish to demonstrate Method 3 to the class. Regardless of the method chosen, you should help students connect this work to their previous work graphing straight lines.

If you choose to have students work cooperatively in Example 1, then you may want to specify that students complete the Your Turn question using a different method than they did with their small group. Alternatively, you may wish to have students work independently to solve the linear system, and then discuss the advantages and disadvantages of each method with their classmates.

Example 2

Before beginning this example, you could have students brainstorm methods for graphing lines, in particular methods that do not rely on tables of values as they have used so far in this section. You may wish to have students recall the work they did in Chapter 7 and briefly summarize the different forms of writing linear equations and how to graph a line in each form. Consider allowing some time for students to explain the method they prefer.

When considering the three solution methods shown, you could ask the following questions:

- Will one method always be more efficient than the other?
- Are all methods equally precise?
- In which circumstances would you choose to use each method?
- Does every graph have an x-intercept and a y-intercept?
- Could you create the graph using points other than the intercepts?
- Does every graph have a slope?

When students are verifying their solutions, ensure that they understand they are simply checking that the solution point lies on both lines. They can check this using any representation they choose—such as a table of values, a graph, or substituting into the equations—other than the one they used to solve the linear system.

You may wish to have students use **TM 8–1 How to Do Page 420 Example 2 Using TI-Nspire™** or **TM 8–2 How to Do Page 420 Example 2 Using Microsoft® Excel** as they work through part b).

Example 3

In this example, students do not determine the solution to a system of linear equations; they verify a given solution. In this case, the given point satisfies the equation of one line but not the other line. Students should note that the line containing the point corresponds to the equation that the coordinates satisfy.

You may wish to ask students, "Are there other situations where a point is not a solution to a system of linear equations?" Elicit the possibility that the point could be on neither of the lines.

Example 4

This example presents an opportunity for problem solving. Students need to visualize the travel of the trams and the fact that once during each trip, the trams pass each other. The trams are at the same altitude at the same time. Because the trams travel at different speeds, the altitude where they pass depends on which tram is going up and which is going down. The opening paragraphs provide information about the number of passengers on the trams, which may be interesting to some students but is not required to solve the problem.

To help students work through the problem, you could ask questions such as the following:
- What quantity will be represented on each axis of the graph?
- How do you know that tram travel is linear?
- How will the graphs representing each tram be similar?
- How will the graphs representing each tram be different?
- Why do the trams not meet exactly halfway up the mountain?
- What properties can we expect for each graph? For example, what can we predict about the slope and the intercepts? How can you check to see if your answer is reasonable?

Encourage students to read the Mental Math and Estimation box beside the solution to part b). The reasoning described in the box allows students to double-check that their answer is consistent with information provided in the problem. Emphasize to students the importance of checking the reasonableness of their answers.

Key Ideas

The Key Ideas summarize the concepts that students need to understand in this section. Allow time for students to incorporate the Key Ideas for this section into their Foldable or their math journal. Encourage students to write the Key Ideas in their own words and to annotate them with their own notes regarding their personal preferences and strategies.

Meeting Student Needs

- You may wish to have money manipulatives available for Example 1.
- Example 1 Method 3 could be illustrated on a computer. You could have students enter data and then highlight and extend the chart. Then, you or a student could demonstrate how to create the graph from the given data. Some, but not all, students will not have experience with that step.
- Inform students that solving algebraically will be covered in Chapter 9 and remind students that they currently have the tools to verify their work algebraically once they are checking a possible solution point.
- Students could create one poster for Example 1 and one poster for Example 2 to illustrate the various methods of solving and verifying solutions to linear systems. Students should create new examples to be used on the posters.
- Draw students' attention to the table of values in the solution to Example 2b). Make sure students understand that the first and second columns represent one equation, and the first and third columns give the data for the other equation.
- If it is available, you may wish to allow students to have access to dynamic algebra/geometry software.
- Allow students to create a graph of the trams in Example 4 travelling in opposite directions. This may help students visualize that the altitude and time at which the trams pass each other depend on the directions in which the trams are travelling.
- Create an exit slip based on the Key Ideas for section 8.1. The exit slip can contain fill-in-the-blank questions, true/false questions, short examples, or even pictures beside each outcome of thumbs up/thumbs down that students can circle.

ELL

- The *dimensions of the graph* on a graphing calculator refer to how much of the x-axis and y-axis can be seen on the display. Practise adjusting calculator screens with students.
- Make sure students understand that *altitude* in Example 4 refers to the vertical distance of the tram above a reference point.
- Have students use words and diagrams to distinguish between *vertical* and *horizontal* axes as referred to in Example 4 Method 1.

Enrichment

- Provide students with the following information: On the Apollo 13 mission to the moon in 1970, the astronauts dealt with a life-threatening explosion and frantic return trip to Earth. They had to use paper-and-pencil calculations in order to navigate, including the dangerous re-entry route into Earth's atmosphere. During the calculation process, engineers on Earth checked the astronauts' calculations. Challenge students to describe how being able to calculate the intersection of two simultaneous equations by hand might be valuable. Have them show their expertise by finding the intersection of the following equations using substitution.

$$2x + \frac{1}{3}y = 1$$

$$y \quad \frac{1}{2}x = 0$$

(Checking answers by hand can confirm the work of a calculator, especially if an incorrect value was entered. $x = \frac{6}{13}$, $y = \frac{3}{13}$)

Common Errors

- In the verification step, students often verify the solution in only one equation, not both.

R_x Emphasize that students need to substitute values into both equations, or look at both tables of values, depending on their choice of method.

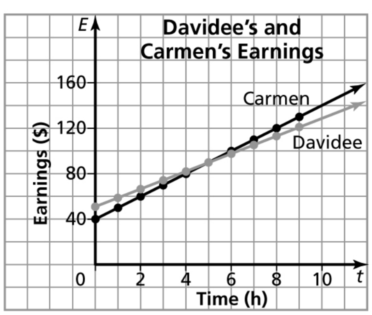
Answers

Example 1: Your Turn

a)

Davidee	
Time (min)	**Cost ($)**
0	40
1	50
2	60
3	70
4	80
5	90
6	100
7	110
8	120
9	130

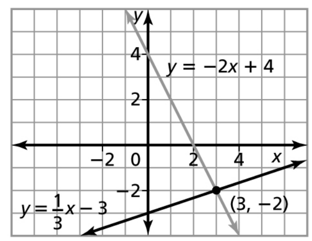
Carmen	
Time (min)	**Cost ($)**
0	50
1	58
2	66
3	74
4	82
5	90
6	98
7	106
8	114
9	122



b) The solution is (5, 90). Both Davidee and Carmen earn $90 for 5 h of work.

Example 2: Your Turn

Substitute $x = 3$ and $y = -2$ into both equations.



In $x - 3y = 9$:

Left Side	Right Side
$x - 3y$	9
$= 3 - 3(-2)$	
$= 3 + 6$	
$= 9$	

Left Side = Right Side

In $2x + y = 4$:

Left Side	Right Side
$2x + y$	4
$= 2(3) + (-2)$	
$= 6 - 2$	
$= 4$	

Left Side = Right Side

Since the ordered pair (3, –2) satisfies both equations, it is the solution to the linear system.

Example 3: Your Turn

a) Substitute $x = 2$ and $y = 5$ into both equations.

In $3x - y = 2$:

Left Side	Right Side
$3x - y$	2
$= 3(2) - 5$	
$= 6 - 5$	
$= 1$	

Left Side ≠ Right Side

In $x + 4y = 32$:

Left Side	Right Side
$x + 4y$	32
$= 2 + 4(5)$	
$= 2 + 20$	
$= 22$	

Left Side ≠ Right Side

Since both equations do not result in true statements, the point (2, 5) is not the solution to this linear system.

Answers

b) Substitute $x = -3$ and $y = -2$ into both equations.

In $2x + 3y = -12$:

Left Side	Right Side
$2x + 3y$	-12
$= 2(-3) + 3(-2)$	
$= -6 - 6$	
$= -12$	

Left Side = Right Side

In $4x - 3y = -6$:

Left Side	Right Side
$4x - 3y$	-6
$= 4(-3) - 3(-2)$	
$= -12 + 6$	
$= -6$	

Left Side = Right Side

Since the ordered pair $(-3, -2)$ satisfies both equations, it is the solution to the linear system.

Example 4: Your Turn

The point of intersection represents when Eric and Nathan will be at the same level at the same time. This point appears to be approximately (18, 21). After about 18 s, Eric and Nathan will pass each other at about floor 21, with Eric on the stairs and Nathan in the elevator.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • For students having difficulty formulating their equation, ask them: – Which of the dollar amounts are fixed numbers? – How much does each person make per hour? after 2 h? after 3 h? Have them verbalize the equation before writing it. • Draw student's attention to Example 1 and ask them: – Which parts of the example and the Your Turn are similar? – Which unit will go on the x-axis of the graph? – Which unit goes on the y-axis? Encourage students to use patterning in creating their equation and graph. • Ask students to identify what each ordered pair on their graph represents (hours, amount earned). • Have students verbalize what the intersection point represents. Encourage them to visualize the labels on their axis and use them in their explanation. • Have students identify the link between the table of values and the intersection point. Ask how one can be used to verify the other.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to review writing equations in slope-intercept form. Have students verbally identify the slope and *y*-intercept. Ask them to explain their meanings. • You may wish to have students who are struggling use technology to assist them in graphing. • Allow students choice in their method: slope-intercept form, *x*- and *y*-intercepts, technology. Encourage them to use two of the three methods. • Remind students of the importance of verifying using not just one but both equations.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Review the meaning of *verifying* and *substituting values into an equation*. • Ask students to identify what the ordered pair has in common with graphing the two lines. Ask them what they would expect to see if the ordered pair is a solution to the system.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work with a partner • Encourage the use of tables for solving problems. • Have students identify how the information in the table in Example 4 is translated into ordered pairs for graphing. Have them identify how they will label their graphs. Ask them what the intersection point would represent. • Encourage the use of patterning between the problem and the example.

Check Your Understanding

Practise

Question #1 asks students to determine the solution to two different linear systems in two different ways. It is worth noting that the two systems are different, but have the same solution. For enrichment, students could be asked the following questions:

- How many different systems could have the same solution?
- Can you provide a third linear system that has this same solution?
- How can you prove that your system has this solution?

A natural result of #2 is to discuss situations when technology presents a clear advantage in solving linear systems. Students can be invited to verify their answer without the use of technology.

Questions #3 and 4 require students to solve linear systems using different methods and to look back at their work. As well, these questions help students make the connection between graphical and algebraic representations of linear systems.

Question #5 has students check to see if a given point is a solution to a system. You may wish to ask students to use a different method to answer each part of this question. This will help them with the metacognitive goal of deciding which method to use in a given situation, based on their own strengths and the characteristics of the system.

In #6 to 8, students solve linear systems graphically. Note that in #6, students are instructed to make a graph without the use of technology, while in #7 and 8, no such stipulations are made. Depending on the technology available, you may wish to have students use different technology tools in #7 and 8, and discuss the advantages and disadvantages of each tool.

In #9, students are asked to check a potential solution to a system of linear equations. However, they are asked to go a step further and describe the graph of the system and the relationship between the point specified and the graphs in the system. Students need to be able to state whether the point is a solution or not a solution. In this case, students should be able to decide if the point is on one of the lines or neither of the lines in the system. You may wish to refer students to Example 3 if they need help with this question.

For #10, some students may benefit from working with a partner and comparing their methods and the systems of equations they develop.

Apply

Question #11 is a problem-solving question. The equations in the system are given. You may want to have students make sure that they understand why those equations are the correct ones for the context given. Part c) asks students how to use the graph to determine profit—a skill that is not specific to systems of equations, but a review of their previous work with functions.

In #12, students solve a linear system graphically, by hand and with technology. This presents another opportunity to address the appropriate use of technology and situations in which the use of technology offers a clear advantage.

Question #13 has students address a shortcoming of tables of values. The tables of values given for the system do not show the solution to the system. Students need to explain why the tables do not yield a solution. You may wish to use the Think-Pair-Share strategy to have students answer the following questions:

- Is it possible to use tables of values to solve this system?
- Is another method preferable for solving this system?

The slopes and y-intercepts in #14 are very similar, requiring care in obtaining a solution. If students graph the system by hand, it may be difficult to work precisely. If students use technology, they will likely not be able to easily read the solution from the graph, but will need to use the functions of the technology to find the intersection point.

Question #15 gives students an opportunity to explain how a graph relates to a given situation as part of the solution to the system.

Question #16 is related to the unit project. This problem should be accessible to all students; the equations are given.

Question #17 requires students to construct a system with a small amount of information and very little scaffolding. You might suggest that students look back to Example 4 if they experience difficulty with this question.

Question #18 is an enrichment opportunity. Students need to translate the information given into a graph of the linear system. If students require help you could suggest that they consider the time and distance at the beginning and end of each trip for each rider. This gives the endpoints of the line segment representing each rider's travel.

Extend

Question #19 involves a system of three linear equations graphed on the same coordinate grid. Some students may not recognize this. You may want to suggest that students consider the equations two at a time as they answer the question.

To help students answer #20, you may want to ask them, "Does it matter how long the truck travels before reaching the car?" If students need help answering this question, you may want to suggest that they perform a few experimental calculations with several values for the time the truck travels before reaching the car.

Create Connections

Questions #21 and 22 give students opportunities to create their own systems of linear equations and applications. You may want to provide time for students to work in small groups to discuss their answers. Alternatively, #21 provides an opportunity for a gallery walk. Students can (anonymously) post their systems throughout the classroom and then take a few minutes to tour the class and see the linear systems and situations created by their peers.

For #23, if students are having difficulty beginning the question, you may wish to suggest that they rewrite the equations in slope-intercept form. This might enable students to analyse and compare the slopes and y-intercepts.

Unit Project

The Unit 4 project questions give students an opportunity to solve problems involving graphs and systems of equations. Students will be asked to prepare a presentation that convinces people to make changes to help reduce water use. You may wish to discuss why we are concerned about our water use and what its importance is in our future. It may give you a better sense of students' understanding of the issues.

Question #16 is a Unit 4 project question. The question provides the system of equations that represents the movement of two different types of ducks to water. It is important for students to be able to identify which equation represents each type of duck and how to evaluate an equation for a specific value, which, in this case, is 25 min. Students have several methods available to them for graphing, including using the slope-intercept form, table of vales, x- and y-intercepts, and technology. Ask students to identify which method is easiest for them to understand; however, encourage them to use more than one method in solving the question. Ask how they could check their work. Point out how two methods can be used to verify their solution.

Meeting Student Needs

- Provide **BLM 8–5 Section 8.1 Extra Practice** to students who would benefit from more practice.
- Allow students to brainstorm possible examples of situations that could be represented with a system of linear equations. You may wish to compile a list ahead of time.
- For #11, you may wish to allow students to create a beaded necklace as an example of the activity.
- Suggest that students develop a question similar to #11. Encourage students to research a fundraising project of their own and use the data. For example, they could sell cases of oranges. There may be a minimum order requirement plus a shipping cost. Students would need to determine the retail price. They could graph the two relations to determine the break-even point.

- For #16, when discussing the Unit 4 project on water conservation, you may wish to invite an Elder or Knowledge Keeper to speak about the Aboriginal teaching of water. Have students discuss in groups the importance of water. For part b), encourage students to explain how the equations relate to the situation.
- Provide time for students to work through at least one part of each of #1 to 8 before continuing with #9 to 22.
- Some students have trouble solving linear systems graphically by hand. Encourage students to develop their skills using technology—graphing calculators or spreadsheet software. You may wish to have students work in pairs or small groups.

ELL

- For #11, students may not be familiar with the terms *revenue*, *break even*, or *profit*. Explain these terms using an example.

Common Errors

- Some students forget what their variables represent and then cannot state their answers accurately. For example, when specifying the time of an event, students may not pay attention to what time = 0 represents and then are not able to interpret what time a number actually means in the context.

R_x Encourage students to clearly identify what each variable represents, including any specific details they need to remember.

- Some students will have difficulty interpreting how the point of intersection relates to the situation.

R_x Suggest that students carefully consider the variables for each axis and visualize the relationship between them. Encourage students to think about what it means in the situation if one point satisfies both equations and if that point contains a coordinate for each variable.

- In #8, students may recognize that the equations in part a) are in slope-intercept form for graphing, but they may have difficulty determining the slope on the graph and finding appropriate points to plot.

R_x Have students convert each decimal slope to a fraction and then count the $\frac{\text{rise}}{\text{run}}$ squares to determine the slope.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3, 5, 6, 7b), d), 8a), 10, 12, and 22. Students who have no problems with these questions can go on to the remaining questions.	• For #1, students may need clarification regarding what is being asked. Prompt students to explain what the intersection point on the graph means. Ask them to explain how a table of values might be the same as a graph. Ask them what they would expect to see in the graph and the table of vales if the solutions were the same. • Reinforce the process for graphing an equation in slope-intercept form. Have students verbalize the process and then have them complete one question that models their understanding. This will help reinforce their confidence for #3, 6, and 7. • Ensure that students are able to change the forms of the equations into slope-intercept form. Again, have them verbalize and model one of the questions before completing # 6 and 7. • Students having difficulty with #10 should review Example 1 and the Your Turn they completed. Ask students to identify what the fixed amount is ($35) and what can change ($5/day and $12/day). Some students will find it easier to generate a table of values by repeated addition and then use these points to graph the system. Although correct, encourage them to write a system that represents the problem.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Unit 4 Project
If students complete #16, which is related to the Unit 4 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• Students may benefit form working with a partner to solve the problem.
• Most students are able to complete a table of values from the given information. Encourage them to complete the table and use the points to generate the graph, if necessary.
• Encourage students to graph from the given system in slope-intercept form. Point out that the graph will verify their solution.
• Some students may need prompting to describe the trip the ducks take to the water. Have students verbally describe what is immediately different about the two ducks' journey. Ask students:
– If they both start at the same time, what would an early start mean for one breed of duck?
– Which duck flies faster?
– How would we know if the canvasbacks catch up to the green-winged teals?
– Where might this show up in a table of values or a graph?
• You may wish to provide students with BLM 8-4 Chapter 8 Unit 4 Project, and have them finalize their answers.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete selected questions from #21 to 23 depending on their level.</td><td>• Question #22 serves as an excellent assessment question. All students should be able to explain the difference between solving and verifying. Prompt students to consider where each occurs in the process of finding a solution to a system of equations. This question is appropriate for all levels of learners.
• Some students may be assigned #21. Prompt them for ideas for #21, or have them refer back to Example 1.
• For #23, students need to have a good knowledge of parameters and their effects on the graphs created.</td></tr>
</table>

8.2 Modelling and Solving Linear Systems

***Mathematics 10*, pages 432–445**

Suggested Timing

100–120 min

Materials

- map of Canada
- ruler
- grid paper or computer with graphing software

Blackline Masters

BLM 8–3 Chapter 8 Warm-Up
BLM 8–4 Chapter 8 Unit 4 Project
BLM 8–6 Section 8.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF7 Determine the equation of a linear relation, given:
- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3–5, 8, 9, 12, 23
Typical	#1 or 2, 4, 6, 7, 9, 10 or 11, 12, 13, 15 or 16, 22, 23
Extension/Enrichment	#9, 11, 12, 14, 17 or 18, 19–22, 24

Unit Project Note that #9 and 12 are Unit 4 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, read and discuss the opening text about travelling and modes of transportation. You may wish to brainstorm some modes of travel and then rate their costs, travel times, and effects on the environment. Challenge students to research and find out how accurate their predicted ratings were. Ask students to list any other factors that travellers should consider when selecting a mode of travel. Invite students who have travelled on *The Canadian* or another train to share their experiences.

Investigate Creating a System of Linear Equations

You may wish to begin the Investigate by having a class discussion about distances to local places and then expanding to talk about the distances from students' city or town to major Canadian cities.

As students work through the Investigate, ensure they have the correct system of equations before they proceed with graphing. If students have difficulty obtaining the equations for the linear system, you can prompt them by asking the following questions:
- What is the dependent variable? the independent variable?
- What is the relationship between distance, speed, and time?
- How can you find the distance if you know the time and speed?
- How far did the train travel before the car started driving?

As students consider the reasonableness of their answers and the changes to the times of departure, you may wish to have them consider the types of answers that are acceptable in this situation:
- What number system(s) can the answers be elements of? Why?
- Is this true for all systems of linear equations?

Meeting Student Needs

- Some students may have difficulty locating cities on a map and determining the distances between them. Allow students to use distances that seem reasonable even if they are not accurate. Ensure that students use units when stating the distance measurements.
- Allow students to use a clock as necessary to calculate times.
- Encourage students to complete the Investigate with any method available to them. If students have difficulty representing the situation with a system of linear equations, allow them to solve the problem numerically.

Answers

Investigate Creating a System of Linear Equations

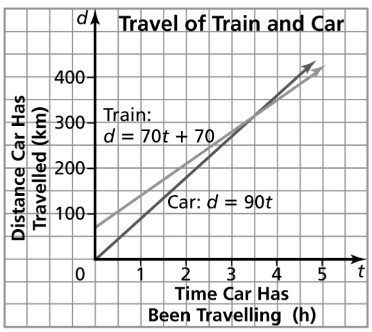
1. Example: Edmonton, AB, to Winnipeg, MB, is 1357 km.

2. Example: Car: about 15.1 h or 15 h 5 min
Train: about 19.4 h or 19 h 24 min

3. a) Let d be the distance the car has travelled, in kilometres.
b) Let t be the time the car has been travelling, in hours.

4. a) $d = 90t$ **b)** $d = 70t + 70$

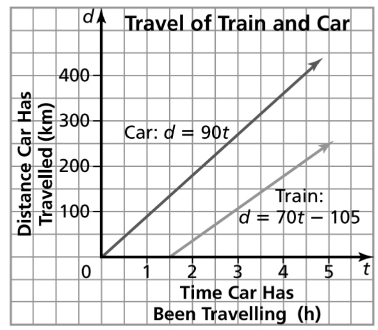
5.



The solution is (3.5, 315). The car will catch the train 3.5 h after the car starts travelling and they will be 315 km away from their start.

6. a) $d = 90t$ and $d = 70t - 105$

b)



The lines do not actually intersect on the graph. They appear to intersect if the lines are extended to negative values, but time cannot be negative so the graph is restricted by the domain.

7. a) Example: I think the first system was easier to write because the difference in start times was already given in hours.
b) Example: Both of their distances depend upon the product of their speed and time travelling. Their speeds are different and the length of time travelled changes.
c) Example: Known information creates constants in the equations and varying information becomes variables. The graphs are dependent on what the results of speed × time give for distances. The speeds are slopes of the lines and the later start of one vehicle results in a different d-intercept.

8. Example: There are two different variables, d and t, so two equations can be written. The vehicles both travel at constant speeds so they are linear equations.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about writing a system of linear equations.	• You may wish to have students complete #1 to 5 independently or with a partner and have some groups model their graphs on the board or share with another group. • As a class, discuss the questions in #5. It may help clarify what students have graphed and assist students in moving forward with #6. • Some students may need to be prompted to understand the significance of the different starting times for the two modes of travel. Ask students what it means in terms of distance. You may wish to talk in terms of a *head start* for the car. Ask what formula they can use to determine the distance travelled. • Question #7c) is a key question. Listen for students' level of understanding and ability to tie the responses to what they learned in section 8.1.

Link the Ideas

You may wish to follow up on the discussion in this section. For instance, you could have students work in pairs, where one student writes a simple expression or equation and the partner writes several situations that could be represented by that expression or equation. Alternatively, you could have one student from each pair write a phrase in words and have the partner write the expression or equation that represents the phrase.

Example 1

In this example, students compare two rental charges. Ensure that students make the connection between the equations and the slope and intercept of each line. For Option A, the y-intercept is 30 and the slope is 8. For Option B, the slope is 14 and the y-intercept is 0. To help students interpret the information in the graph, you could ask, "Why does knowing the solution to this linear system not give enough information for you to choose a rental option?" Of course, the intent of this question is to help students understand that one rental option costs less for rentals less than 5 h, the other option costs less for rentals longer than 5 h, and for exactly 5 h, the cost is the same.

Students may not see the importance of assigning variables at the beginning of a solution. You may wish to emphasize to them that this is a crucial step to their success. Some students may need to be reminded to avoid assigning letters for variables that might be confusing, such as S or l.

The solution provides two methods of graphing. You may wish to have students compare and contrast the two methods. You may wish to ask the following questions:

- Is one method likely to be faster than the other? Explain.
- Is one method likely to be more precise than the other? Explain.
- Is one method likely to be more convenient than the other? Explain.

Example 2

This example involves two grain bins that are being emptied, so the volume decreases over time. Unlike the Investigate where one traveller starts before the other, in Example 2, both bins start emptying at the same time. The bins contain different amounts of grain when they start emptying.

As students verify their answers, you may want to ask the following questions:

- Why is the term containing *time* negative in each equation? (Note that this is one of the margin questions in the student resource.)
- What does each of the equations in the linear system represent?
- What does the point of intersection represent?
- How would you determine the volume of grain that has been removed from each bin?

When completing the Your Turn, encourage students to consider these questions:

- How can you determine reasonable scales and values for the axes of your graphs?
- What does the slope of each line mean in the context of the problem?
- What do the intercepts of each line represent?
- Would you prefer to solve this system by hand or using technology? Why?
- What does the intersection point mean in the context of this situation?
- How did you verify your solution to the system of linear equations? Is there any doubt that your answer is correct?

Example 3

Students may find this example challenging to grasp, because one unknown is not dependent on the other, and the points on the lines, other than the intersection point, have no meaning in the context. You may want to suggest that students work with smaller numbers and use patterning. Ask them the following questions:

- What two different quantities are given in this problem?
- Since there are two quantities, does it make sense that each would lead to one equation in the system?
- What would the revenue be if 5 adult tickets and 10 student/senior tickets were sold?
- What if 8 adult and 12 student/senior tickets were sold?
- What if n adult tickets and k student/senior tickets were sold?

As students consider the reasonableness of their answers and the changes to the linear system if the ticket prices change, encourage them to consider the types of answers that are acceptable in this situation. Ask the following questions:

- What number system(s) can the answers be elements of? Why?
- Is this true for all systems of equations?

If students need assistance completing the Your Turn, you can ask them to begin by defining the variables they will use in the system. Asking questions like the following should help students develop the equations they need.

- How does the total time of 5.25 h lead to one equation?
- How does the total distance of 440 km lead to one equation?
- How do you determine the distance a person has driven? What quantities do you need to use?

Key Ideas

Have students read the Key Ideas with a partner. The last bullet contains two key points. Have students find or create an example to clarify each of the five points. When they are satisfied with the examples they have chosen, they can work with another team and compare examples. Students may wish to include the Key Ideas and the examples they have added in their Foldable or math journal.

Meeting Student Needs

- This section has many word problems dealing with money. You may wish to bring in money manipulatives to demonstrate an activity.
- You may wish to remind students of the following strategy for translating words to mathematical expressions and equations: Read line by line and see if a mathematical statement can be written from the information given. Use the given variables or assign variables then.
- Write common operations on the whiteboard; then, have students suggest other words or phrases that mean the same thing. For example, for *subtraction*, students might write *difference* or *is less than*. Emphasize that students need to pay close attention when writing subtraction statements.
- Students will likely benefit from a discussion of the window of a graph, whether they are graphing by hand or using technology.
- Discuss with students what expenses they have when they rent a movie and watch it at home. Compare them to the costs involved in going to a movie in a theatre.
- In Example 2, allow students to explore the equations by asking them to generate a table of values for the amount of grain remaining after the first several minutes. Have students look for a pattern in how they found the values.

ELL

- Some students may not be familiar with the concept of rent. Explain the meaning of rent to students and emphasize that rent is based on a specified unit of time. You may wish to brainstorm examples of things that are rented on an hourly, daily, weekly, monthly, and yearly basis.

Enrichment

- Ask students to create a scenario that can be modelled by two simultaneous equations. (Students' questions will vary. Check to ensure their equations and solutions are accurate. Example:

  Ten years ago, Madge was $\frac{3}{2}$ times as old as Fran.

  In ten years, Fran will be $\frac{5}{6}$ times as old as Madge.

  How old are they now?

Time	Fran	Madge
10 years ago	$F - 10$	$M - 10$
This year	F	M
10 years from now	$F + 10$	$M + 10$

$M - 10 = \frac{3}{2}(F - 10)$ and $F + 10 = \frac{5}{6}(M + 10)$

Madge is 30 and Fran is 23 and 4 months.)

Gifted

- Challenge students with this question: The numerals of a two-digit number add to seven. If the number is written with the numerals reversed, the new number is nine greater than the original number. What is the original number? (34) Encourage students to create their own number question.

Answers

Example 1: Your Turn

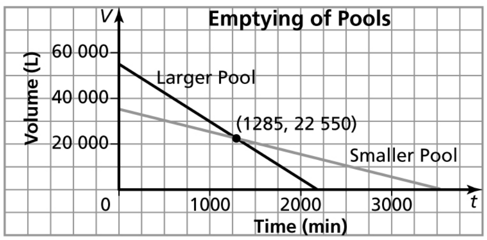
a) Let m represent the duration of the main act, in minutes. Let a represent the duration of the opening act, in minutes. $2a - m = 3$ and $m + a = 132$

b) The solution to the linear system is (87, 45). It represents the duration of each act. The main act was on stage for 87 min and the opening act was on stage for 45 min.

Example 2: Your Turn

a) Let V represent the volume of water remaining in each pool, in litres. Let t represent the time, in minutes. $V = 54\ 675 - 25t$ and $V = 35\ 400 - 10t$

b)



Example: The graph shows that the volume of water remaining in each pool decreases over time. Both lines stop at the horizontal axis because the volume of water left is 0 L. The two lines intersect at (1285, 22 550). This is the only point when the two pools contain the same volume of water at the same time. At 1285 min, each pool has 22 550 L of water remaining. Before 1285 min, the larger pool contains more water. After 1285 min, the smaller pool contains more water.

Example 3: Your Turn

Let f represent the time that Jamie's father drives, in hours.
Let c represent the time that Jamie's cousin drives, in hours.
$f + c = 5.25$ and $90f + 80c = 440$
Jamie's father drives for 2 h and her cousin drives for 3 h 15 min.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may require remediation in translating the math into algebraic expressions. Prompt students to verbalize what variables they will choose and the meaning of *less than* and *twice the time*. • Have students identify how they will label the axes. • You may suggest that students compare their responses to part b). Have students explain their thinking about what the solution represents. Discuss this important point as a class to clarify any misinterpretations.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • If students are having difficulty, ask them what parallels exist between this question and Example 2. Have them verbalize what variables could be used in setting up the system. • Encourage students to use a table similar to the one in the example. This will assist them in setting up their equations and the system. • You may wish to discuss, as a class, how students will decide on an appropriate scale for their graph as the values are quite large. • Prompt students to verbalize how they would label the axes. Ask what the intersection point would mean, given their labels. • Remind students of the importance of verifying their solution.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may find this Your Turn challenging because one unknown is not dependent on the other. The intersection point will be a solution to both systems but requires students to interpret it in terms of the labels given to the axes and in terms of the context. • Students may require some coaching in identifying the significance of the values given in the problem (440 km, 90 km/h, 80 km/h, 5.25 h). • Prompt students to verbally identify variables that could be used in the problem. • Before students graph the system, ensure that they have written a correct slope-intercept form.

Check Your Understanding

Practise

For #2, students translate words into equations. This question may be slightly more challenging for students than #1, because some calculations are required. Students need to be more mindful of assigning variables in this question.

Question #4 has a monetary context. Some students may find this type of question difficult. A table of values has been provided to help students develop the system of equations.

Apply

Question #5 is similar to Example 2. If students have difficulty with this question, you can refer them to the example. A solution to the system is not required, but students need to be able to interpret the solution.

In #6, students write and graph a system of equations, which they do not need to solve. Students explain what the solution represents within the given context.

In #7, students are asked to solve a system of equations. This system involves a linear rate, so you can assist students by asking them to work through this question using the slope-intercept form of the equation of a line.

Question #8 is similar to Example 1. Students need to pay attention to units, because values are stated in cents and in dollars and cents at different points in the question.

Question #9, which involves retrofitting, is related to the unit project. It is likely that students have some experience with the idea of water conservation. You may want to allow a brief discussion at this point, to emphasize the link to the project. For example, students could be invited to check the flow rate of shower heads in their home, if possible. Students may have knowledge of other considerations or costs when retrofitting. Prompt students to discuss whether the solution to the linear system is reasonable based on the length of a typical shower. If other flow rates are available, students could write a new linear system using those data and compare the solution to that system to the solution to the original question.

Question #11 asks students to go a step beyond finding a solution to a linear system and decide whether the solution fits within the context of the question. It is not enough to find the intersection point. Students need to decide whether the domain of the system includes that point, though the question is not posed in those terms. You may want to have students identify any assumptions they are making, for example, that both cyclists can continue at their current speeds.

The topic of #12 is wildlife, which links to the unit project. If students need assistance writing the system of linear equations, refer them to Example 1. Many students will likely have prior knowledge of wetlands, which may provide an interesting topic for a brief group discussion.

For #13, students use the distance-speed-time relationships to write and solve the system of equations. In particular, the equation distance = speed × time is needed to write the correct system of linear equations.

In #14, students use rates, specifically the rate at which two people can paint a fence, in linear feet per hour. It may not be obvious to students that each person will paint for the same amount of time to complete the task. Students may also be initially surprised that one of the equations in the system is a constant function.

In #15 and 16, students use the endpoints of the given intervals to help determine the lines that comprise each system of equations. Students need to solve these linear systems, and they may choose to create the graphs directly without first determining the equations.

Extend

For #19, students form a linear system by translating words into equations. This question requires careful reading by students, because the question involves subtraction.

You can assist students with #20 by asking them, “What two factors affect the net speed of a person swimming when a current is present?” Further, you can ask how those two factors combine to produce the net speed when the person is swimming with and against the current.

Question #21 is commonly described as a *mixing problem*. You may want to ask students how they convert percents to decimals.

Create Connections

Question #23 offers students two options for modelling the situation. Students may choose to use a table of values or a diagram.

Question #24 gives students the solution to a system of linear equations, and has them determine the equations in the system.

Unit Project

You may wish to have a group discussion about recent renovations in students' homes. Prompt the discussion with the following questions:

- What types of renovations have taken place?
- What was the motivation for the changes?
- Did you install energy-efficient furnaces? on-demand hot water? low-flow shower heads? Emphasize that these relationships can be illustrated graphically to show the cost saving for homeowners.

Meeting Student Needs

- Provide **BLM 8–6 Section 8.2 Extra Practice** to students who would benefit from more practice.
- Have students research the purpose and meaning of the Métis sash and try to have a student bring in a sash. Most Métis families have a sash. Students can create a mini sash using yarn and braiding the threads.
- Questions #1 and 2 allow students opportunities to develop equations without the pressure of also needing to solve them. Students should discuss appropriate variables for each question in addition to the writing of the equations.
- You may wish to display posters listing the steps involved in problem solving. (Read it, Plan it, Solve it, Look back)
- You may wish to have students complete two or three questions from the Apply section. Encourage students to list the step-by-step process they followed when working through each question. Ask the following questions:
  - What was difficult? What was easy?
  - Was there a section that was confusing?
  - What linear system did they use to solve the equations?

  Students could share this information with other students at the end of the section.

ELL

- In #9, to perform a *graphical analysis* means to explain your answer by creating a graph and referring to it to support your answer.

Common Errors

- In #16, students may forget to divide by 4 for the number of weeks in the period.

R_x Remind students to carefully consider the information in the question and to have a clear idea of the problem they are solving.

WWW Web Link

For information about the Test of Metal mountain bike race referred to in #11, go to www.mhrmath10.ca and follow the links.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1, 3–5, and 8. Students who have no problems with these questions can go on to the remaining questions.</td><td>
You may wish to have students work with a partner.
For #1, students can refer back to the examples they have completed. The translation required in the question is at an entry level. Remind students that assigning a variable that is appropriate for the context of the question makes it easier to work with and remember.
If students are having difficulty with #3, prompt them to consider what the two ways are for Molly to get points (goals and assists). Ask what variables could represent these values. Ask how these variables and the 32 points could make up one of the equations. Finally, ask them how Molly gets most of her points and how this could be used to write a second equation.
You may wish to have students add another row to the table in #4 for totals. Ask them what total values should appear in the Number of Coins column and in the Value of Coins column.
Some students may need coaching in changing the value of the coins to cents.
You may wish to have students review Example 2 for #5. Suggest they set up a table of values for the problem to assist them in writing their system.
Students should find #8 to be a straightforward question. Refer them to Example 1 if they require a pattern. Ensure that they can understand and verbalize the meaning of $19 (y-intercept). Encourage students to fully explain their reasoning for their choice of plan. Some students may need to be explicitly asked to address both the plans up to the point of intersection, and after this value as well.
</td></tr>
<tr><td>Unit 4 Project
If students complete #9 and 12, which are related to the Unit 4 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.</td><td>
Some students may require prompting for #9a). Have them identify the cost of heating 1 L of water. Ask how they can determine the cost for 170 L of water. Once students have completed part a), ensure they understand how this information can be used to set up their system. Have them verbalize their process or have them work with a partner to complete part b).
It may be beneficial to take a few minutes to discuss the value in conserving water by using a shower head that uses less water. Talk about whether it is a cost-saving decision.
For #12, it is important to coach students to use appropriate variables. Suggest they use a table, as in Example 1, to set up their system.
Some students may require coaching to interpret the meaning of critical in the question and how it plays a role in this situation. Remind students that justify means to support their explanation mathematically.
You may wish to provide students with BLM 8-4 Chapter 8 Unit 4 Project, and have them finalize their answers.
</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #23. If they have no difficulty with this question, have them complete #24.</td><td>
Students may find it helpful to refer back to the Investigate to assist them with #23. For students who could benefit from a more scaffolded approach, encourage them to use a table.
Note that #23 asks students to describe how they would determine whether James will catch up to Gavin. You may wish to have them follow through and actually complete and solve it.
In #24, student to write a system from a given graph. Assign this question only if students have a good understanding of systems and can translate their understanding of slopes and y-intercepts into a context.
</td></tr>
</table>

8.3 Number of Solutions for Systems of Linear Equations

Mathematics 10, **pages 446–459**

Suggested Timing

100–120 min

Materials

- stopwatch or other timer or clock showing seconds
- measuring tape
- grid paper and coloured pencils
- computer with graphing software
- graphing calculator

Blackline Masters

BLM 8–3 Chapter 8 Warm-Up
BLM 8–7 Section 8.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 4, 5, 7, 9, 12, 19, 20
Typical	#1, 3, 4, 6, 9, 10, 12, 13, 19–22
Extension/Enrichment	#6, 8, 11, 12, 14–17, 18, 21, 22

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce prerequisite skills needed for this section.

You may wish to have students brainstorm situations like a race that involve time and distance and can be represented with a linear system. After students have brainstormed situations involving time and distance, have them think of other situations that could be represented with a linear system.

You may wish to have a class discussion about the question "Does every system of linear equations have one solution?" You could use a Think-Pair-Share strategy in which students answer the question independently, then discuss their answer with a partner, then some pairs share their conclusions with the whole group. As most students are surprised by mathematical problems that have no solution, you could also have students try to find equations (not systems) that do not have solutions. For example, $x^2 + 1 = 0$ or $\sqrt{x} = -5$ suggest that there are simple equations that are not solvable. This activity could take the form of a brief game or challenge, where a student or pair of students tries to find an equation that an opposing team cannot solve.

Investigate Number of Solutions for Systems of Linear Equations

As students work through the Investigate, circulate to monitor their progress. Help students think of several possibilities for races, such as two racers starting at the same time, in the same position, with one winner. In particular, students may need help to build race scenarios that end in a tie or have one person beginning with a head start, either by beginning the race earlier than his/her opponent or by beginning the race partway along the course. Ensure that students know that the racers do not need to travel at the same speed as each other; they just each need to travel at a constant speed. You can help students think of different scenarios by posing questions like the following:

- Does every race have only one winner?

- How could a race start so that one racer has an advantage?
- Does each racer have to begin at the starting line?
- Does each racer have to begin at the same time?

It is not necessary that each group arrive at all three possibilities: no solution, exactly one solution, and infinitely many solutions. As you work with individual groups, make sure that each of the three possibilities is found by at least one group in the class.

You can also help students think of different scenarios by referring to the properties of the lines that represent each racer. For example, you could ask questions such as the following:

- Do both lines have an x-intercept? What does this tell you about the race?
- Do both lines have the same y-intercept? What does this tell you about the race?
- Do both lines have the same slope? What does this imply about the race?
- Are the lines corresponding to the two racers always different? What does this imply about the race?

After all groups have conducted their races and solved the resulting systems of equations, provide an opportunity to compare results. You could have each group post its linear systems, graphs, and solutions, and have students circulate around the class examining the displays. Or, you could have each group report to the class on one race and system of equations of their choice, detailing the number of solutions to the system and the conditions of the race that led to this result. Try to make sure that all three possibilities are brought forward by the students.

By answering #6, students will make the connection that knowing the slopes and y-intercepts of the lines in a system of equations allows them to determine the type of system and number of solutions to the system.

Meeting Student Needs

- Most students may not have seen a snowshoe, especially if they are from an urban setting. Bring an actual snowshoe into the classroom, if available. You may wish to seek out and invite someone to come to the classroom and explain how snowshoes were created traditionally.
- Students will benefit from being actively involved in the discovery of the number of solutions to different linear systems.

Enrichment

- Have students consider the equations $2x + 3y = 1$ and $2x + 3y = 2$. Then, ask them the following questions:
  - What do you notice about the left and right sides of each equation?
  - What would you know about the solutions if this was a system of equations?
  - How would graphs of the equations compare? How do you know?

  (There is no point that satisfies both equations. A graph would show parallel lines, because the two lines are on the same plane and never cross or have a solution.)

Common Errors

- Some students may plot time on the vertical axis and distance on the horizontal axis.

R$_x$ Remind students to think about whether one variable is dependent on the other. Have students recall that the dependent variable (distance) is plotted on the vertical axis and the independent variable (time) is plotted on the horizontal axis.

Answers

Investigate Number of Solutions for Systems of Linear Equations

1., 2. Examples:

Race	Racers	Start Time (s)	End Time (s)	Distance (m)
1	Bev	3	15	36
	Ajay	0	15	45
2	Don	0	15	42
	Caryn	4	15	44

3. Check that students' graphs reflect the data collected for each race.

4. Examples: Race 1: $d = 3(t - 3)$ and $d = 3t$. Race 2: $d = 2.8t$ and $d = 4(t - 4)$

5. Example: There is no solution for race 1 and one solution for race 2, about (13, 37).
In race 1, Bev started 3 s after Ajay. Since they moved at the same constant speed, Bev never passed Ajay. Therefore, there is no solution. In race 2, Caryn started 4 s after Don. Since Caryn moved at a faster speed than Don, she passed him. Therefore, there is one solution.

6. Example: If the starting times are different but the speeds are the same, there will be no solution. If the starting times are different and the speeds are different, there may be one solution (if the faster racer started later).

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about writing a system of linear equations.	• Students will complete the Investigate in groups but the discussions might best be completed as a class. It may be helpful for the discussions if each group's graphs are projected for the class to see. • As a class, discuss: – why the solutions vary – whether they have the number of solutions in their group that they expected and why • It may help students to understand the different solutions if they can verbally describe each group's race. Ask whether there are any assumptions that they made (running at a constant rate). • It may benefit some students to review the distance-formula calculation. • Ideally, all three possible scenarios for solutions will occur. If not, prompt students to consider the missing alternatives. Ask them what the graph would need to look like to represent each solution.

Link the Ideas

The case where lines are coincident is the one most likely to surprise students. Some may find it unsatisfying to think of a system of equations containing two copies of the same line. If this happens in your class, you can help students understand that this is possible by referring to the case of a race where two racers begin at the same time and have the same speed, thus finishing the race in a tie.

There is an accepted terminology for referring to types of systems. This terminology is not required, but is shown below for reference. Notice that one set of terms refers to the system of equations, the other to the graphs of the lines in the systems.

Number of Solutions	Type of System	Description of Lines
Exactly one	Independent	Intersecting
None	Inconsistent	Parallel
Infinitely many	Dependent	Coincident

Example 1

This situation presents six different systems of equations. Students are asked about three of them. You may want to suggest that students graph the lines in the order requested, rather than beginning by graphing all four lines. Graphing each competitor's line in a different colour may also help students.

After students have written equations describing each competitor's travel—and before graphing—you may want to ask students to predict what type of system will result for each pair. Students can discuss how they arrive at these predictions and then create the graphs in order to check their predictions. Note that in this example, the number of solutions to the system is determined and then interpreted in the context of the problem. For example, if a system has no solution, this means that those competitors will never meet during the race.

You may wish to have students work in groups of three to complete the Your Turn. Each student could solve one of the systems in the question and then report the results to the other two members of the group. The group would need to achieve consensus on the number of solutions for each system of equations and the meaning in the context of the problem.

Example 2

Predicting the number of solutions to each system may be easier for some students depending on the form each system of equations is written in. In Chapter 7, students explored writing equations in slope-intercept form, general form, and slope-point form. Many students will express the equation in slope-intercept form to graph using paper and pencil. Students may also wish to graph using a table of values or the x-intercept and y-intercept.

Consider having students work in groups of three to complete the Your Turn. Each student could determine the type of linear system for one system and report the results to the rest of the group.

Example 3

Students may have difficulty understanding the concept described in part a). Explain to students that the left sides of the equations contain the variables and the right sides contain constants. Any ordered pair that satisfies one equation and is therefore on that line cannot possibly satisfy the other equation, because the right sides are not equal. Infinitely many points can lie on each line, but no point lies on both lines. You may want to have students spend a few minutes answering the margin question referring to the first system in this example: “How else could you confirm that the lines are parallel?” Students could work individually and then report their results to the whole group. Some of the possible strategies are listed below:

- Rewrite the equations in slope-intercept form and compare the slopes.
- Find the x-intercept and y-intercept for each line and determine the slopes.
- Use any two points on each line to determine the slopes.

Key Ideas

As there are several possible strategies for predicting the number of solutions to a system of linear equations, you may want to allow a few minutes for students to detail their strategies, perhaps in their Foldable or math journal. Students should describe not only their personal strategy, but also why they have made that choice. You may wish to have each student create an example for each type of linear system and show how their strategy could be used to determine the number of solutions.

Meeting Student Needs

- For Example 1, some students may benefit from using an advanced organizer or table. The main headings could be No Solution**,** One Solution, and Infinitely Many Solutions. Under each heading, have students list the key components. For example, under One Solution they would list “different slopes, one point in common.”
- Use the organizer completed for Example 1 to work through Example 2.
- You may wish to allow students to use the organizer described above as they work through Examples 2 and 3.
- Students may benefit from having the information presented in the Key Ideas displayed in the classroom.
- Remind students that, when using technology, relations and functions are input with x and y or $f(x)$ as the variables. Students may need to substitute for the variables they have been using. Make sure students enter the correct equations. In Example 1, x is input for t and $f(x)$ is input for d.
- There are several methods of isolating y in an equation. Encourage students to perform the steps on their own and then to compare their final equation with that of a classmate.

ELL

- Help students deepen their understanding of the word *infinite* by asking them to consider all the numbers between 0 and 1. Students will likely come up with limited responses, such as 0.1, 0.2, and 0.3. Help them see that numbers such as 0.11, 0.12, and 0.13 also lie between 0 and 1.
- Mention to students that the term *coincident* has the same root as *coincide*, which some students may be familiar with or may find easier to remember the meaning of.
- For the Your Turn at the end of Example 3, explain to students that *by inspection* means just by looking (and maybe performing a simple operation). Students are not to graph the systems of equations to determine the number of solutions.
- Some students may need to be reminded of the meaning of *reducing to lowest terms* as mentioned in the Key Ideas.

Gifted

- Challenge students with this question: A kayaker paddles 6 km down the Fraser River in $\frac{3}{4}$ h. Travelling against the current, it takes her 90 min to cover the same distance. Find what the speed of the kayak would be if there were no current. What is the speed of the river current? (The kayaker travels 6 km/h and the speed of the current is 2 km/h.)

Common Errors

- Students may determine that two lines have the same slope and conclude that there is no solution to the system of equations.

R_x It is common to ignore the y-intercept in making this determination. You may need to remind students to check both parameters. Ask students if knowing the slope alone is enough to determine what the graph of a line looks like.

Answers

Example 1: Your Turn

Let d represent distance travelled, in kilometres. Let t represent time, in hours.

a) $d = 40 + 90t$ and $d = 25 + 90t$. The lines are parallel. They have no points in common, so the linear system has no solutions. The cars never meet.

b) $d = 40 + 90t$ and $d = 40 + 90t$. The lines are coincident, so they share all the same points. The linear system has an infinite number of solutions. The vehicles are always driving next to each other.

c) $d = 30 + 110t$ and $d = 40 + 90t$. The lines intersect at one point, so the linear system has one solution. The truck will pass the RV once.

Example 2: Your Turn

Check that students include graphs for parts a) to c) that justify their answers.

a) Rearrange the first equation to slope-intercept form, $y = -\frac{1}{2}x + 2$. Since the lines have the same slope and different y-intercepts, the graph will result in parallel lines. The lines will never intersect. Therefore, this linear system has no solution.

b) Rearrange the first equation to slope-intercept form, $y = \frac{2}{3}x + 1$. Since the lines have the same slope and y-intercept, the graph will result in coincident lines. Therefore, this linear system has an infinite number of solutions.

c) The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

Example 3: Your Turn

a) Rewrite the first equation in lowest terms to get $x + y - 8 = 0$, which is identical to the second equation. Therefore, the equations are equivalent and the graph will be a pair of coincident lines. The linear system has an infinite number of solutions.

b) The x- and y-coefficients of the two equations are the same, resulting in the same slope. However, the constant values are different. Therefore, the graph will be a pair of parallel lines. The linear system has no solution.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Consider suggesting students use a different colour for each line graphed. • Graphing them in the order of vehicle may assist students in identifying the commonalities between equations and systems. • Prompt students to verbalize what they notice about the slope of their graphs drawn and where they would find this value in the equation. • Prompt students to identify the solution for each system Ask them whether the solution involves one point, no points, or more than one point.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • This example links directly to Example 1. Students should have a good understanding from Example 1 how to determine lines that are parallel, coincident, or intersecting. • Students may wish to use technology to graph the system for a quicker visual image. • Remind students of the importance of writing the equations of a system in the same order in order to make comparisons. • There may be a need to refresh students' skills writing equations in slope-intercept form. Provide students with additional problems to practise, if necessary. • You may wish to have students work with a partner.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students who are more visual to graph the systems before answering. • Reinforce the importance of writing the equations in a common format. For students who find it easier to compare slopes and intercepts, encourage them to write the equations in slope-intercept form. Point out how they could determine the same response by comparing the coefficients of x and y.

Check Your Understanding

Practise

For #1, students choose systems of equations that have one solution and no solution, given a graph of several equations. In order to help students consider all of the possibilities, you may wish to ask them to list all the possible systems of equations. It may be helpful for students to organize their possibilities using a tree diagram and eliminating the duplicate linear systems.

In #2 and 3, students apply their personal strategies to determine the number of solutions to systems written in different forms.

For #4, students describe linear systems that have a specified number of solutions. One way for students to check the accuracy and effectiveness of their descriptions is to exchange work with a classmate and have the classmate give feedback about whether the explanation is clear.

In #5, students describe a strategy for predicting the number of solutions to a linear system. As there are many different ways to answer this question, it may be productive for students to compare their methods with one or more classmates.

Apply

Question #6 gives students one equation from a system and asks them to produce an additional equation that causes the system to have a specified number of solutions. This is another opportunity for students to collaborate with classmates. As answers will vary, students could exchange work with a classmate to see if they agree on the results.

Question #7 is similar to Example 1. Students that have difficulty with this question can be referred to that example to help them get started.

For #8, students need to make a judgment about the work of two people. Students should be able to determine from the equations that Sal is correct and a single solution exists. It may be more difficult to determine why Jeff thought an infinite number of solutions exist. Making the graph with technology may help students with this higher-order task. You could also ask them to express the slopes of the lines in the system as decimals, in order to highlight the fact that the slopes are similar to each other. This question may also provide an opportunity to revisit the advantages and disadvantages of graphing by hand and with technology.

In #9, the error made by Stephanie is fairly obvious. Students should quickly determine that the lines are not parallel, but that Stephanie has not made the best choice of graphing window. In this case, the solution to Stephanie's problem is to change the graphing window so that the intersection of the two lines is visible.

Question #10 is a conceptual question in which students create graphs to represent systems of equations, but without algebraic representations. Students can be referred to Example 1 in order to emphasize the concept being addressed.

Question #11 is simply stated, but requires some thought. If students have trouble evaluating the veracity of the statement, you may wish to suggest that they take a system that has infinitely many solutions, perhaps one from an example, and choose arbitrary points to see if they are solutions to the system.

To complete #12, you may want to suggest that students use inductive reasoning. That is, to evaluate the truth of the global statements given, it may help to construct a small number of examples with convenient values for the slopes and intercepts of lines. This question also provides a natural setting for collaborative work. Students could work in small groups to debate the statements and discuss examples that would help them make decisions. A representative of each group could report conclusions to the whole class.

Extend

To answer #14, you may wish to suggest that students do the following, as necessary:

- write each equation in slope-intercept form
- try to reduce each equation to lowest terms
- determine the x-intercept and y-intercept of each line

The strategy chosen by students should reflect their own personal strategy for determining the number of solutions to a linear system.

It may not be obvious to some students that #15 is a rate problem. You may need to assist students by asking, "How can you rephrase the rates so that the fuel consumption for each vehicle is expressed in the same form using the same units?"

Create Connections

To answer #18, students need to understand and explain that the point of intersection of the two lines occurs outside the domain of the system. In particular, the lines are not parallel, but the intersection occurs at a point before the beginning of the problem. Another way to express this is that the system of equations is represented by two line segments, not two straight lines, and the two segments do not intersect.

Question #19 should help students see that the description *linear* is important in the systems of equations explored. You may want to ask, "If you are not limited to straight lines, can you draw a system of equations that has exactly two solutions?"

In #20, students' answers may vary. You may wish to prepare some systems of linear equations that appear "unfriendly" in order to help students decide if it is always possible to make decisions by inspection only. For example, you could use the system $2(x - 9) + 3y - 7 = 0$ and $0.5(x + 6y) = 2.1$.

You may wish to allow collaboration for #21. Alternatively, you could ask students to create the required systems to be used on a future assessment, perhaps an assignment or exam. Students should be expected to provide solutions to the systems they create, where that is possible.

Question #22 provides an opportunity to use a graphing calculator and/or computer technology to explore and extend the concepts of this section. In this case, technology is used to quickly explore how several changes to the equations in a system change the number of solutions. The use of technology allows students to quickly test different values and equations to find patterns and principles.

Meeting Student Needs

- Provide **BLM 8–7 Section 8.3 Extra Practice** to students who would benefit from more practice.
- Have students refer to the posters of Key Ideas or the advanced organizer they might have created. The organizer should be on their desks for quick reference.
- Students should ensure that they are comfortable graphing systems, both by hand and using technology. You may wish to assign specific questions to be graphed by hand. Once students have predicted the number of solutions, the system could be graphed using technology.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2, 4, 5, 7, 9, and 12. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to have students work in pairs. • Encourage students to draw the three possibilities for #4 and have them verbalize their thinking. Use prompts like the following questions: – What makes lines parallel? – What characteristic would you look for in parallel lines? – What do we know about they y-intercept of parallel lines? • Students having difficulty with #7 can review Example 1. Allow them to use their own personal strategy in identifying their solution. • If students are unable to extrapolate in #9, have them use two rulers (or a folded piece of paper, or the edge of a book, or any straightedge) overlaid on the lines to help them visualize that the lines will intersect. Ask them how else they know they will intersect (they do not have the same slope). • For #12, it may assist students to see the slope-intercept form written out as $y = mx + b$. Remind them that slope = m and the y-intercept is b. Encourage them to substitute their own values into the variables based on the conditions given in each part of the question. Encourage them to verify their solutions by graphing with or without technology.
Assessment *as* Learning	
Create Connections Have all students complete #18 to 21.	• You may wish to have students work in pairs to complete #19. Listen for their discussion and encourage them to graph systems in response to the question. Remind them to explain and draw a diagram to support their conclusion. Remind them that they are dealing with *linear* systems. (Point out that the word *line* is in the word *linear*.) • Use #20 as an assessment *as* learning question. Encourage students to use whatever personal strategy they feel most comfortable with in identifying solutions from a system.

Chapter 8 Review

8

Mathematics 10, pages 460–462

Suggested Timing

60–90 min

Materials

- grid paper
- ruler
- graphing calculator or computer with graphing software

Blackline Masters

BLM 8–5 Section 8.1 Extra Practice
BLM 8–6 Section 8.2 Extra Practice
BLM 8–7 Section 8.3 Extra Practice

Planning Notes

Have each student complete the review for the chapter individually. Students should note the questions that they are unable to complete. Encourage students to use their chapter Foldable and/or their notes and student resource to go back and work through any questions they had difficulty with. Have students work through any questions that remain unsolved in groups of two or three students. This allows students to diagnose their own areas of strength and weakness. Suggest to students that they use previously unassigned Check Your Understanding questions in the relevant sections of the student resource to practise any areas that require remediation.

Have students who are not confident discuss strategies with you or a classmate. Encourage students to refer to their summary notes, worked examples, and previously completed questions in the related sections of the student resource. Have students make a list of questions that they need no help with, a little help with, and a lot of help with. They can use this list to prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 8–5 Section 8.1 Extra Practice**, **BLM 8–6 Section 8.2 Extra Practice**, and **BLM 8–7 Section 8.3 Extra Practice**.
- Have students refer to the student learning outcomes posted in the classroom. Given approximately 10 min, students should create a summary of what they have learned for each outcome, listing the key steps when possible. At this time, students should highlight the areas that they are having difficulty with. Once these are identified, students will know which section(s) of the review to focus on.
- Some of the word problems involve a lot of detail and reading. Allow students enough time to work through these problems and visualize the situations and how the equations relate to the context; for example, #4, 8, and 11.
- Emphasize to students the importance of drawing diagrams to help organize the information in a problem. This skill will be especially useful for #5, 7, 8, and 11. A diagram of the situation may provide students with an extra way to check the reasonableness of their answers.

ELL

- For #7, some students may not be familiar with the term *atmospheric pressure*.
- Some students may need you to explain the meaning of *investment* and *rate of interest* when working on #13.

Enrichment

- Ask students to solve the following, using a system of linear equations: Suppose a son is $\frac{1}{13}$ the age of his dad. In ten years, the dad will be three times as old as his son is in ten years. How old is the dad now? ($d = 13s$ and $d + 10 = 3(s + 10)$. The dad is 26 now.)

Gifted

- Ask students to create a question similar to the age question above. The method and solution must be shown.

WWW Web Link

Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 8 Review The Chapter 8 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against the answers in the back of the student resource.	• Encourage students to express their thinking by identifying the initial values and rates or slopes of the relationships in the problems. • Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test. • Consider assigning students #1–5, 10, and 12 as the minimum questions that will cover the chapter.

Chapter 8 Practice Test

8

***Mathematics 10*, pages 463–465**

Suggested Timing

60–90 min

Materials

- grid paper
- ruler

Blackline Masters

BLM 8–8 Chapter 8 Test

Planning Notes

Students should complete the practice test individually. Have them write the question numbers in their notebook and indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions. Tell them that this test is for practice and that identifying which questions are most challenging will show them what concepts they need to revisit.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–9.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1	8.1	Example 2	✓ solve systems of linear equations by creating graphs, with and without technology
#2	8.3	Link the Ideas	✓ explain why systems of linear equations can have different numbers of solutions
#3	8.1	Link the Ideas	✓ explain the meaning of the point of intersection of two linear equations
#4	8.3	Example 2	✓ identify how many solutions a system of linear equations has
#5	8.2	Examples 1, 2, 3	✓ translate word problems into systems of linear equations
#6	8.1	Examples 2, 3	✓ verify solutions to systems of linear equations using substitution
#7	8.1	Key Ideas	✓ solve systems of linear equations by creating graphs, with and without technology
#8, 13	8.1	Examples 1, 2	✓ solve systems of linear equations by creating graphs, with and without technology ✓ verify solutions to systems of linear equations using substitution
#9	8.3	Example 3	✓ identify how many solutions a system of linear equations has
#10	8.3	Link the Ideas	✓ identify how many solutions a system of linear equations has
#11	8.2	Example 2	✓ interpret information from the graph of a linear system ✓ solve problems involving systems of linear equations
#12	8.2	Key Ideas	✓ interpret information from the graph of a linear system
#14	8.3	Example 1	✓ solve problems involving linear systems with different numbers of solutions

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 8 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their Foldable.	• Have students use their responses on the practice test and work they have completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach students in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 8 Test After students complete the practice test, you may wish to use **BLM 8–8 Chapter 8 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable.

9 Solving Systems of Linear Equations Algebraically

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

By the end of this chapter, students will be able to

Section	Understanding Concepts, Skills, and Processes
9.1	✓ solve systems of linear equations algebraically using substitution
9.2	✓ write equivalent equations to eliminate a variable
	✓ solve systems of linear equations algebraically using elimination
9.3	✓ choose a strategy to solve a problem that involves a system of linear equations

Assessment	Supporting Learning
Assessment *as* Learning	
Use the Before column of **BLM 9–1 Chapter 1 Self-Assessment** to provide students with the big picture for this chapter and help them identify what they already know, understand, and can do. You may wish to have students keep this master in their math portfolio and refer back to it during the chapter.	• During work on the chapter, have students keep track of what they need to work on in the What I Need to Work On section of their Foldable. They can check off each item as they develop the skill or process at an appropriate level.
Assessment *for* Learning	
Method 1: Use the introduction on page 466 in *Mathematics 10* to activate student prior knowledge about the skills and processes that will be covered in this chapter. **Method 2:** Have students develop a journal entry to explain what they personally know about systems of linear equations and linear equations. You might provide the following prompts: • How might systems of linear equations apply to real life? • What are some examples of systems of linear equations? • What are some ways you can represent systems of linear equations? • How do you solve a system of linear equations graphically? • How do you solve a linear equation algebraically?	• Have students use the What I Need to Work On section of their Foldable to keep track of the skills and processes that need attention. They can check off each item as they develop the skill or process at an appropriate level. • Students who require activation of prerequisite skills may wish to complete **BLM 9–2 Chapter 9 Prerequisite Skills**. This material is on the Teacher CD of this Teacher's Resource and mounted on the www.mhrmath10.ca book site.
Assessment *as* Learning	
Chapter 9 Foldable As students work on each section in Chapter 9, have them keep track of any problems they are having in the What I Need to Work On section of their Foldable.	• As students complete each section, have them review the list of items they need to work on and check off any that have been handled. • Encourage students to write definitions for the Key Terms in their own words, including reminder tips that may be helpful for review throughout the chapter. • Encourage students to write examples of their own into their notebook or math portfolio. They should have at least one example for each method that is covered in the chapter.
Assessment *for* Learning	
BLM 9–3 Chapter 9 Warm-Up This reproducible master includes a warm-up to be used at the beginning of each section. Each warm-up provides a review of prerequisite skills needed for the section.	• As students complete questions, note which skills they are retaining and which ones may need additional reinforcement. • Use the warm-up to provide additional opportunities for students to demonstrate their understanding of the chapter material. • Have students share their strategies for completing math calculations.

Chapter 9 Planning Chart

Section/ Suggested Timing	Prerequisite Skills	Materials/Technology	Teacher's Resource Blackline Masters	Exercise Guide	Assessment		
					Assessment *as* Learning	Assessment *for* Learning	Assessment *of* Learning
Chapter Opener • 20–30 min (TR page 339)	Students should be familiar with • solving linear equations • solving linear systems graphically		BLM 9–1 Chapter 9 Self-Assessment BLM 9–2 Chapter 9 Prerequisite Skills BLM 9–4 Chapter 9 Unit 4 Project BLM U4–1 Unit 4 Project BLM U4–2 Unit 4 Project Checklist		TR page 338 Chapter 9 Foldable, TR page 338	TR page 338	
9.1 Solving Systems of Linear Equations by Substitution • 60–75 min (TR page 341)	Students should be familiar with • solving linear equations • solving linear systems graphically • multiplication of a linear equation by a constant • numerical substitution and evaluation of a linear equation • the relationship between distance, velocity, and time		BLM 9–3 Chapter 9 Warm-Up BLM 9–4 Chapter 9 Unit 4 Project BLM 9–5 Section 9.1 Extra Practice	**Essential:** #1–3, 6–9, 12, 13, 15, 19, 26 **Typical:** #1–8, three of 9–13, 15–17, 19, 25, 26 **Extension/Enrichment:** #3–5, 7, 8, 15, 18–26	TR pages 342, 346 Chapter 9 Foldable, TR page 338	TR pages 344, 346	
9.2 Solving Systems of Linear Equations by Elimination • 60–75 min (TR page 347)	Students should be familiar with • solving linear equations • solving linear systems graphically • multiplication of a linear equation by a constant • numerical substitution and evaluation of a linear equation • the relationship between distance, velocity, and time		BLM 9–3 Chapter 9 Warm-Up BLM 9–4 Chapter 9 Unit 4 Project BLM 9–6 Section 9.2 Extra Practice	**Essential:** #1–4, 7–9, 13, 14, 20, 21 **Typical:** #1–3, 5, three of 7–12, 14–16, 20, 21 **Extension/Enrichment:** #2, 5, two of 10–13, 14– 21	TR pages 348, 353 Chapter 9 Foldable, TR page 338	TR pages 351, 353	
9.3 Solving Problems Using Systems of Linear Equations • 60–75 min (TR page 354)	Students should be familiar with • solving linear equations • multiplication of a linear equation by a constant • solving linear systems graphically • numerical substitution and evaluation of a linear equation variable • the relationship between distance, velocity, and time	• graphing calculator or spreadsheet software	BLM 9–3 Chapter 9 Warm-Up BLM 9–4 Chapter 9 Unit 4 Project BLM 9–7 Section 9.3 Extra Practice	**Essential:** #1–3, 5, 8, 9, 14 **Typical:** #1–7, 9, 11, 14, 15 **Extension/Enrichment:** #2, 9–13, 15	TR pages 355, 359 Chapter 9 Foldable, TR page 338	TR pages 356, 357, 359	
Chapter 9 Review • 60–75 min (TR page 360)		• graphing calculator	BLM 9–5 Section 9.1 Extra Practice BLM 9–6 Section 9.2 Extra Practice BLM 9–7 Section 9.3 Extra Practice	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapter 9 Foldable, TR page 338	TR page 361	
Chapter 9 Practice Test • 50–60 min (TR page 362)		• graphing calculator	BLM 9–8 Chapter 9 Test	Provide students with the number of questions they can comfortably do in one class. Choose at least one question for each concept, skill, or process. **Minimum:** #1–6, 8, 9	TR page 362		TR page 362 BLM 9–8 Chapter 9 Test
Unit 4 Project • 90–120 min (TR page 363)			Master 1 Project Rubric BLM U4–3 Unit 4 Project Final Report BLM 8–4 Chapter 8 Unit 4 Project BLM 9–4 Chapter 9 Unit 4 Project				TR page 363 Master 1 Project Rubric
Unit 4 Review and Test • 60–90 min (TR page 365)		• graphing calculator • ruler • grid paper	BLM 9–9 Chapter 9 BLM Answers	Have students do at least one question related to any concept, skill, or process that has been giving them trouble.	Chapters 8 and 9 Foldables	TR page 366	TR page 366

Solving Systems of Linear Equations Algebraically

Opener

Mathematics 10, pages 466–467

Suggested Timing

20–30 min

Blackline Masters

BLM 9–2 Chapter 9 Prerequisite Skills
BLM 9–4 Chapter 9 Unit 4 Project
BLM U4–2 Unit 4 Project Checklist

Key Terms

substitution method   elimination method

What's Ahead

In this chapter, students continue their work with linear systems from Chapter 8. Students construct a system of linear equations from a problem and then solve the linear system algebraically. The two algebraic methods students use are substitution and elimination. Also, students compare these two methods with the graphical method from Chapter 8 and use strategies to determine which method is most appropriate and efficient for a particular problem.

Planning Notes

With the class, discuss the images in the collage and how they may relate to a linear system. Ask guiding questions to assist their thinking:

- What variables would be involved in these different occupations and activities?
- How do some of these variables relate?

Read through the section on air traffic controllers. Ask students what variables an air traffic controller might have to deal with and how they interact with each other.

Unit Project

You might take the opportunity to discuss the Unit 4 project described in the Unit 4 opener. See TR page 298. Throughout the chapter, there are individual questions for the unit project. These questions are not mandatory but are recommended because they provide some of the work needed for the final report for the Unit 4 project assignment.

You will find questions related to the project in the Check Your Understanding in sections 9.1, 9.2, and 9.3.

Foldables™ Study Tool

Discuss with students the benefits of keeping a summary of what they are learning in the chapter. If they have used Foldables before, you may wish to have them report on how useful they found various designs.

- What designs have they used?
- Which designs were the most useful?
- Which, if any, designs were hard to use?
- What disadvantages do Foldables have?
- What other method(s) could they use to summarize their learning?

Discuss the Foldable design on page 467 and how it might be used to summarize Chapter 9. Point out to students that the design of this Foldable allows them to add additional pages of blank or graph paper if needed. Encourage students to suggest revisions for this Foldable, or to replace this Foldable with another design of their choice. Allowing personal choice in this way will increase student ownership in their work.

Give students time to develop the summary method they have chosen. Ask them to include some method of keeping track of what they need to work on; discuss the advantage of doing this.

Students may wish to use the fronts of the shutters to highlight some of the methods they preferred to use in solving systems of equations in Chapter 8. Remind them that this chapter is an extension of what they have learned in Chapter 8 and will provide additional methods to solve linear systems. You may wish to suggest to them that they staple the Foldable from Chapter 8 to the back of the centre page of the Foldable for Chapter 9, which will then provide them easy reference for all concepts related to systems of equations. On the right and left sides of the Foldable, students have a sufficient number of pages to personalize their strategies and examples for the substitution method and the elimination method.

The large open centre in the middle allows students to include several different approaches to problems involving systems of equations.

As students progress through the chapter, provide time for them to keep track of what they need to work on, which they can record on the back of the Foldable. This will assist them in identifying and solving any difficulties with concepts, skills, and processes. Have them check off each item as they deal with it.

Meeting Student Needs

- Consider having students complete the questions on **BLM 9–2 Chapter 9 Prerequisite Skills** to activate the prerequisite skills for this chapter.
- Consider having students staple a copy of **BLM U4–2 Unit 4 Project Checklist** to the back of the Foldable. This master provides a list of all the related questions for the Unit 4 project. Students can use it to keep track of the questions they have completed.
- Some students may benefit from completing all unit project questions.
- **BLM 9–4 Chapter 9 Unit 4 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 4 project analysis.
- If you have students do the Unit 4 project questions, consider offering them the option of working on these alone or with a partner.
- Tell students that as they work through the first two sections of the chapter, they should think about reasons why each method of solving might be more appropriate for certain questions than others. They will explore this concept in detail in section 9.3.
- Refresh students' understanding of the vocabulary for this chapter, and explain what the new concepts will be.
- Post the outcomes in the classroom where they can be seen by all students.
- To build students' math vocabulary, consider maintaining a math wall of new terms.
- With the class, read the introduction and Big Ideas, and explain the Key Terms. Use real-life examples to assist students in understanding the Key Terms.
- Regarding the Did You Know? on page 467, students may be interested to know that commercial aircraft that fly over the Arctic are likely to be headed to or from Asia. Improved technology and increasesd extended range capabilities have opened up the Arctic, with air routes now passing over the North Pole.

ELL

- For each section of the student resource, you may want to consider the following approach as a way to assist students:
  - Read the opening paragraph as a class, and discuss any definitions given in the paragraph. Identify any words with which students are unfamiliar. Suggest that students restate the meaning of the material in their own words. You may even consider having students translate the material into their own language first, and then restate it in English.
  - Discuss and identify examples of any definitions.
  - Work through the investigation as a class exercise. It may assist students to work in pairs to respond to the Reflect and Respond question. You may have the class share their findings and then construct a class response.
  - Work through each example and solution as a class, demonstrating the process of solving each problem. Ask students to work in pairs to solve the Your Turn questions.
- As students work through the chapter, have them record definitions for the Key Terms and write into their Foldable examples that model the various approaches used in the chapter.

Enrichment

- Challenge students to create and solve a system of equations that represents the following situation: The numerator of a fraction is four less than the denominator. Increasing both by one makes the fraction $\frac{1}{2}$. Hint: Let n be the numerator and d the denominator. $\left(\frac{3}{7}\right)$

Gifted

- Challenge students to explain why an algebraic solution to a system of equations might be more precise than a graphical one. Have them give an example of why the precision of algebra might be needed or applied in a real-life situation. (Example: The programs that aim lasers for industrial applications need to be precise.)

Career Connection

Use the photograph and the text to start a discussion about the career of air traffic controller. Invite students to research training and qualifications, employment opportunities, and career outlook. You might have them address how math concepts and skills are important in what air traffic controllers do. They may find the related Web Link in the student resource helpful.

Solving Systems of Linear Equations by Substitution

9.1

Mathematics 10, pages 468–479

Suggested Timing

60–75 min

Blackline Masters

BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–5 Section 9.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 6–9, 12, 13, 15, 19, 26
Typical	#1–8, three of 9–13, 15–17, 19, 25, 26
Extension/Enrichment	#3–5, 7, 8, 15, 18–26

Unit Project Note that #15 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Solving Systems of Linear Equations by Substitution

In this investigation, students get an opportunity to explore the concept of algebraic substitution through the process of substituting equivalent expressions. For students who may not be familiar with a balance scale, it may be worthwhile to borrow one from the science department or display a picture from the Internet.

Allow students to work in pairs to answer #1 to 6. As you circulate around the classroom, ask the following questions related to the diagrams:
- What does the mass of each block represent in the context of the problem?
- How is equality represented in the diagram?
- Why is it not possible to replace the blocks in Diagram 1 with cones?
- Using Diagram 2, how can you determine the cost of one light bulb? the cost of one T-shirt?

As students translate the diagrams into algebraic equations, observe whether they use appropriate letters to represent their variables (i.e., the light bulb and the T-shirt.) Check that students write correct expressions to represent the shapes in each pan of the balance scale.

Have students complete the Reflect and Respond section. Take up these questions with the class. Have a student come to the board and model the solutions to #7 and 8. Have students share their ideas for #9.

Meeting Student Needs

- Ensure that students understand what it means to *solve for y*. Explain that it means to rearrange an equation so that it has $y =$ on one side. If students have a choice between two equations, such as $4x + 5y = 23$ and $3x = y - 9$, they need to see that it is easier to solve for y in the second equation. Show students what happens when they solve for y in the first equation. The result is $y = -\frac{4}{5}x + \frac{23}{5}$.
  This equation complicates the substitution due to the fractional coefficients. In the second equation, the result is $y = 3x + 9$. This equation is less complicated to use for substitution due to its integer coefficients.
- Discuss the meaning of *substitution*, which is replacing one variable with an equivalent expression. If $c = 2a + 3$, the expression $2a + 3$ can be substituted for c in another equation. This must be done using brackets, since the single

variable c is being replaced with a two-term expression. For example,

$$5c + 4a = 12$$
$$5(2a + 3) + 4a = 12$$

When demonstrating to students, use a different colour for c and $2a + 3$ to emphasize the substitution.
- Introduce substitution by using coins; for example, five nickels can be substituted for one quarter.
- Bring a double-pan balance to the classroom. Allow students to place classroom objects, such as pens, pencils, erasers, markers, and planners, on the balance to determine objects that they can substitute for other objects and maintain balance. Ensure that students can replace one type of object with a certain number of another object; for example, one eraser can be replaced with three pencils.

ELL

- Have students work with a partner on the Investigate. Ensure students understand that a block represents a light bulb, and a cone represents a T-shirt.

Answers

Investigate Solving Systems of Linear Equations by Substitution

1. In diagram 2, the two cones from diagram 1 have each been replaced by three blocks.

2. Since the mass of seven blocks equals 42 g, dividing by 7 will give the mass of one block as 6 g.

3. Each cone has a mass of 18 g because the mass of three blocks equals the mass of one cone.

4. Let x represent the mass of one cone. Let y represent the mass of one block. For the first balance, $2x + y = 42$. For the second balance, $x = 3y$.

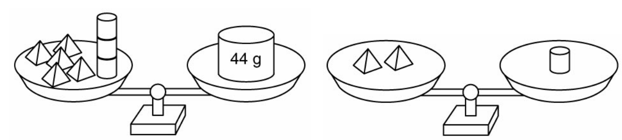
5. $7y = 42$

6. Let y represent the cost of one light bulb. Let x represent the cost of one T-shirt.

$$7y = 42$$
$$\frac{7y}{7} = \frac{42}{7}$$
$$y = 6$$

Substitute 6 for y in $x = 3y$.
$x = 3(6)$
$x = 18$
The cost of one T-shirt is \$18. The cost of one light bulb is \$6.

7.



8. For the first balance, $5p + 3c = 44$. For the second balance, $2p = c$. Substitute $2p$ for c in $5p + 3c = 44$ and solve for p.

$$5p + 3(2p) = 44$$
$$5p + 6p = 44$$
$$11p = 44$$
$$p = 4$$

Substitute 4 for p in $c = 2p$.
$c = 2(4)$
$c = 8$
The mass of one pyramid is 4 g, and the mass of one cylinder is 8 g.

9. Example: If the situation involves large numbers of objects, the diagram would be less effective than an algebraic method.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Ask students to determine the algebraic equation for each diagram. • Have students follow the steps in the Investigate for the Reflect and Respond problem. Suggest they show each step in diagram form and follow through progressively to the end. • Some students may find it easier to translate the diagrams into algebra if the shapes are labelled with *p* for pyramid and *c* for cylinder. • For #9, listen and watch for those students who suggest the visual is a better format. These students may not be ready to move into an algebraic form and should be given additional examples to complete.

Link the Ideas

In the example provided, y is isolated in the second equation because it has a coefficient of 1. Emphasize that students should use brackets when substituting the expression equivalent to y, $3x + 9$.

Example 1

For this example, you may wish to refer students to the Did You Know? Students may be interested to learn that the Abbotsford Airshow is one of the largest of its kind in the world.

Emphasize the proper modelling of the solution, including the definition of each variable. Direct students to the questions in the margin of the solution. Some students might benefit from further elaboration of the general questions:

- What does the 2 represent in the first equation?
- What does the value $3C$ represent in the first equation?

In Method 2, point out that the entire expression $2A$ has been replaced with 50. Ensure students understand that it is not always necessary to determine the value of $1A$ before substituting.

Emphasize the process of checking the solution, as shown in the student resource.

As students solve the linear system in the Your Turn section, observe how many students directly substitute 12 for $3x$ in the first equation. For students who solve for x, ask how they could solve this problem by substituting for $3x$ instead of x. Follow up by asking these students when it might be even more beneficial to substitute for $3x$ if the situation presents itself. For example, if $3x = 17$, a direct substitution of $3x$ means avoiding working with a fractional number.

Example 2

As you define variables and set up equations, emphasize to students what the terms $9a$ and $6c$ represent. Clarify the notation for numbering an equation in a linear system with a circled number. Explain that this type of notation becomes more important in larger linear (and non-linear) systems involving three or more variables.

Ask students which variable (in which equation) they would recommend for isolation. Discuss with the class how they can reach a solution more efficiently by selecting variables with coefficients of 1 or 2. Point out that Method 2 includes improper fractions because of the decision to isolate c.

For the linear system in the Your Turn section, observe whether any students multiply the second equation by 10 in order to eliminate the decimal coefficient of y. You may wish to identify this option at this point. Focus students' attention on this topic by asking how the problem is different from the linear system in the example.

Key Ideas

This section contains a summary of the process of solving a linear system using the substitution method. If you use the example provided, check the solution at the end of the question.

Meeting Student Needs

- Emphasize that the substitution method is usually most appropriate when the value for x or y can be determined easily to be substituted into the other equation of the system.
- Some students may not be familiar with what an air show is. You may wish to have students research the Abbotsford Airshow or other air shows in Canada to gain a better understanding. See the related Web Link that follows in this Teacher's Resource.
- You may wish to model Example 1 using manipulatives, similar to what was done in the investigation. You might create magnets for the white board, which could show students in a highly visual way how this method works. Cut out squares of coloured paper and attach a small piece of magnet to each. Label each of them with a variable or number; for example, label all red papers with A, all yellow papers with C, and all green papers with numbers. These manipulatives are useful for solving all types of equations and can be stored in small plastic containers.
- For Example 2, you may wish to demonstrate the solution on a graphing calculator. Be sure to discuss the importance of selecting the correct window. Students could also determine the intersection of the two lines in order to check their work. This allows students to compare two methods of solving a system.
- Ensure that students compare the two methods demonstrated in Example 2. Ask them to write down which method they would use and why.

- Post an example of substitution. Colour code the two equations throughout the solution. This will allow students to visualize that part of one equation is substituted into another equation during the process.
- Have students create a poster stating the steps for solving linear systems using substitution. They may wish to use the Key Ideas as a guide.

ELL

- Ensure that students add the term *substitution* to their vocabulary dictionaries.
- For Example 1, students may need assistance in understanding what *admission* is. Ensure they see that it is an amount of money. Explain, using diagrams, that in this example, each admission cost mentioned in the question applies to all of the people in the car.

Common Errors

- Some students may struggle with setting up equations similar to those in Example 2, which include unit rates/prices.

R_x Coach students through additional problems where the focus is only on setting up the two linear equations.

- Some students will not necessarily choose the easier variable to isolate. For example, the easier variable may appear on the right side of the equation and the student is more comfortable isolating on the left side.

R_x Provide students with additional systems where they are encouraged to isolate a variable on the right side of the equation.

WWW Web Link

For more information about the Abbotsford Airshow, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

$x = 4$ and $y = 3$

Example 2: Your Turn

$x = -6$ and $y = 25$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. Some students may question why there is a need to solve for A if the two equations both contain $2A$. Have students recognize that they can substitute 50 in immediately, as indicated in the blue sidebar. Prompt discussion by asking students to identify when it is appropriate to make this substitution and when solving for the variable is necessary. Note that the Your Turn provides an opportunity for students to first solve for the single variable and then substitute. • Remind students about the importance of verifying their solution.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may need to refresh their skills in simplifying equations that contain fractions or decimals. Model several examples. • As students work on the Your Turn, ask them to identify which variable they believe would be easier to solve for and why. • The skill of recognizing by inspection which variable is the easier to isolate will not come immediately to all students. Provide sufficient examples where one variable would clearly be a better choice and several where either variable could be used. Try to use variables other than x and y in example equations, so students become accustomed to identifying any variable.

Check Your Understanding

Practise

In the first three questions, there is at least one variable that has a coefficient of 1.

In #4, some students may need assistance with eliminating fractional coefficients. You may wish to have a student model solving this type of linear system for the class.

In #7, students discover one of the disadvantages of a graphical solution for a linear system: determining non-integer coordinates of the point of intersection.

Apply

In #5, students are shown two approaches to solving a linear system. Using Helen's method results in fractions that cannot be represented with terminating decimals. However, students should feel comfortable working with fractions as they solve linear systems.

For #13, encourage students to check their answer graphically as a review of Chapter 8.

For #16, some students may start part a) by using the value of a nickel and of a dime. Remind them that the question has to do with the number of coins rather than their values.

Extend

For #22, it may be necessary to remind students to work with one unit of time (hours or minutes).

In #23, students work with some physics formulas for simple circuits, which they may have seen in science class.

Encourage all students to try #24.

Create Connections

For #26, you may wish to guide students to #13 for a question that lends itself to a graphical solution.

Unit Project

The Unit 4 project question, #15, provides an opportunity for students to assign variables to a bush and a tree and set up a system of equations to solve for the cost of each. Students may wish to review either Example 1 or 2 and model their system using the same format of identifying variables and writing equations. Reinforce that students must solve the system. Ask them what it is that they are determining in this problem. Ask what units their answer will require.

Encourage students to sketch a diagram to help visualize what the problem is asking. For example, they may wish to sketch and label a scale.

Meeting Student Needs

- Provide **BLM 9–5 Section 9.1 Extra Practice** to students who would benefit from more practice.
- Algebra tiles or other manipulatives may be useful in helping students to work through #1 to 3. The movement of the manipulatives and actual physical substitution will solidify the concept for some students. Not all students need to use this method, but it may be good to demonstrate one question for the entire class.
- For verbal learners, #3 and 5 are useful questions. Ensure that these students have the opportunity to share their writing.
- Have all students complete #5 individually, as it is an effective question for Assessment *as* Learning. A discussion should follow this question, so that students may learn from errors before they complete the rest of the Check Your Understanding questions.
- For #10 and 12, you may wish to have students research data about snowfall and TV-watching habits in their own community. They might then create questions based on the information they find.
- Some students will benefit from completing the unit project question, #15.
- You may wish to organize students into groups and vary the assigned questions for each group. Have students produce an answer key once all questions have been answered.

ELL

- For #9, show students visuals of cable so they understand it is a long piece of material that can be cut into pieces.
- Since #15, 18, and 23 include challenging language, you may wish to have students work with a partner on these questions.
- For #16 and 19, you may wish to provide actual coins for students to handle if Canadian money is new to them.

Enrichment

- Present the following scenario: A fraction's value in lowest terms is $\frac{5}{6}$. Adding 11 to the numerator of the fraction creates a fraction whose value is $\frac{6}{5}$. Challenge students to use a system of linear equations to determine the fraction. $\left(\frac{25}{30}\right)$

Gifted

- Have students solve the following problem using a system of linear equations and substitution as the method: A fraction has a two-digit numerator and denominator whose place values are reversed (such as $\frac{13}{31}$). If the value of the fraction is $\frac{7}{4}$, determine at least two fractions that meet the criteria. (Examples: $\frac{21}{12}$, $\frac{42}{24}$, $\frac{63}{36}$, $\frac{84}{48}$)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 6–9, 12, 13, and 19. Students who have no problems with these questions can go on to the remaining questions.	• For the initial Practise questions, you may wish to have students work with a partner of similar ability. • For students who find #1 challenging, prompt and coach them through part a) of this question. Then, have them solve parts b) and c) on their own. • For #3 and 8, students may find it easier to eliminate the decimals. Coach students through #3 and have them complete #8 to check for understanding. This question provides prompts so students should be able to complete it alone. • Some students may benefit from drawing a model diagram, for example, for #6 and 12. Prompt them to select variables that are easy to remember for the problem. Have them translate the diagram into a system of equations. Ask students to verbalize what they are looking for and what the variables represent. • For #7, ask students to explain how they know the graph is a solution to the system. Challenge them to support their explanation with two different approaches. • For #9, students who have difficulty writing the linear system may benefit from first drawing a diagram and labelling the parts with variables. • Some students are confident using the graphing method but have difficulty writing a linear system. For a question such as #13, ask them which part of the question would represent the slope of the line and which part the *y*-intercept. Ask them to write the equations in slope-intercept form and then determine the system and use substitution to solve. Remind them that they can use graphing to verify the solution.
Unit 4 Project If students complete #15, which is related to the Unit 4 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 9–4 Chapter 9 Unit 4 Project** and have them finalize their answers. • Prompt students to select variables appropriate for the question. They may wish to draw a diagram first and then translate it into a system. • Students may find that the approaches they used in #6 and 12 may help them complete this question. • Remind students to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • For #25, you may wish to have students revisit #8 before responding, since this question also involves a graphical representation and solving using substitution. Ensure that students answer with both similarities and differences. Have students use this comparison as a springboard to explaining the advantage of an algebraic approach. • For #26, some students may be uncertain which question is appropriate to choose. Write a list of possible questions on the board or direct them to any part of #1. You might have students record their response to this question in their Foldable and use it as a study tool.

Solving Systems of Linear Equations by Elimination

9.2

Mathematics 10, pages 480–491

Suggested Timing

60–75 min

Blackline Masters

BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–6 Section 9.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcome

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 7–9, 13, 14, 20, 21
Typical	#1–3, 5, three of 7–12, 14–16, 20, 21
Extension/Enrichment	#2, 5, two of 10–13, 14–21

Unit Project Note that #14 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

If your school has an environmental club, you may wish to discuss ways in which the club raises money for their projects. Also, show students images or samples of organic coffee, reusable bags, and reusable food and drink containers.

Investigate Solving Systems of Equations by Elimination

As students work in pairs to answer the questions, circulate among them and ask questions.

To assist students with #1, ask how the shapes have changed on scale B from Diagram 1 to Diagram 2.

To assist students with #2, ask the following questions:
- In Diagram 2, how does dividing the 17-g mass into two 7-g masses and one 3-g mass make the two scale balances similar?
- What else is similar about the two scale balances in Diagram 2?
- What extra items are found on scale A compared to scale B in Diagram 2?

For students struggling with #3, check that they have correctly determined the mass of the block and then guide them back to Diagram 1.

Once students begin working on the algebraic section of the investigation, check that they translate the diagrams correctly and do not confuse the variables.

In the Reflect and Respond section, check that students construct the diagrams correctly to model the problem. With the class, go over #8 to 10. Ensure that students understand from the diagram why the substitution method would not work for this investigation (both variables appear on the same pan).

Meeting Student Needs

- Explain that a person's *ecological footprint* is a measure of that person's demand on Earth's ecosystems. It represents the amount of biologically productive land and sea area needed to regenerate the resources a person consumes.
- You may wish to further the discussion about ecological footprints. Ask students about the kinds of garbage they see when they are out in nature. Discuss whether the garbage has any effect on wildlife. Ask where the garbage might come from. Discuss whether switching to reusable shopping bags might be better for the environment and why.

- Some students may not be familiar with *hybrid vehicles*. Explain that a hybrid is a vehicle that runs off both an electric motor and a gas- or diesel-powered engine.
- When preparing students for this section, emphasize the notion of equivalent equations. Refer to doubling or tripling a recipe (all items in the recipe must be doubled or tripled, and so will the quantity), mixing gas and oil to a specific ratio (the amount of each will be determined by the total amount required), or other relevant examples. You can also demonstrate the concept of equivalent equations using algebra tiles and other manipulatives, including blocks.
- Work through the investigation by bringing reusable bags, packages of coffee, and play money to the classroom. This would be a helpful hands-on learning experience for some students.
- Depending on the needs of your class, you may wish to write a couple of equations on the board as examples for students prior to having them determine the equations for the Investigate.

ELL

- Show students samples of reusable bags, organic coffee packets, and reusable food and drink containers.
- Have students work with a partner on the Investigate.

Common Errors

- Some students may struggle with modelling #9 of the investigation.

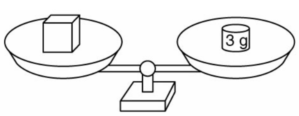
R_x Guide students through translating the words into the diagram. Some students may benefit from translating into the algebraic equation first.

Answers

Investigate Solving Systems of Equations by Elimination

1. The mass of one cone and one block is 7 g so the mass of two cones and two blocks is twice as much.

2. From scale A in diagram 2, take two cones and two blocks from the left side and the two 7-g masses from the right side.



3. The mass of one cone added to the mass of one block is 7 g. Since the mass of one block is 3 g, the mass of one cone must be 4 g.

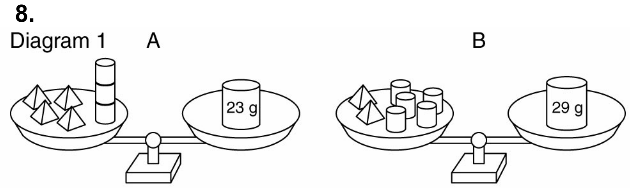
4. Let c represent the mass of one cone. Let b represent the mass of one block. The equations are $2c + 3b = 17$ and $c + b = 7$.

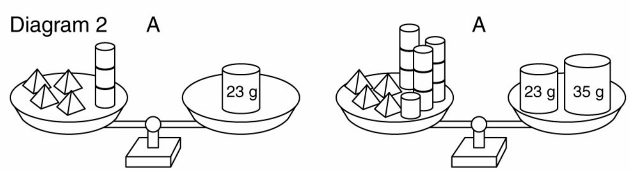
5. Multiplying both sides of the equation $c + b = 7$ by 2 gives $2c + 2b = 14$.

6.
$$\begin{array}{r} 2c + 3b = 17 \\ \underline{-\,(2c + 2b = 14)} \\ b = 3 \end{array}$$

7. Double the equation for scale B in diagram 1. Subtract this equation from the equation for scale A. The cost of a grocery bag is \$3, which equals the mass of a block, in grams. The cost of a packet of coffee is \$4, which equals the mass of a cone, in grams.

8.





9. The mass of one pyramid is 2 g, and the mass of one cone is 5 g.

10. Yes. Example: You could isolate p or c from either equation and substitute this expression into the other equation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner. • Encourage students to label the pyramids *p* and the cylinders *c* to assist them in writing the system of equations. • Have students verbalize what *opposites* are. You may need to go over opposites of integer values and the sum of a number and its opposite value. • Remind students of the importance of multiplying every term on both sides of the equation. A common mistake is to multiply only one side of the equation.

Link the Ideas

Discuss with students that it may be easier to multiply one of the equations by a negative integer in order to eliminate the variable by addition.

This may be an appropriate point to introduce a system with fractional coefficients and explain how it may be necessary to multiply the equation in order to create integral coefficients.

Example 1

Discuss with students that an acceptable answer must be a whole number for this type of question, since the variables represent the number of downloads of each type. Focus students on using appropriate variables for a song (S) and a game (G).

After setting up the equations, ask students the following questions:

- Which variable should you eliminate? Why?
- Could the other variable be eliminated instead? Explain.

After students have determined the variable to be eliminated, ask them what constant multiplier(s) should be used, if any. Discuss whether the multiplier(s) should be negative or positive and what the implication is for each choice.

Reinforce the final step of checking the solution by substitution into the original two equations.

For the Your Turn question, the numbers are larger but should not cause a problem for students.

Example 2

Using the Did You Know?, discuss the concept of carbon offsets and their connection to global warming.

Help students understand why a farmer may choose to convert some cropland into woodland by discussing the work of organizations such as the Pacific Carbon Trust.

The Pacific Carbon Trust expects to purchase between 700 000 and 1 000 000 tonnes of carbon-dioxide equivalent offsets each year. Offsets or carbon savings are generated from changes made to avoid or absorb carbon dioxide. Typically, carbon offsets fall into three categories:

- renewable energy such as hydro power generation using natural river flow, wind farms, or solar installations
- energy efficiency such as increasing industrial energy efficiency by improving pollution control equipment, switching from oil to natural gas, or performing energy retrofits
- emissions storages or sinks, such as forestation of lands not previously forested

Land owners can develop projects that reduce carbon emissions and sell these carbon offsets to individuals and industry. As people become more concerned about climate change, more businesses and individuals are purchasing carbon savings to offset the environmental cost of such things as major sporting events (e.g., World Cup Soccer), travel, conferences (e.g., United Nations World Climate Research Programme), industrial operations (e.g., Google and Nike), and even weddings.

This provides opportunities for land owners, such as the farmer in this question, to invest in carbon reduction projects. They then sell the carbon savings they have generated to others to offset activities that increase carbon emissions. For more information on carbon offsets, use the Web Link at the end of this section.

In this problem, students use a table to organize the information. On the completed table, the two equations in this linear system appear vertically in the last two columns.

The numbers are large in these equations. Careful selection of the variable that will be eliminated can help minimize the size of the coefficients in the equivalent equations.

When taking up the problem in the Your Turn section, discuss whether students multiplied to eliminate the decimal coefficients. Discuss the advantages and disadvantages of this practice.

Example 3

For this question, emphasize the importance of drawing a diagram and labelling it with the assigned variables and given information. Students will need to rearrange one of the equations to ensure that both equations are in the same form, $ax + by = c$. You may wish to discuss with students that this form is not mandatory. What is mandatory is that both equations be in the same form. In other words, the form could be $ax = by + c$.

Key Ideas

As you go through the Key Ideas, ask students the following questions:

- In the second bullet, what does *elimination* mean?

- In the third bullet, what would be an alternative way to arrange the two equations in the same form? (See the Example 3 notes above.)
- In the fourth bullet, what multipliers would you use if you wished to eliminate the variable y instead?
- In the last bullet, how would you check your solution? Explain.

Meeting Student Needs

- You may wish to encourage students to always add the two equations. Discuss the idea of opposites having a sum of zero. In Example 1, draw attention to equation ② being multiplied by –2 resulting in $-4s$, which is the opposite of $+4s$ in equation ①. This allows students to add the two equations. If you have students who repeatedly subtract incorrectly, the option of multiplying to produce opposites may be helpful.
- Ensure that students understand what *common multiples* are.
  Multiples of 2 are 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, …
  Multiples of 3 are 3, **6**, 9, **12**, 15, **18**, …
  The multiples that are in boldface are the common multiples. When using the elimination method, a common multiple is necessary.
- For Example 2, assist students in understanding what *revenue* is: the entire amount of income before deductions. *Gross* is another word often used to describe revenue.
- Examples 2 and 3 demonstrate two methods of creating the equations necessary to solve the problem. Encourage students to try both methods to allow for individual learning styles.
- In Example 3, work through the translation of the words into algebraic equations. For example, "2.46 m longer than five times the width" needs to translate to $5w + 2.46$.
- For Example 3, give students a chance to practise rearranging equations. To do this, students perform the same operation on both sides of the equation. For example, the following equation needs to be rewritten in the form $ax + by = c$:

$$\begin{aligned} 3x + 5 &= 2y \\ 3x + 5 - 2y &= 2y - 2y \\ 3x + 5 - 2y &= 0 \\ 3x + 5 - 2y - 5 &= 0 - 5 \\ 3x - 2y &= -5 \end{aligned}$$

- Have students create a poster stating the steps for solving linear systems using elimination. They may wish to refer to the Key Ideas.

ELL

- Ensure that students add the term *elimination* to their vocabulary dictionary.
- For the Example 1 Your Turn, have a volunteer explain basketball to students who may not be familiar with the sport. Ensure that pictures or Internet video footage is available to assist their understanding.
- In Example 2, students may not be familiar with a number of terms: *crop*, *converting*, *cropland*, *woodland*, *carbon sink*, *revenue*, etc. Use verbal descriptions, examples, and visuals to assist student understanding.
- For Example 3 and the Your Turn problem, encourage students to draw their own diagram to assist them as they work through the questions.

Web Link

For more information on carbon offsets, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

The price for an adult was \$14. The price for a student was \$9.

Example 2: Your Turn

Let m represent the number of muffins sold. Let y represent the number of yogurts sold. Solve the linear system $m + y = 160$ and $1.5m + 2y = 273.5$. 93 muffins and 67 yogurts

Example 3: Your Turn

The length is 3.4 m. The width is 2.7 m.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students verbalize what variables they will use. • Some students may find it easier to add integers than to subtract integers, especially where negative values are concerned. In this case, explain that the elimination method can also be referred to as the addition method since you add opposite terms to eliminate them. Review the notion of opposite integers and provide several examples. • The side notes beside the worked example ask important questions that should be addressed and discussed as a class before students go on to the Your Turn. Have students model their solution after the example. • Remind students to multiply both sides of an equation when multiplying by a constant value.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use a table or other graphic organizer to help organize their thinking. Draw attention to the table in Example 2, which enables you to generate the system from the last two columns. Some students may find it easier to fill in the total row first and then work backwards. Encourage them to use whichever approach works best for them. • Assist students in developing a table for the Your Turn question. • Ask them to verbalize the selection of variables and what the system would be, using inspection.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may benefit from a discussion of the meaning and implications of the words *double*, *triple*, *quadruple*, *half*, *one third*, etc. • Encourage the use of diagrams to solve the problem. • Ensure that students understand the importance of writing the equations in the same form. It does not need to necessarily be $ax + by = c$; it could be $by = ax + c$ or $by + ax = c$ or $ax = by + c$. Emphasize that what is important is that the forms match. • Remind students of the importance of verifying their solutions.

Check Your Understanding

Practise

In #1, none of the equations require rearranging to solve with elimination. Then, in #2, students need to rearrange in each problem.

In #3, the data appear in a table in order to remind students how data can be organized for a word problem.

In #4, both equations in each problem must be multiplied by a constant before elimination can occur.

For #5c), refer students to the work they did for #24 of section 9.1. Have them compare the two solutions and then explain their solution for #5c).

In #6, students will find that there is no solution. Students may wish to graph the two equations with their graphing calculator in order to better visualize the problem.

Apply

For #7, you may wish to assist students in setting up the second equation. Students should realize that $b + t = 30$, since each vehicle will have only one seat. For setting up the second equation, ask what $2b$ and $3t$ would represent.

For #8, refer students to the Did You Know? in the margin for some context to the Communities in Bloom project. You may wish them to learn more about this project. See the related Web Link that follows in the Teacher's Rescource.

For #12, students may benefit from creating a table to arrange their information. The table might include speed, time, and distance as headings.

Extend

In #15, students have an opportunity to practise previously learned algebraic skills of simplifying expressions within equations.

For #17, students complete a typical mixture-type problem. Students need to convert the percents to decimals before setting up the equations in the linear system.

Create Connections

At this point in the section, it might be beneficial to have students take a question that they have solved using graphing and substitution and now complete it using elimination. They could include this work in their Foldable.

Unit Project

The Unit 4 project question, #14, provides an opportunity for students to determine water usage and the number of loads of laundry of two women in a week based on their habits and the type of washing machine they use. Some students will find this easier to complete using a table. Have them review Example 2 and use it as a model. Other students may find it easier to interpret graphically, especially part b). Although the section focuses on the elimination method, encourage students to use whatever approach works best for them. Emphasize that using more than one method allows them to verify their solution.

Meeting Student Needs

- Provide **BLM 9–6 Section 9.2 Extra Practice** to students who would benefit from more practice.
- Have students work in pairs to demonstrate their understanding prior to doing individual work on the Check Your Understanding questions.
- Students may wish to check their work by entering the equations into a graphing calculator.
- For one part of #1 or another question of your choice, do the following activity: If you have a tiled floor in your school, use masking tape to outline the axes of a graph. Then, after writing the equations in $y = mx + b$ form, have one student start at the y-intercept and move the number of squares indicated by the slope. Have a different student create a straight line of tape that forms the line of equation 1. Follow the same procedure with two other students and equation 2. Students can then determine the point of intersection. The graph on the floor would serve as a reminder of the definition of the solution of a system of equations.
- For #4 and 5, students multiply both equations by a constant. You may wish to refer students back to Example 3 prior to assigning these questions.
- For #6, students need to understand that if there is no solution, the two lines are parallel. A system of parallel lines will never intersect. You may want students to solve this problem graphically once they have tried solving it algebraically. They will then be able to see why there is no solution.
- Before students begin the Apply questions, show a couple of examples of solving word problems. Students, particularly those with language difficulties, may not understand what is expected of them. You may wish to have students create a word problem to help them in their understanding.
- Some students will benefit from completing the unit project question, #14.

ELL

- For #7, show students pictures of a bicycle and a tricycle to ensure they understand the number of wheels that each one has.
- For #8, refer students back to the Investigate, which is a similar question.
- For #11, explain to students that an example of a passenger vehicle is a car and that a ferry is a type of boat that carries cars and trucks.
- Since #12–14 and 16 include challenging language, you may wish to have students work with a partner to complete them.

Enrichment

- Ask students to examine the following system and predict whether it has one solution, no solution, or an infinite number of solutions. Have them check their prediction using the elimination method.

$$0.1x + \frac{2}{5}y = 1000$$

$$\frac{1}{10}x - 0.4y = 1000$$

(An infinite number of solutions. Since they are equivalent equations, elimination eliminates everything, showing that any value for x or y solves both equations.)

Web Link

For more information about Communities in Bloom, go to www.mhrmath10.ca and follow the links.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Practise and Apply
Have students do #1–4, 7–10, 13, 20, and 21. Students who have no problems with these questions can go on to the remaining questions.</td><td>For #1, students complete an entry level set of questions in which the variables do not require multiplication by a constant. Have students verbalize which variable would be the easier one to eliminate and whether they would add or subtract.
For #2, ensure that students understand the importance of writing the equations in the same form when they begin a question. Clearing up any misunderstandings at this point will assist them in writing their systems for the Apply questions.
In #3, students have the opportunity to use a table to set up and then solve a system. Many students find a table helpful in organizing their information. For students who are overwhelmed by the language in a problem, suggest that they use a table.</td></tr>
<tr><td>Unit 4 Project
If students complete #14, which is related to the Unit 4 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.</td><td>You may wish to provide students with BLM 9–4 Chapter 9 Unit 4 Project and have them finalize their answers.
Students may wish to refer to Example 1 to assist them in setting up the equations. Alternatively, since it may help students to use a table, they might review Example 2.
Have students identify variables and what the variables will represent in the answer.
Remind students that they can use a variety of approaches. Some students may find answering parts b) and c) easier from a graphical representation. Ask students how they would have to change their equations to represent them as graphs.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #20 and 21.</td><td>For #20, students use the substitution method on questions they solved using the elimination method. Good choices to direct them to are #1 or 2. In part c), if students have difficulty identifying what they are looking for, suggest that they refer to #4 and 5 before answering.
As an assessment question, #21 is useful for identifying whether students have a good grasp of the methods in Chapter 9. You may wish to collect their responses to get an idea of their understanding at this point and to identify methods that students are not comfortable with. You might have students include their responses in their Foldable for future reference. Students need a clear understanding of each method before moving on to section 9.3.</td></tr>
</table>

9.3 Solving Problems Using Systems of Linear Equations

Mathematics 10, pages 492–501

Suggested Timing

60–75 min

Materials

- graphing calculator or spreadsheet software

Blackline Masters

BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–7 Section 9.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
Visualization (V)

Specific Outcome

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 8, 9, 14
Typical	#1–7, 9, 11, 14, 15
Extension/Enrichment	#2, 9–13, 15

Unit Project Note that #9 is a Unit 4 project question

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Solving a Problem Involving a System of Linear Equations

Read through the opening paragraph with students. Ask students questions to activate their thinking:

- What is your ecological footprint?
- Do you know anyone who owns a hybrid vehicle? What were their reasons for purchasing a hybrid?

In the opening paragraph of the problem, there is a reference to the operating cost per kilometre. As a class, discuss the components of this figure: insurance costs, maintenance costs, fuel costs, and depreciation costs.

Have students work through the Investigate in pairs. Students may find it challenging to solve this question graphically due to the large values and similar slopes. It may benefit students to try solving the linear system algebraically first. However, seeing the visual relationship between the two graphs is important for students to understand the cost savings of the hybrid over the long term.

For #2, you may wish to ask these guiding questions:

- What values are you using for your window in the graphing calculator?
- What is the cost to operate each vehicle for 100 km, 1000 km, and 10 000 km?
- What is the difference in costs over 10 000 km?

At the end of the Investigate, discuss with the class which method was quicker and which method was more helpful for conceptually understanding the problem.

Meeting Student Needs

- Have students refer to the posters in the classroom illustrating the methods of solving. Ask students to predict when they would use each method.
- Have one or two students make a brief presentation about hybrid cars. Encourage them to include pros and cons as part of the presentation. You may wish to choose students who have strong presentation skills but are weaker in math.

- Depending on your class, you may wish to show examples of word problems before beginning this section. Students, particularly those with language difficulties, may find this section challenging. Consider having students work in pairs or small groups.
- Once students have completed the investigation, allow time for them to compare their summaries. Some students will benefit from verbally presenting their knowledge.

ELL

- You may wish to explain to students what hybrid cars are by comparing them to other cars.

Common Errors

- Some students may struggle with an appropriate window to see the two linear graphs on the calculator.

R_x Ensure that students have the correct equations entered. Then, help them to understand that appropriate sections of the graphs lie in quadrant I. Also, help them establish appropriate maximum values for the horizontal and vertical axes.

Answers

Investigate Solving a Problem Involving a System of Linear Equations

1. Let C represent the total cost, in dollars, of operating a car.
Let k represent the number of kilometres driven.
Hybrid: $C = 28\ 000 + 0.18k$.
Gas powered: $C = 21\ 500 + 0.22k$

2. 162 500 km

3. a) 162 500 km

b) Example: I used the substitution method because the solution occurs when the costs are equal. So, one equation can be substituted easily into the other.

4. Example: To solve a system graphically, you must draw two graphs, either manually on a grid or on a graphing calculator, and determine the point of intersection. The coordinates of this point give the solution to the system. When solving by substitution, you must isolate one of the variables from one of the equations, substitute this expression into the other equation, and solve. This solution is then substituted into the other equation to solve for the second variable. When using elimination, the two equations are manipulated so that when they are added together one of the variables will be eliminated. Once you solve for the remaining variable, you then substitute that value into either equation to determine the value of the second variable.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner. • Some students may need prompting in writing the system for #1. Ask students how they will need to rewrite their equations to graph them. • As a class, discuss student responses to #3. The discussion will assist students in answering #4. You may want to write the student responses to #3 on the board in point form for students to refer to. • For #4, you may want to have students refer back to #21 in section 9.2, as they have some similarities. Use students' response to #4 to determine whether they have clarified their thinking.

Link the Ideas

With the class, discuss what graphic organizer(s) students could use to display the advantages and disadvantages of the graphical method and the algebraic method. Have students work individually or in groups, but develop and complete individual organizers. You may wish to discuss with students how making their own organizers aids their concept development.

Example 1

You may wish to assist students in determining the two initial equations in the linear system by prompting them with the following questions:

- Which unit of time will you use?
- How many trees will Jeremy plant in 1.5 h before Shilan starts?
- If time requires one variable, what will require the other variable for this system?

Note that once the equations are written, a graphical solution will likely provide students with a quicker insight into the problem. Encourage students to discuss with classmates their boundaries for the window on their graphing calculator.

Example 2

You may wish to discuss with your class what bannock is.

Solve the system algebraically using one of the two methods. Then, have students use the alternative algebraic method to solve the same problem. Discuss with the class which method was more efficient.

Key Ideas

The first bullet is the most important. Emphasize that students can use any of these methods to solve a linear system. However, there are problems that lend themselves to one particular method. Go over the other points in the Key Ideas, and then discuss examples of linear systems that lend themselves better to each method.

Meeting Student Needs

- For Example 2, have students research the Folklorama Festival in Winnipeg, Folkfest in Saskatoon, or other multicultural festivals in northern and western Canada. See the related Web Link that follows in this Teacher's Resource. You may wish to have students determine which cultural groups that are of interest to them participate in these festivals and what their pavilions offer to visitors.
- Organize students into groups. Have each group work through the examples provided in this section, discuss the various methods illustrated, and create a graphic organizer for use during the Check Your Understanding and Chapter 9 Review. Enlarge one copy of the graphic organizer and post it in the classroom for reference. Alternatively, students can use their chapter Foldable.

ELL

- For Example 1, you may need to clarify what a *fundraiser* and a *seedling* are. Also, students may not be familiar with the word *plant* as a verb. Use pictures and descriptions to explain these terms.
- For Example 2, show students pictures of bannock and buffalo stew so they understand that these are types of food.

WWW Web Link

For more information about the Folklorama Festival in Winnipeg and Folkfest in Saskatoon, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

$x = 160$ and $y = –160$. Check that students' graphs show the correct equations and intersection point (160, –160). Example: I prefer the algebraic method (substitution) because the second equation needs to be rearranged before it can be graphed.

Example 2: Your Turn

$x = 9$ and $y = 2.5$. Example: The substitution method is more complicated because there are several steps required to isolate either variable, and the result involves a complicated decimal expression. The elimination method is much simpler.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may be unable to determine where the values of 90 and 1.5 came from. Prompt students with these questions: – How much time passes between when Jeremy starts and when Shilan starts? (90 min) – How many trees can Jeremy plant in 1 min? (1) – What does 90 represent for graphing an equation? (y-intercept) – How many trees can Shilan plant in 1 min? (3 trees in 2 min; therefore, 1.5 trees in 1 min) – What does 1.5 represent for graphing an equation? (slope) • Remind students of the importance of putting the equations in the same form before solving algebraically. You may wish to ask whether the form they use to graph could also be used to solve algebraically. Pointing out this possibility may save students some work when writing the terms in the same form.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may use addition and subtraction as two different methods of solving the system. You may wish to indicate that you want them to use substitution and elimination. • An alternative approach to the Your Turn is to ask students to solve the system using two different methods and leave it open. This allows the student who prefers a visual representation to use graphing as an approach. • In the example, the rationale given for solving for variable *B* and not *S* is that it leads to fractional numbers. However, ensure students understand that it is acceptable to have fractional numbers. Some students believe that if they end up with a fractional expression, they have made an error. It would be helpful to complete an example so students see that they can arrive at the answer through multiple approaches.

Check Your Understanding

Practise

For some parts of #1 and 2, one algebraic method will result in fewer steps than the alternative algebraic method.

Apply

In #3, using an algebraic method is the most straightforward approach.

In #4, students must convert percents, which results in coefficients that are decimal numbers.

In #9, students write one linear equation for the number of trout in the lake as a function of time and a second equation for the number of trout eaten as a function of time.

For #10, you may wish to note that the relationship between depth and time is only linear between 60 m and 90 m. Outside of this range, the relation is no longer linear.

Extend

In #12, students need to determine the intersection points of each pair of lines. These three intersection points will correspond to the vertices of the triangle bounded by the three lines.

In #13, the solutions are very different, even though the numbers are quite similar in the two linear systems. Students will get some insight into this phenomenon when they graph the systems in part c).

Create Connections

In #15, to assist students in creating a linear system with a solution that cannot be expressed exactly on the graphing calculator, encourage them to start with a solution that has a non-terminating decimal representation.

Unit Project

The Unit 4 project question, #9, presents students with a contextual question to be solved graphically. You might encourage students to also solve it algebraically using a method of their choice. Some students may have already demonstrated a preference for one method over another. Encourage them to use the method that would be the most appropriate or that they are most comfortable with.

If students find this question challenging, suggest that they draw the graph first. Students can plot the points in the table and proceed from there. They may wish to refer back to Chapter 8, their Foldable for Chapter 8, or section 9.1. Remind them that the slope-intercept form can also be used as a system of equations.

For the algebraic approach, remind them to identify the meaning of their variables and what they are trying to determine, and to write the equations in the same form.

Meeting Student Needs

- Provide **BLM 9–7 Section 9.3 Extra Practice** to students who would benefit from more practice.
- For #1, ask each student to choose one of either part a), b), or c) and write a problem containing the information presented in the two equations given. Have them exchange the problem with another student to check that the material is accurate. Save these problems to use as examples or assignments in future classes.
- For #3 and 4, you may wish to have students research mean temperatures and transportation habits in their own community. They might then create questions based on the information they find.
- As an alternative to #5, if a class in the school is running a fundraiser based on sales, gather the data from that class on the cost and selling price of each item being sold. Then, present a problem to the class using the information gathered.
- For #7, have students research the history of Cirque du Soleil. They may be interested to know that the organization has its roots in Québec. See the related Web Link that follows in this Teacher's Resource.
- Allow students to choose three questions from #3 to 8. Tell students to solve each question using a different method—one graphically, one algebraically using substitution, and one algebraically using elimination. Ensure that students can justify the method of choice for each question.
- Some students will benefit from completing the unit project question, #9.

ELL

- Since #6 and 9 contain some difficult language, you may wish to have students work with a partner on these questions.
- In #8, some students may not be familiar with car rental companies. Use descriptions and pictures to assist their understanding.
- For #11, you may wish to have a student volunteer explain cross-country skiing and squash to students who are not familiar with these sports. Make available pictures and Internet video footage to help them visualize.

Enrichment

- Have students solve the following problem, then create several of their own, including complete solutions: A school store ordered 13 packages of paper and 4 packages of pencils, which came to $48.70. A second order of 6 packages of paper and 2 packages of pencils arrived, which cost $23.20. Determine the cost of one package of paper and one package of pencils. ($2.50 for paper and $4.05 for pencils)

Gifted

- Challenge students to create and solve a problem that meets the following criteria:
  - It involves two sizes of drink bottles.
  - The two sentences that follow must be filled in and included in the problem.
    _______ small bottle(s) and one large bottle can hold 4 L of water.
    One large bottle subtract ______ small bottle(s) results in 1 L of water.
  - The question must be "How many litres of water does each bottle hold?"

  (Example: Two small bottles and one large bottle can hold 4 L of water. One large bottle subtract one small bottle results in 1 L of water. How many litres of water does each bottle hold? Answer: The small bottle holds 1 L of water and the large bottle holds 2 L of water.)

WWW Web Link

For more information about Cirque du Soleil, go to www.mhrmath10.ca and follow the links.

For graphing calculator activities related to systems of equations, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, and 8. Students who have no problems with these questions can go on to the remaining questions.	• For #1, remind students of the importance of writing the equations in the same form. Emphasize that they are checking their equations graphically. You may wish to go through all three and have students suggest which algebraic method might be the most appropriate to use for each question and explain why. • For #2, you may wish to review how to multiply out the decimal and fractional values before solving. Again, have students verbalize which method they feel would work best for solving each system. • For #3, ask students what they are looking for. Have them identify what the variables will represent. Ask which variable will have a larger number. Coach them through the first equation. These prompts should also assist them in explaining the variables in #5. • For #8, some students may find the question easier to approach if they think of it graphically. Ask them to identify the slope of each line. Then, ask them to identify the fixed number, or y-intercept, for each line. Have them verbalize what their window setting will be and compare with a partner before solving.
Unit 4 Project If students complete #9, which is related to the Unit 4 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 9–4 Chapter 9 Unit 4 Project** and have them finalize their answers. • Some students will find it easier to solve the project question by graphing first. Prompt students to consider what they have learned about graphing that would help them. You may wish to use the following prompts: – Is the trout population in the lake increasing or decreasing? – Plot the points for year and number of trout in the lake. Is the slope of this line positive or negative? – How can you determine the slope? – What is the slope? – What is the y-intercept, or starting value, of trout in the lake? – How can this information be used in an equation of the form $y = mx + b$? Have students follow the same process for the second equation. Then, on the same graph, have them graph the points and line for the number of trout eaten by osprey. • Students should now be able to answer parts a) and b). They can use their graph to answer what the point of intersection means.
Assessment *as* Learning	
Create Connections Have all students complete #14 and most students complete #15.	• For #14, guide students not to write a system of equations that has coefficients that are all 1. Suggest that only one coefficient can have that value. This will allow you and students themselves to better assess their understanding of the methods for solving systems and of the steps involved. • You may wish to brainstorm with the class rational numbers that cannot be expressed exactly on a graphing calculator (e.g., $\frac{1}{3}$, $\frac{1}{7}$, $\frac{2}{9}$).

9 Chapter 9 Review

Mathematics 10, **pages 502–503**

Suggested Timing

60–75 min

Materials

- graphing calculator

Blackline Masters

BLM 9–5 Section 9.1 Extra Practice
BLM 9–6 Section 9.2 Extra Practice
BLM 9–7 Section 9.3 Extra Practice

Planning Notes

Consider having students work in pairs to solve this set of problems.

Some students may rely more heavily on one algebraic method than the alternative method. If you notice a student solving problems with a method that is clearly less appropriate, you may wish to discuss why the alternative method would be more efficient and/or you may wish to re-teach the alternative method.

If students encounter difficulties, provide an opportunity for them to discuss strategies with you or other students. Encourage them to refer to their chapter Foldable, the worked examples, and previously completed questions in the related sections of the student resource.

Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the practice test.

Meeting Student Needs

- Students who require more practice on a particular topic may refer to **BLM 9–5 Section 9.1 Extra Practice**, **BLM 9–6 Section 9.2 Extra Practice**, and **BLM 9–7 Section 9.3 Extra Practice**.
- Ask students to complete an exit slip explaining the content of the chapter. Have them define a system of linear equations and then list an example of each method for solving that was demonstrated in this chapter.
- Use a station approach for reviewing the section. Choose ten questions and post them on large pieces of paper around the classroom. Students move from question to question, in any order. Since students encounter different groups of students as they work on each question, they have an opportunity to exchange ideas with a variety of classmates. Assist at the stations when required.

ELL

- For #8, ensure students understand what carbohydrates are and that they are something that is found in foods such as fruit. You may need to show pictures of grapes and oranges if students are unfamiliar with these fruits.
- Both #9 and 11 refer to campgrounds and their facilities. You may want to show students pictures and diagrams to assist them in their understanding. You might have students work with a partner on these questions.

Enrichment

- Encourage students to create a problem that involves the use of two coins of different denominations, the ratio comparing the number of each type of coin, and the value of the coins added up. The problem must be solved using a system of linear equations. (Example: Sam has five times as many nickels as dimes. If he has $0.70 in coins, how many of each coin does he have? Answer: two dimes and ten nickels)

Gifted

- Challenge students with the following situation, and then ask them to create their own similar question: A swimmer covers 400 m in 4 min with the current and takes 5 min to cover the same distance against the current. Determine the speed of the swimmer and the speed of the current. (The speed of the swimmer is 90 m/min and the speed of the current is 10 m/min.)
- Some students may already be familiar with the skills handled in this review. To provide enrichment and extra challenge for gifted students, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Chapter 9 Review The Chapter 9 Review is an opportunity for students to assess themselves by completing selected questions in each section and checking their answers against answers in the back of the student resource.	• Have students check the contents of the What I Need to Work On section of their Foldable and do at least one question related to each listed item. • Have students revisit any section that they are having difficulty with prior to working on the chapter test. • You may wish to assign #1–3, 5, 6, 9, and 13 as minimum questions to be covered.

9 Chapter 9 Practice Test

***Mathematics 10*, pages 504–505**

Suggested Timing

50–60 min

Materials

- graphing calculator

Blackline Masters

BLM 9–8 Chapter 9 Test

Planning Notes

Have students start the practice test by writing the question numbers in their notebook. Have them indicate which questions they need no help with, a little help with, and a lot of help with. Have students first complete the questions they know they can do, followed by the questions they know something about. Finally, suggest to students that they do their best on the remaining questions.

This practice test can be assigned as an in-class or take-home assignment. Provide students with the number of questions they can comfortably do in one class. These are the minimum questions that will meet the related curriculum outcomes: #1–6, 8, and 9.

Study Guide

Question(s)	Section(s)	Refer to	The student can …
#1, 5, 7, 8	9.1	Examples 1, 2	✓ solve systems of linear equations algebraically using substitution
#2–4, 6	9.1	Example 1	✓ solve systems of linear equations algebraically using substitution
#5, 9	9.2	Example 1	✓ write equivalent equations to eliminate a variable ✓ solve systems of linear equations algebraically using elimination
#5	9.2	Example 3	✓ write equivalent equations to eliminate a variable ✓ solve systems of linear equations algebraically using elimination
#9, 10	9.1	Example 2	✓ solve systems of linear equations algebraically using substitution
#11, 12	9.3	Example 1	✓ choose a strategy to solve a problem that involves a system of linear equations

Assessment	Supporting Learning
Assessment *as* Learning	
Chapter 9 Self-Assessment Have students review their earlier responses in the What I Need to Work On section of their chapter Foldable.	• Have students use their responses on the practice test and work they have completed earlier in the chapter to identify areas in which they may need to reinforce their understanding of skills or concepts. Before the chapter test, coach them in the areas in which they are having difficulties.
Assessment *of* Learning	
Chapter 9 Test After students complete the practice test, you may wish to use **BLM 9–8 Chapter 9 Test** as a summative assessment.	• Consider allowing students to use their chapter Foldable.

UNIT 4

Unit 4 Project

Mathematics 10, page 506

Suggested Timing

90–120 min

Blackline Masters

Master 1 Project Rubric
BLM U4–3 Unit 4 Project Final Report

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

Specific Outcomes

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Planning Notes

Begin by brainstorming ways in which individuals can reduce water use in their homes. Encourage students to share what they know about retrofitting homes to reduce water use. Then, ask students how some of this information might be shown using what they learned in Chapters 8 and 9 about solving systems of linear equations graphically and algebraically.

You may wish to list and post students' thoughts from the brainstorming session. Encourage students to use the ideas as a springboard to develop their project. Explain that they will complete an analysis of reducing water use in homes and retrofitting homes with water-efficient plumbing fixtures. Review the expectations for the project outlined on page 506 of the student resource. Clarify with the class that the analysis should include

- data involving the effect of our water use on populations of wildlife
- information about costs and flow rates of various low-flow plumbing fixtures
- a comparison using linear systems (represented multiple ways) of the cost of keeping conventional fixtures and the cost of retrofitting

Encourage students to think about what format they might use to present their work. Students may wish to create a written report, oral presentation, PowerPoint presentation, or video presentation, or they may prefer another format of their choice. Explain that the presentation should outline the environmental and economic benefits of retrofitting and reducing water use. The student resource lists the questions that each presentation should address. Read and discuss this list of questions so that all students are clear on the assignment.

If students completed #8 in section 6.1 of Chapter 6, you may wish to refer them to this question since it may be useful to them as they work on their project.

Suggest that students review the contents of their project portfolio and ensure that they have completed all of the required components for their final analysis and presentation.

Assessment	Supporting Learning
Assessment *of* Learning	
Unit 4 Project The unit project gives students an opportunity to apply and demonstrate the concepts, skills, and processes learned in Unit 4. **Master 1 Project Rubric** provides a holistic descriptor that will assist you in assessing student work on the Unit 4 project.	• You may wish to have students use **BLM U4–3 Unit 4 Project Final Report**, which provides a checklist for students to identify where in their project they demonstrate the concepts, skills, and processes explored in Unit 4. • Reviewing **Master 1 Project Rubric** with students will help clarify the expectations and the scoring. It is recommended to review the scoring rubric at the beginning of the unit, as well as intermittently throughout the unit to remind students about the project assessment.

The Specific Level Notes below provide suggestions for using **Master 1 Project Rubric** to assess student work on the Unit 4 project.

Score/Level	Specific Level Notes
5 (Standard of Excellence)	• provides a complete and correct response with clear and concise communication; may include a minor error that does not affect the understanding of the overall project; may include weak communication in no more than one calculation or part
4 (Above Acceptable)	Demonstrates *one* of the following: • provides a complete response to all parts of the project, demonstrating a thorough understanding of concepts; may have weak or missing justification in no more than two parts; includes good communication; demonstrates a clear understanding of the meaning of the solution to answer contextual problems and is able to use solutions in making meaningful predictions • provides a complete response with one error that is carried through correctly (i.e., an incorrect system is written for one context, solved, and interpreted correctly for that context); includes good communication that addresses which strategy for solving a system may be the best in a given context • provides a response that addresses all parts of the project but is difficult to follow and lacks organization; does not provide support for the costs associated with retrofitting; includes good communication
3 (Meets Acceptable)	Demonstrates *one* of the following: • makes correct initial start to all sections of the project • correctly completes questions in the project; solves consistently, using two methods; demonstrates a basic understanding of the meaning of the intersection point or the solution to a system; communication is generally correct but may include some errors or omissions; can consistently write the system of equations from a context; includes good communication with some connections • provides answers to all parts without supporting work or justification; weak or missing presentations links
2 (Below Acceptable)	• makes initial starts to various sections of the project; provides some correct links • draws a distance-versus-time graph, with or without technology, and solves the system graphically, but has difficulty interpreting the solution • is able to describe changes in a population from a table of values • demonstrates the ability to isolate a variable and use this in solving a system but has difficulty with interpreting the solution • demonstrates some success in writing equations from contexts • demonstrates a method for solving systems of equations with relative consistency, can correctly identify the solution but has difficulty with interpretations or explanations • includes some communication
1 (Beginning)	• makes initial starts to various sections of the project but is unable to carry through or link concepts together • draws a distance-versus-time graph given the equations, with or without technology, using appropriate scales • is able to describe the changes in a population from a table of values; communication is weak • demonstrates the ability to isolate a variable but has no success in solving the system • demonstrates limited to no success in writing equations from a context • attempts to use various methods to solve systems but is unable to demonstrate one method with any consistent success • includes little or no communication

UNIT
4

Review and Test

***Mathematics 10*, pages 507–511**

Suggested Timing

60–90 min

Materials

- graphing calculator
- ruler
- grid paper

Planning Notes

Have students work independently to complete the review and then compare their solutions with a classmate's. Assign the Unit 4 Review to reinforce the concepts, skills, and processes learned so far. If students encounter difficulties, provide an opportunity for them to discuss strategies with other students. Encourage them to refer to their notes in their chapter Foldables, and then to the specific sections in the student resource and/or their notebook. Once they have determined a suitable strategy, have them add it to the appropriate chapter Foldable.

Consider having students make a list of the questions that they found difficult. They can then use the list to help them prepare for the unit test.

Meeting Student Needs

- Encourage students to draw and label diagrams, when appropriate.
- Encourage students to use their chapter Foldables and to add new notes if they wish.
- For #14 of the Unit 4 Review, you may wish to have students research the Thelon River and the canoe expeditions offered by tour companies. Have students create their own question using data that they discovered. See the related Web Link that follows in this Teacher's Resource.
- For #1 of the Unit 4 Test, suggest that students research Batoche, SK, and its importance to the history of Métis peoples. See the related Web Link that follows.
- For #6 of the test, students may be interested to learn that the artist, Mary Kuutsiq Mariq, grew up living off the land in the traditional nomadic Inuit lifestyle. The wall hanging is made of hand-sewn wool, felt, and embroidery floss tapestry, and shows Inuit gesturing at birds and fish, which they plan to hunt. As soon as the weather permits in the summer, many Inuit travel out onto the land where they camp and eat traditional food that they catch and prepare themselves. They live in tents made of hide and eat raw fish and meat, preparing tea made from water boiled in kettles set over rock fires. Students may wish to research other works by this artist. See the related Web Link that follows.

ELL

- You may choose not to assign #1 of the Unit 4 Review to new English language learners, since it is a language-based question.
- For #10 of the review, you may need to explain the game of lacrosse and what an assist is.
- For #14 of the review, explain what it means to go with and against the current.

Enrichment

- Have students write questions that are based on the ones in the review and test, that are based on the questions in the chapters, or that are completely original. Students can then exchange questions and answer them for further practice.

WWW Web Link

For more information about the Thelon River, go to www.mhrmath10.ca and follow the links.

For more information about Batoche, SK, go to www.mhrmath10.ca and follow the links.

To see more wall hangings by Mary Kuutsiq Mariq, go to www.mhrmath10.ca and follow the links.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Unit 4 Review
The cumulative review provides an opportunity for students to assess themselves by completing selected questions pertaining to each chapter and checking their answers against the answers in the back of the student resource.</td><td>• Have students review their notes from each Foldable and the tests from each chapter to identify items that they had problems with, and do the questions related to those items. Have students do at least one question that tests skills from each chapter.
• Have students revisit any chapter section they are having difficulty with.</td></tr>
<tr><td colspan="2">Assessment of Learning</td></tr>
<tr><td>Unit 4 Test
After students complete the unit review, you may wish to use the unit test on pages 510 and 511 as a summative assessment.</td><td>• Consider allowing students to use their chapter Foldables.</td></tr>
</table>